



SWANSEA METROPOLITAN UNIVERSITY  
PRIFYSGOL METROPOLITAN ABERTAWE

# School of Automotive Engineering

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## Introduction to Mass and Inertial Effects

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# Linear Motion

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## Linear Motion

Position  $x$

Velocity  $v$

Acceleration  $a$

Motion equations  $x = \bar{v} t$

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2ax$$

Mass (linear inertia)  $m$

Newton's second law  $F = ma$

Momentum  $p = mv$

Work  $Fd$

Kinetic energy  $\frac{1}{2}mv^2$

Power  $Fv$

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# Rotation – Linear Parallels

## Linear Motion

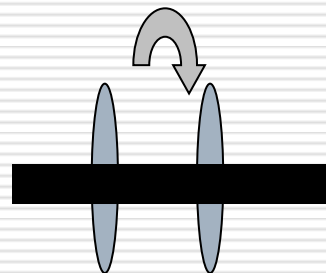
## Rotational Motion

|                       |                              |                                              |                      |
|-----------------------|------------------------------|----------------------------------------------|----------------------|
| Position              | $x$                          | $\theta$                                     | Angular position     |
| Velocity              | $v$                          | $\omega$                                     | Angular velocity     |
| Acceleration          | $a$                          | $\alpha$                                     | Angular acceleration |
| Motion equations      | $x = \bar{v}t$               | $\theta = \bar{\omega}t$                     | Motion equations     |
|                       | $v = v_0 + at$               | $\omega = \omega_0 + \alpha t$               |                      |
|                       | $x = v_0t + \frac{1}{2}at^2$ | $\theta = \omega_0t + \frac{1}{2}\alpha t^2$ |                      |
|                       | $v^2 = v_0^2 + 2ax$          | $\omega^2 = \omega_0^2 + 2\alpha\theta$      |                      |
| Mass (linear inertia) | $m$                          | $I$                                          | Moment of inertia    |
| Newton's second law   | $F = ma$                     | $\tau = I\alpha$                             | Newton's second law  |
| Momentum              | $p = mv$                     | $L = I\omega$                                | Angular momentum     |
| Work                  | $Fd$                         | $\tau\theta$                                 | Work                 |
| Kinetic energy        | $\frac{1}{2}mv^2$            | $\frac{1}{2}I\omega^2$                       | Kinetic energy       |
| Power                 | $Fv$                         | $\tau\omega$                                 | Power                |

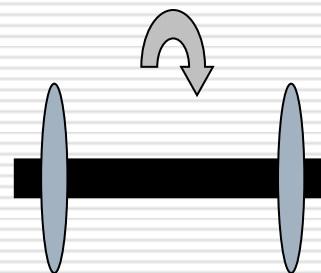
# moment of inertia

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- ❑ Force required to twist long dumbbell is higher than required for the short dumbbell
- ❑ Longer dumbbell has higher inertia
- ❑ Same force is required to lift both dumbbells
- ❑ Both dumbbells are the same mass
- ❑ Need to know mass and inertia properties to fully understand system behaviour



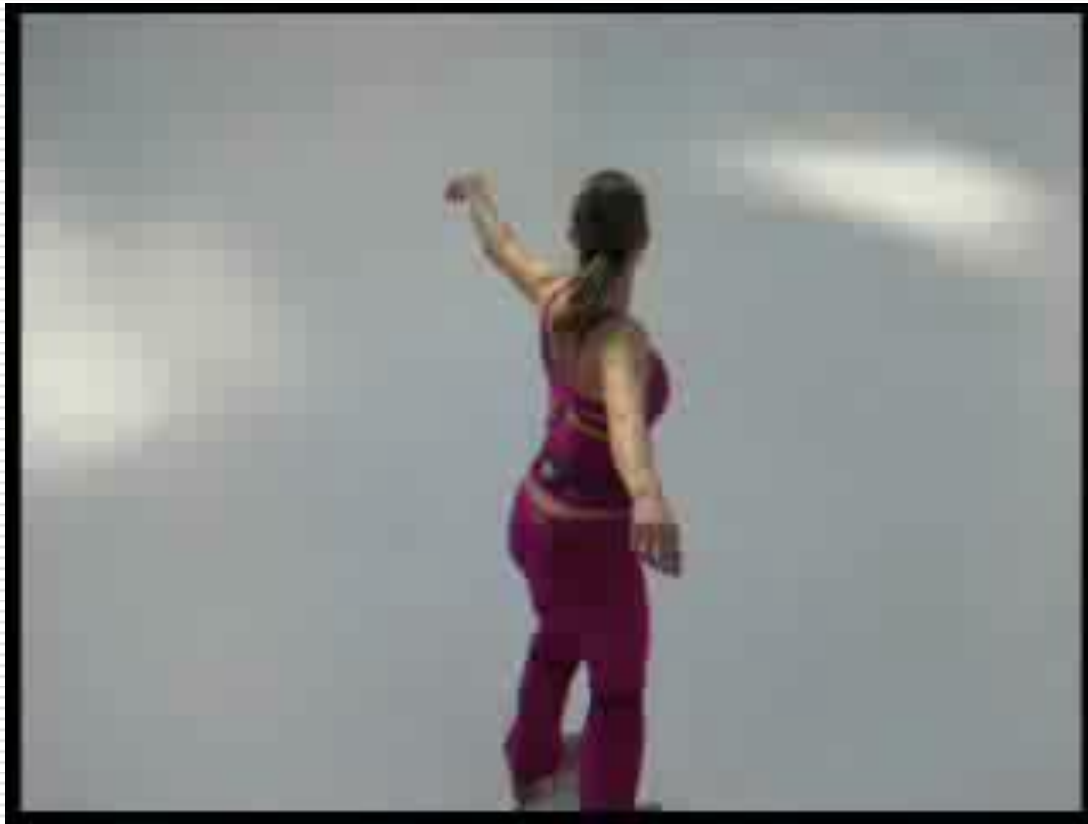
Short Dumbbell  
Easy to Twist



Long Dumbbell  
Hard to Twist

# Spinning Skater

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# Moment of Inertia

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$$I = mr^2$$

For a point mass the moment of inertia is just the mass times the radius from the axis squared. For a collection of point masses (below) the moment of inertia is just the sum for the masses.

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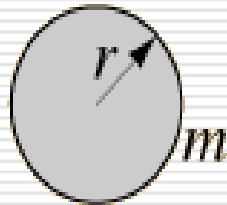
# Moment of Inertia

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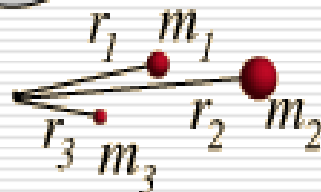
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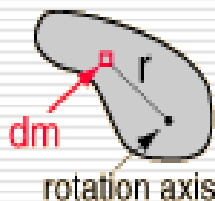
$$I = kmr^2$$

For an object with an axis of symmetry, the moment of inertia is some fraction of that which it would have if all the mass were at the radius  $r$ .



$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$

Sum of the point mass moments of inertia.



$$I = \int_0^M r^2 dm$$

Continuous mass distributions require an infinite sum of all the point mass moments which make up the whole. This is accomplished by an integration over all the mass.

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Linear  $F = ma$

Newton's Second  
Law

Angular  $\tau = I\alpha$

Linear  $p = mv$

Momentum

Angular  $L = I\omega$

Moment  
of Inertia  
 $I$

Linear  $KE = \frac{1}{2}mv^2$

Kinetic Energy

Angular  $KE = \frac{1}{2}I\omega^2$

Linear  $F_{net}d = \Delta \left( \frac{1}{2}mv^2 \right)$

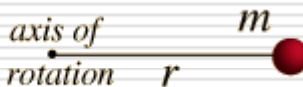
Work-Energy

Angular  $\tau_{net}\theta = \Delta \left( \frac{1}{2}I\omega^2 \right)$

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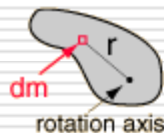
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Since the moment of inertia of an ordinary object involves a continuous distribution of mass at a continually varying distance from any rotation axis, the calculation of moments of inertia generally involves calculus, the discipline of mathematics which can handle such continuous variables. Since the moment of inertia of a [point mass](#) is defined by

$$I = mr^2$$


The diagram shows a horizontal line representing the axis of rotation. A point is marked on this line with the label "axis of rotation" above it. To the right of this point, a red dot represents the mass  $m$ . A horizontal line segment connects the axis of rotation to the mass, labeled  $r$  below it.

then the moment of inertia contribution by an infinitesimal mass element  $dm$  has the same form. This kind of mass element is called a [differential element](#) of mass and its moment of inertia is given by

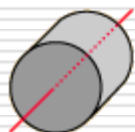


$$dI = r^2 dm$$

The "d" preceding any quantity denotes a vanishingly small or "differential" amount of it.

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Solid cylinder or disc, symmetry axis



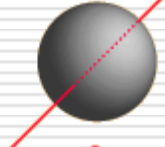
$$I = \frac{1}{2} MR^2$$

Hoop about symmetry axis



$$I = MR^2$$

Solid sphere



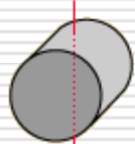
$$I = \frac{2}{5} MR^2$$

Rod about center



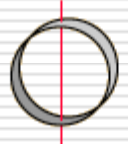
$$I = \frac{1}{12} ML^2$$

$$I = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$$



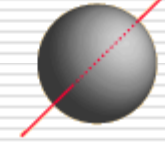
Solid cylinder, central diameter

$$I = \frac{1}{2} MR^2$$



Hoop about diameter

$$I = \frac{2}{3} MR^2$$



Thin spherical shell

$$I = \frac{1}{3} ML^2$$



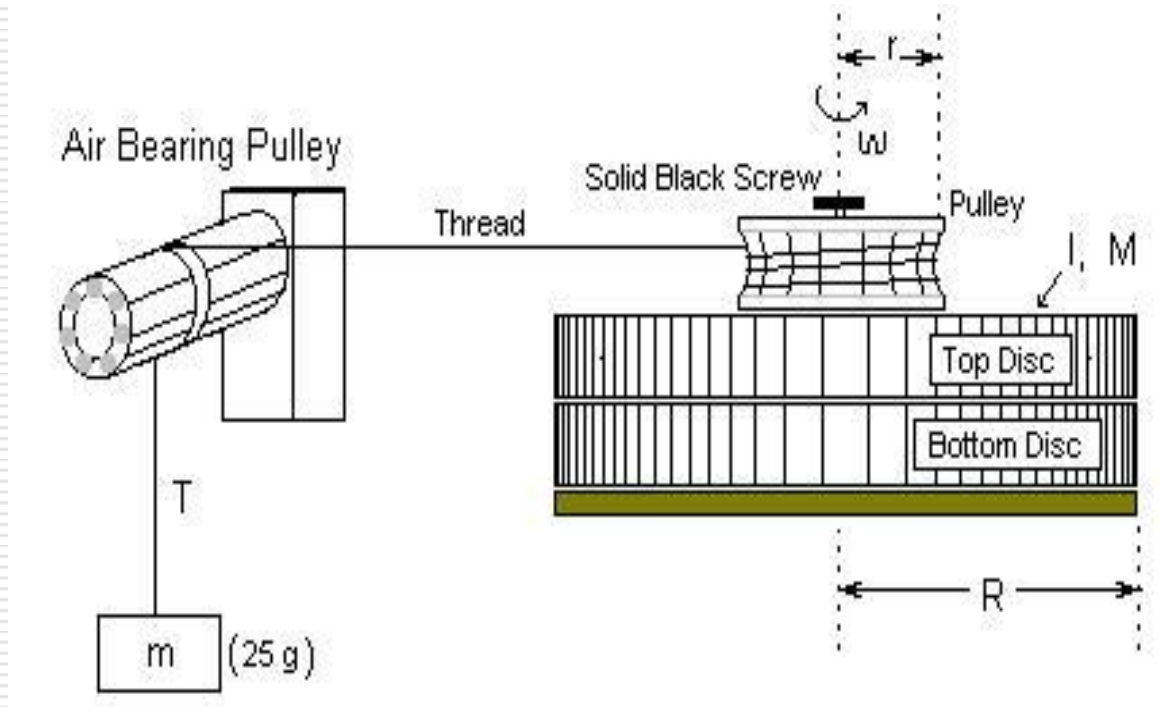
Rod about end



# Inertia measurement

## Torsional force

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Applied torque = Inertia x angular acceleration

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# Vehicle Inertial measurement

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CIC's Moment of Inertia Facilities provide measurement of:

- 3-Axis, Moment and Product of Inertias
- Principal Inertias and their Direction Cosines
- Total Mass, Centre of Gravity Location and Mass Distribution

|                 |                                                 |
|-----------------|-------------------------------------------------|
| Mass range      | From 1kg to 2,500kg                             |
| Dimension range | Nominally 6500 (L) x 2800 (W) x 2000 (H) mm max |

# Inertial Measurement Centre of Gravity And Inertial Measurement System

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- The vehicle is clamped to the 'centre table' of the SPMM via multi-axis load cells. The centre table is then automatically driven through a sequence of manoeuvres to enable the Centre of Gravity and Moments of Inertia of the body to be determined. A test sequence typically lasts less than 15 minutes.

