

**LOCATION: SWANSEA**

**LLEOLIAD: ABERTAWE**

**LEVEL 4**

**LEFEL 4**

**EXAMINATION PERIOD**

**CYFNOD ARHOLIADAU**

**MAY 2026**

**MAI 2026**

**HND ELECTRICAL AND ELECTRONIC ENGINEERING**

**MODULE CODE: GOEE4002**

**Dynamics Control and Instrumentation**

**TIME: 2 HOURS**

**ANSWER ALL FIVE QUESTIONS**

Equipment or materials required – Calculators and a formulae booklet are allowed.

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### Question One (20 marks)

An automated control system is being designed to maintain the water level in a storage tank. The system includes a water pump as the actuator, a water sensor for feedback, and a controller that adjusts the pump based on the water level measurements.

- a. The water level is initially designed as an open-loop system where the pump runs for a fixed time to fill the tank, without considering the actual water level in the tank. Using a simple block diagram, illustrate this open-loop system.

An open-loop control system has no feedback-- it operates on pre-set commands regardless of actual water level.

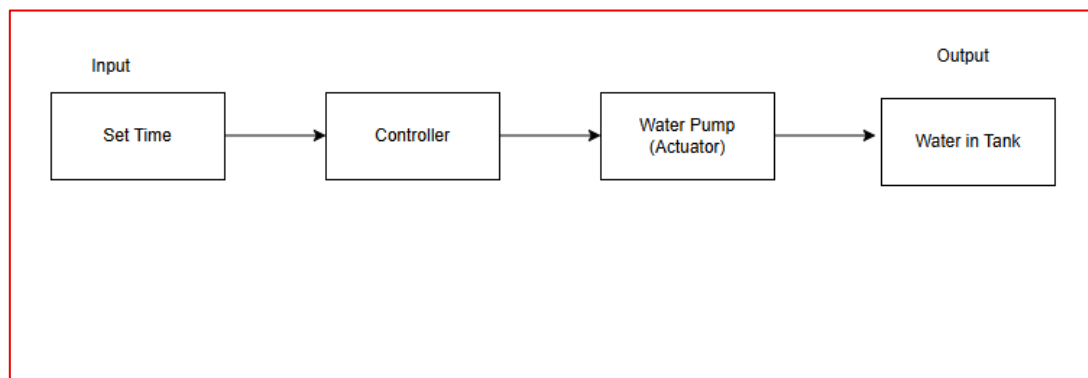
Set Time: Pre-determined filling time.

Controller: Sends signal based on time.

Water Pump: Fills the tank.

Water in Tank: Output water level (not measured).

[2 marks for correctly explaining the key components of an open-loop system]



[2 marks for placing the key components in the diagram]

[4 marks]

- b. If an external disturbance such as unexpected water usage causes the water level to drop during filling, how will the open-loop system respond?

An open-loop system cannot detect or compensate for disturbances like unexpected water usage. So, if water is used while filling, the final water level will be lower than intended.

[1 mark]

- c. To improve performance, a closed-loop system is implemented, where the controller adjusts the pump operation based on real-time water level data from the sensor. Using

a simple block diagram, illustrate the feedback control loop of this closed-loop system, identifying all key components.

Water level in a storage tank. In a closed-loop control system, the controller adjusts the actuator based on real-time feedback from a sensor:

Setpoint ( $L_{set}$ ): Desired water level.

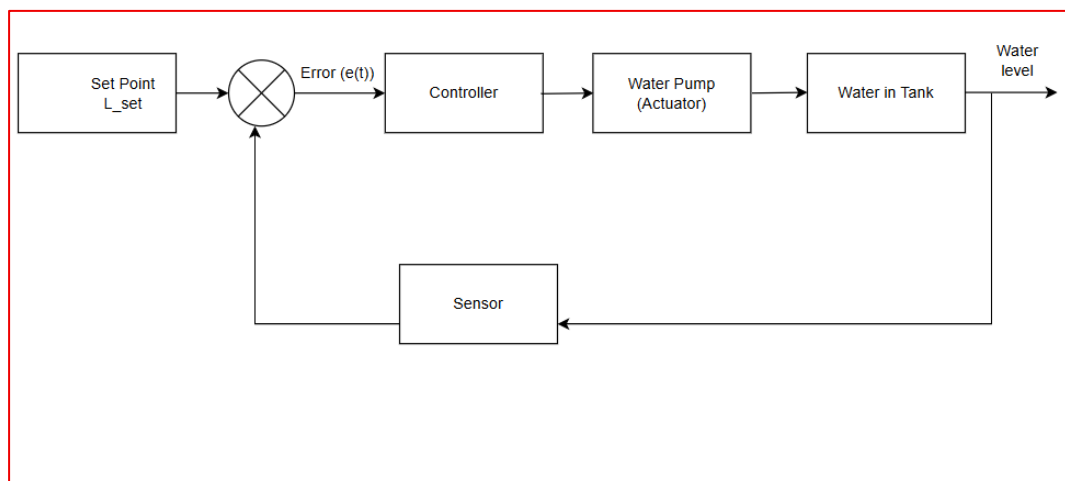
Controller: Compares  $L_{set}$  to measured level, adjusts pump accordingly.

Pump (Actuator): Fills the tank.

Water in Tank: Actual water level (output).

Sensor: Measures water level and sends feedback.

[3 marks for correctly explaining the key components of a closed-loop system]



[3 marks for placing the key components in the diagram]

[6 marks]

- d. The closed-loop system uses a proportional controller with gain,  $K_p$ . The sensor provides unity feedback, and the combined water pump and tank dynamics are modelled by the transfer function:

$$G(s) = \frac{5}{(s + 3)}$$

If the desired water level is changed via a step input of  $L_{set} = 100\text{cm}$ , answer the following:

- i. Derive the closed-loop transfer function.

Given:

Plant transfer function  $G(s)$

$$G(s) = \frac{5}{(s + 3)}$$

Controller: Proportional, gain  $K_p$ .

Unity feedback

Closed-loop transfer function:

$$T(s) = \frac{K_p G(s)}{1 + K_p G(s) H(s)}$$

[1 mark]

Since  $H(s) = 1$  (unity feedback):

$$T(s) = \frac{K_p \cdot \frac{5}{(s + 3)}}{1 + K_p \cdot \frac{5}{(s + 3)}}$$

[1 mark]

$$= \frac{\frac{5K_p}{(s + 3)}}{1 + \frac{5K_p}{(s + 3)}}$$

[1 mark]

Multiply numerator and denominator by  $(s + 3)$ :

$$T(s) = \frac{5K_p}{s + 3 + 5K_p}$$

[1 mark]

[4 marks]

- ii. Determine steady-state water level  $L_{ss}$  for a step input of  $L_{set} = 100\text{cm}$ .

Applying the Final Value Theorem to find the steady-state output:

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \cdot Y(s) = \lim_{s \rightarrow 0} s \cdot [T(s) \cdot R(s)]$$

[1 mark]

$$L_{ss} = \lim_{t \rightarrow \infty} L(t) = \lim_{s \rightarrow 0} s \cdot T(s) \cdot \frac{100}{s}$$

[1 mark]

$$\text{Substitute } T(s) = \frac{5K_p}{s+3+5K_p}:$$

$$L_{ss} = \lim_{s \rightarrow 0} \left( s \cdot \frac{5K_p}{s+3+5K_p} \cdot \frac{100}{s} \right)$$

[1 mark]

$$= \frac{5K_p \cdot 100}{3+5K_p}$$

[1 mark]

$$L_{ss} = \frac{500K_p}{3+5K_p} \text{ (in cm)}$$

[1 mark]

[5 marks]

### Question Two (20 marks)

A crate of mass  $m=6\text{kg}$  is placed on a rough inclined ramp at an angle of  $\theta = 20^\circ$  from the horizontal as shown in Figure 1. The coefficient of kinetic friction between the crate and the surface is  $\mu=0.25$ . A motor applies a cable tension of force  $F = 40\text{N}$  parallel to the ramp to pull the crate upward.

- Using Newton's second law, calculate the acceleration of the crate along the ramp. Assume that the applied force is greater than the downhill component of weight and friction.

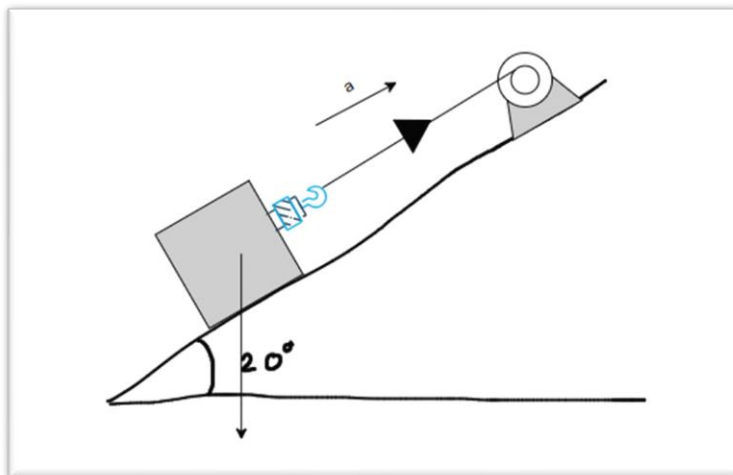


Figure 1

$$F = 40\text{N (motor pulling up)}$$

$$W_{\parallel} = mg\sin\theta \text{ (weight component pulling down ramp)}$$

$$F_f = \mu N = \mu mg\cos\theta \text{ (friction pulling down)}$$

$$F_{\text{net}} = F - mg\sin\theta - \mu mg\cos\theta$$

[1 mark]

Applying Newton's second law:

$$F_{\text{net}} = ma$$

$$= F - mg\sin\theta - \mu mg\cos\theta$$

[1 mark]

$$6a = 40 - (6 \cdot 9.81 \cdot \sin 20^\circ) - 0.25(6 \cdot 9.81 \cdot \cos 20^\circ) = 6.04 \text{ N}$$

[1 mark]

$$a = \frac{40 - 20.13 - 13.83}{6} = 1.01 \text{ m/s}^2 \text{ up the ramp}$$

[1 mark]

[4 marks]

- b. The crate is controlled by a PID controller that adjusts cable force  $F$  to maintain the crate's speed at a constant value of  $1.5\text{m/s}$  as it moves up the ramp. The transfer function representing the system dynamics between applied force and crate velocity is given by:

$$G(s) = \frac{1}{ms + b}$$

- i. If the damping coefficient is  $b = 0.4\text{Ns/m}$ , use the transfer function to derive the steady state applied force required to maintain the velocity at  $1.5\text{m/s}$  in the presence of friction and gravitational force.

At steady state: The velocity is constant ( $a = 0$ ), hence the applied force must balance out opposing forces:

[1 mark]

Opposing forces:

- Gravity component:  $20.13\text{N}$

- Friction component: 13.83N

$$F_{ss} = mg\sin\theta + \mu mg\cos\theta + bv$$

[1 mark]

$$F_{ss} = 20.13 + 13.83 + 0.6 = 34.96\text{N}$$

[2 marks]

[4 marks]

- ii. Calculate  $G(0)$  and explain its physical meaning in determining the steady state-applied force.

$$G(s) = \frac{1}{ms + b}$$

Set  $s = 0$

$$G(0) = \frac{1}{b} = \frac{1}{0.4} = 2.5$$

[2 marks]

$G(0)$  is the DC gain (steady-state gain). It shows how much velocity results from a constant applied force when inertia no longer matters. The steady-state behaviour depends only on damping  $b$ .

[2 mark]

[4 marks]

- c. Now suppose the crate is attached by a rope that passes over a pulley to a hanging mass  $m_2 = 10\text{kg}$ . The system is designed to accelerate at a constant rate of  $1.8\text{m/s}^2$  up the ramp.

- i. Write down the equations of motion for both masses.

State what is given:

$$m_1 = 6\text{ kg (on ramp)}$$

$$m_2 = 10\text{ kg (hanging)}$$

$$a = 1.8\text{ m/s}^2$$

For the crate on the ramp:

$$T - m_1 g \sin \theta - \mu m_1 g \cos \theta = m_1 a$$

[1 mark]

For the hanging mass:

$$m_2 g - T = m_2 a$$

[1 mark]

[2 marks]

ii. Derive the required tension  $T$  in the rope using Newton's second law.

Using the crate equation:

$$T = m_1 a + m_1 g \sin \theta + \mu m_1 g \cos \theta$$

[1 mark]

Substitute values:

$$T = 6(1.8) + 20.13 + 13.83$$

$$T = 10.8 + 33.96 = 44.76 \text{ N}$$

[1 mark]

$$T = 44.8 \text{ N}$$

[1 mark]

[3 marks]

- d. A position sensor is mounted on the ramp and measures the position of the crate, producing a voltage signal that is proportional to the position with a scaling factor of  $0.02 \text{ V/m}$ . If the crate moves at a constant velocity of  $1.5 \text{ m/s}$ , calculate the sensor output voltage after 6 seconds.

Given:

Sensor scale:

$$0.02 \text{ V/m}$$

Constant velocity:

$$v = 1.5 \text{ m/s}$$

Time:

$$t = 6 \text{ s}$$

Position:

$$x = vt = 1.5 \times 6 = 9 \text{ m}$$

[1 mark]

Voltage output:

$$V = 0.02x = 0.02(9) = 0.18 \text{ V}$$

[1 mark]

$$\therefore V = 0.18\text{V}$$

[1 mark]

[3 marks]

### **Question Three (25 marks)**

A unity feedback system has the following open-loop transfer function:

$$G(s) = \frac{(s + 3)}{(s + 4)(s^2 + 3s + 6)}$$

- a. Determine the locations of the poles and zeros of the open-loop system in the complex s-plane.

Given:

$$G(s) = \frac{(s + 3)}{(s + 4)(s^2 + 3s + 6)}$$

Zeros: Zeros come from the numerator:

$$s + 3 = 0 \Rightarrow s = -3$$

$$\text{Zero: } s = -3$$

[1 mark]

Poles: Poles come from the denominator.

First pole:

$$s + 4 = 0 \Rightarrow s = -4$$

[1 mark]

Second-order term:

$$s^2 + 3s + 6 = 0$$

Using the quadratic formula:

$$s = \frac{-3 \pm \sqrt{9 - 24}}{2}$$

$$s = \frac{-3 \pm \sqrt{-15}}{2}$$

$$s = \frac{-3 \pm j\sqrt{15}}{2}$$

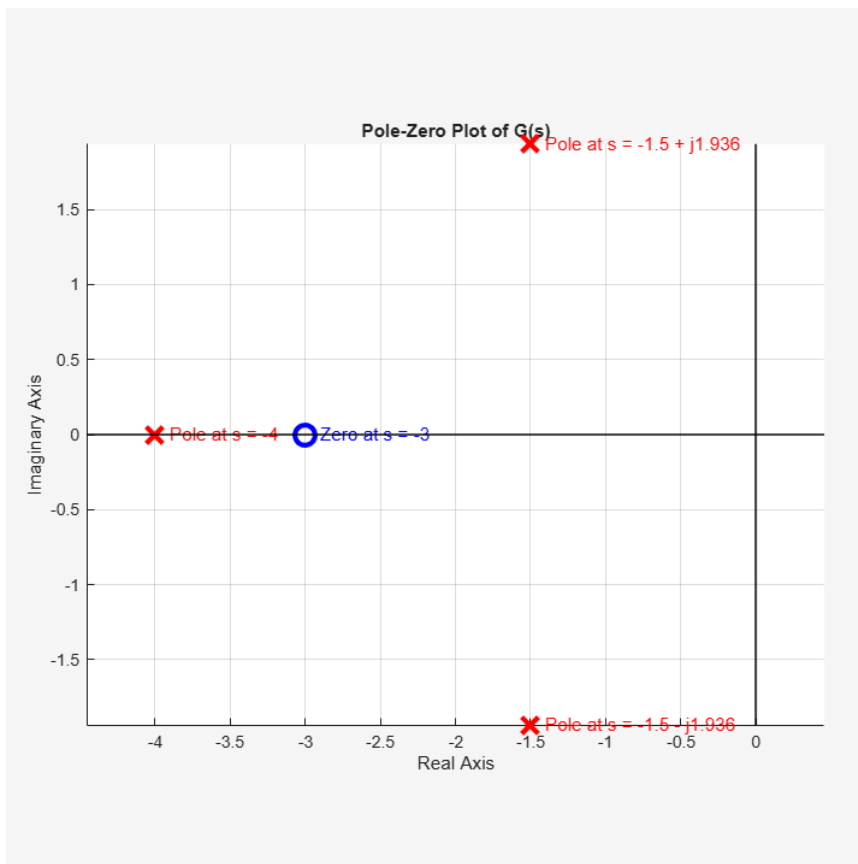
$$s = -1.5 \pm j1.936$$

[2 marks]

[4 marks]

- b. Plot the pole-zero diagram, clearly labelling all poles and zeros.

[4 marks]



- c. Comment briefly on the stability characteristics of the open-loop system.

All poles have negative real parts:  $-4$  and  $-1.5 \pm j1.936$

[1 mark]

Therefore, the system is open-loop stable since the poles lie in the left-half s-plane.

[1 mark]

[2 marks]

- d. Evaluate the magnitude of the transfer function  $G(s)$  at the complex frequency point:

$$s = -1 + 2j.$$

Provide your answer in both rectangular and polar form showing all working.

We evaluate:

$$G(s) = \frac{s + 3}{(s + 4)(s^2 + 3s + 6)}$$

with

$$s = -1 + 2j$$

Step 1: Numerator

$$s + 3 = (-1 + 2j) + 3 = 2 + 2j$$

[1 mark]

Step 2: First denominator term

$$s + 4 = (-1 + 2j) + 4 = 3 + 2j$$

[1 mark]

Step 3: Second denominator term

$$\begin{aligned} & s^2 \\ & (-1 + 2j)^2 \\ & = 1 - 4j - 4 \end{aligned}$$

$$= -3 - 4j$$

[1 mark]

$$s^2 + 3s + 6$$

$$\begin{aligned} &(-3 - 4j) + 3(-1 + 2j) + 6 \\ &= (-3 - 4j) + (-3 + 6j) + 6 \end{aligned}$$

$$= 2j$$

[2 marks]

Step 4: Denominator product

$$\begin{aligned} &(3 + 2j)(2j) \\ &= 6j + 4j^2 \\ &= 6j - 4 \\ &= -4 + 6j \end{aligned}$$

[1 mark]

Step 5: Complex division

$$G(s)|_{s=-1+2j} = \frac{2 + 2j}{-4 + 6j}$$

Multiply by conjugate  $-4 - 6j$ :

$$\frac{(2 + 2j)(-4 - 6j)}{(-4)^2 + 6^2}$$

[1 mark]

Denominator:

$$16 + 36 = 52$$

Numerator:

$$= -8 - 12j - 8j - 12j^2$$

Since  $j^2 = -1$ :

$$\begin{aligned} &= -8 - 20j + 12 \\ &= 4 - 20j \end{aligned}$$

[2 marks]

Hence:

$$G(s)|_{s=-1+2j} = \frac{4-20j}{52}$$

[1 mark]

Rectangular Form:

$$G(s)|_{s=-1+2j} = \frac{4-20j}{52}$$

$$G(s)|_{s=-1+2j} = 0.0769 - 0.3846j$$

[2 marks]

Polar Form:

Magnitude:

$$\begin{aligned}|G| &= \sqrt{0.0769^2 + (-0.3846)^2} \\ &= 0.392\end{aligned}$$

[1 mark]

Angle:

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{-0.3846}{0.0769}\right) \\ &= \tan^{-1}(-5) = -78.7^\circ\end{aligned}$$

[1 mark]

$$G(s)|_{s=-1+2j} = 0.392\angle -78.7^\circ$$

[1 mark]

[15 marks]

#### Question Four (15 marks)

For the given system block diagram:

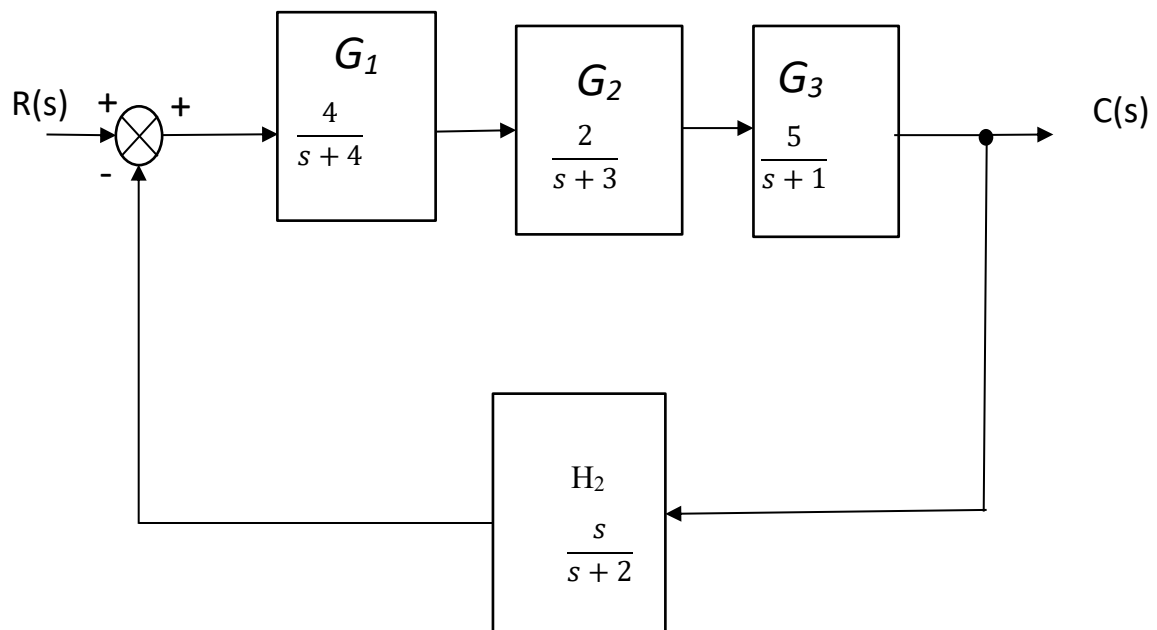


Figure 2

- a. Using block diagram reduction techniques, reduce the system to a single equivalent block and determine the closed-loop transfer function  $\left(\frac{Y(s)}{R(s)}\right)$ . All intermediate steps must clearly be shown.

[10 marks]

Forward-path transfer function

The three forward blocks are in series, so:

$$G(s) = G_1(s)G_2(s)G_3(s) = \frac{4}{s+4} \cdot \frac{2}{s+3} \cdot \frac{5}{s+1} = \frac{40}{(s+4)(s+3)(s+1)}$$

[2 marks]

Feedback transfer function

$$H(s) = \frac{s}{s + 2}$$

The system uses negative feedback, therefore:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

[1 mark]

Substitute G(s) and H(s):

$$\frac{Y(s)}{R(s)} = \frac{\frac{40}{(s + 4)(s + 3)(s + 1)}}{1 + \frac{40s}{(s + 4)(s + 3)(s + 1)(s + 2)}}$$

[2 marks]

Multiply numerator and denominator by:

$(s + 4)(s + 3)(s + 1)(s + 2)$ :

$$\frac{Y(s)}{R(s)} = \frac{40(s + 2)}{(s + 4)(s + 3)(s + 1)(s + 2) + 40s}$$

[2 marks]

Expanding:

$$(s + 4)(s + 3)(s + 1)(s + 2) = s^4 + 10s^3 + 35s^2 + 50s + 24$$

$$\Rightarrow \text{Denominator} = s^4 + 10s^3 + 35s^2 + 90s + 24$$

[2 mark]

Closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{40(s + 2)}{s^4 + 10s^3 + 35s^2 + 90s + 24}$$

[1 mark]

- b. Determine the order and type of the closed loop system and hence the steady state error for a unit step input.

[4 marks]

The order of the system is given by the highest power of s in the denominator:

$$\text{Order} = 4$$

[1 mark]

The system type is determined by the number of pure integrators in (poles at the origin) in the open-loop transfer function:

$$G(s)H(s) = \frac{40s}{(s+4)(s+3)(s+1)(s+2)}$$

There are no poles at  $s = 0$

Type 0 system.

[1 mark]

For a Type 0 system, the steady-state error to a unit step input is:

$$e_{ss} = \frac{1}{1 + K_p}$$

where

$$K_p = \lim_{s \rightarrow 0} G(s)H(s)$$

$$K_p = \lim_{s \rightarrow 0} \frac{40s}{(s+4)(s+3)(s+1)(s+2)} = 0$$

Hence  $e_{ss} = 1$

[2 marks]

- c. Comment on the stability of the closed loop system, stating clearly the criterion used.

[1 mark]

The closed-loop characteristic equation is:

$$s^4 + 10s^3 + 35s^2 + 90s + 24 = 0$$

All coefficients of the characteristic equation are positive, and no sign changes occur. All closed-loop poles lie in the left-half of the s-plane; therefore, the system is stable.

[1 mark]

**Question Five (20 marks)**

- a. An RC circuit consists of a resistor  $R = 2k\Omega$  in series with a capacitor  $C = 10\mu F$  as shown in Figure 3. The input voltage  $v_{in}(t)$  is applied across the series combination, and the output voltage  $v_o(t)$  is measured across the capacitor.

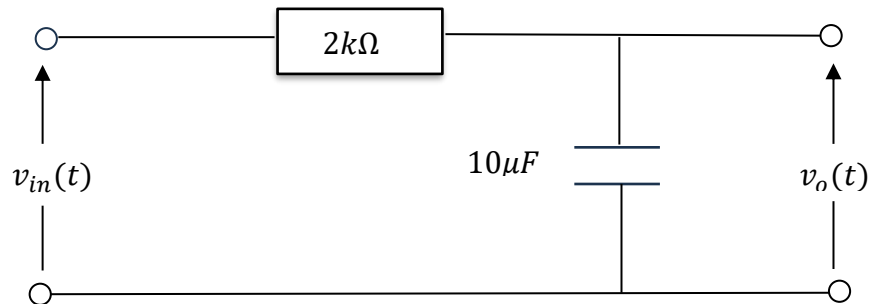


Figure 3.

- i. Derive the transfer function

$$H(s) = \frac{V_o(s)}{V_{in}(s)}$$

for the circuit.

Using KVL:

$$v_{in}(t) = v_R(t) + v_o(t)$$

[1 mark]

Resistor voltage:

$$v_R(t) = Ri(t)$$

Capacitor current:

$$i(t) = C \frac{dv_o(t)}{dt}$$

Substitute into the resistor equation:

$$v_R(t) = RC \frac{dv_o(t)}{dt}$$

Now substitute into KVL:

$$v_{in}(t) = RC \frac{dv_o(t)}{dt} + v_o(t)$$

[1 mark]

Take Laplace transform:

$$\mathcal{L}\{v_{in}(t)\} = \mathcal{L}\left\{RC \frac{dv_o(t)}{dt}\right\} + \mathcal{L}\{v_o(t)\}$$

$$V_{in}(s) = RCsV_o(s) + V_o(s)$$

Factor  $V_o(s)$ :

$$V_{in}(s) = V_o(s)(RCs + 1)$$

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{RCs + 1}$$

[1

mark]

$$RC = 2000 \times 1.0 \times 10^{-5} = 0.02$$

$$H(s) = \frac{1}{0.02s + 1}$$

[1 mark]

[4 marks]

- ii. The input voltage is a step input  $v_{in}(t) = V_0 u(t)$ , where  $V_0 = 5V$ . Determine the time-domain expression for the output voltage  $v_o(t)$

Input:

$$v_{in}(t) = V_0 u(t), V_0 = 5 \text{ V}$$

Laplace transform:

$$V_{in}(s) = \frac{5}{s}$$

Output:

$$V_o(s) = H(s)V_{in}(s)$$

[1 mark]

$$V_o(s) = \frac{5}{s(0.02s + 1)} = \frac{5}{s} - \frac{0.1}{0.02s + 1}$$

[2

marks]

$$V_o(s) = \frac{5}{s} + \frac{5}{s+50}$$

[1mark]

Taking inverse Laplace:

$$\mathcal{L}^{-1}\{1/(s)\} = 1$$

$$\mathcal{L}^{-1}\{1/(s+a)\} = e^{-at}$$

$$v_o(t) = 5 - 5e^{-50t}$$

[1 mark]

$$v_o(t) = 5(1 - e^{-50t})$$

[1 mark]

OR for an RC charging circuit:

$$v_o(t) = V_0(1 - e^{-t/RC})$$

Since  $RC = 0.02$ 

$$v_o(t) = 5(1 - e^{-t/0.02})$$

$$v_o(t) = 5(1 - e^{-50t})$$

[6 marks]

- iii. State whether the circuit behaves as a low pass or high pass filter and briefly justify your answer.

The circuit is a low pass filter since the capacitor blocks high-frequency signals. At low frequency (DC), the capacitor behaves like an open circuit, and the output equals the input

[2 marks]

- b. An RLC circuit has resistance,  $R = 150\Omega$  inductance,  $L = 2H$ , and capacitance,  $C = 25\mu F$ . The current in the circuit is given by:

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = \frac{V}{CL}$$

- i. Determine the natural (undamped) angular frequency  $\omega_0$  and the damping factor  $\alpha$ . Hence state whether the system is overdamped, critically damped, or underdamped?

Given:

$$R = 150 \, \Omega$$

$$L = 2 \, \text{H}$$

$$C = 25 \, \mu\text{F} = 25 \times 10^{-6} \, \text{F}$$

Equation:

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = \frac{V}{CL}$$

Standard form:

$$\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \omega_0^2 i = \text{forcing}$$

[1 mark]

Comparing coefficients:

$$\omega_0^2 = \frac{1}{LC}$$

[1 mark]

Therefore, the natural frequency:  $\omega_0 = \sqrt{\frac{1}{LC}} = 141.4 \, \text{rad/s}$

[1 mark]

$$2\alpha = \frac{R}{L}$$

[1 mark]

$$\alpha = \frac{R}{2L} = \frac{150}{4} = 37.5 \, \text{s}^{-1}$$

[1 mark]

$$\alpha < \omega_0$$

$$37.5 < 141.4$$

Underdamped system since the damping factor is less than the natural frequency.

[1 mark]

[6 marks]

- iii. For a constant DC input voltage, state the steady- state current in the circuit and briefly explain your reasoning.

For DC:

Capacitor  $\rightarrow$  open circuit

Inductor  $\rightarrow$  short circuit

Because the capacitor blocks DC current:  $i_{ss} = 0$

There is no continuous current flow through the capacitor at steady state.

[1 mark]

- iv. If the applied voltage is

$$V(t) = V_0$$

Where  $V_0 = 10V$  is constant, find the steady- state current,  $i_{ss}$  of the circuit?

Because the input is constant DC:

$$i_{ss} = 0 A$$

[1mark]

**Good luck**

**End of the exam**