



HND/C PHASE TEST SPECIFICATION RESIT

Programme:	HND Automotive Engineering / Motorsport Engineering				
Unit:	Mechanical Principles				
Phase Test No:	1	Lecturer:	T Tudor	Internal Verifier:	A Summerfield

Time and Date:	Monday 7 th March 2011	Duration:	110 mins	Contribution to Overall Unit Assessment (%):	25%
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Outcomes being Assessed (from Unit Syllabus):	
Number	Description
Learning Outcome 1:	Investigate complex loading systems
Learning Outcome 2:	{Deflection of beams ('standard cases') { 'Stresses in pressure vessels'

For Attention of Students:

Answer **ALL** questions.

The mark allocation is as indicated – total 50 marks.

A 'formula sheet' is provided for reference.

Assume 'g' = 9.81 m/s².

and 1 bar = 10⁵ N/m² (P_a)

1. Mutually perpendicular stresses of 20MPa (tensile) and 35 MPa (compressive) act at a point in a material. Calculate the magnitude and sense of the strains in the directions of the stresses, given that Poisson's ratio is 0.27 and the Modulus of Elasticity is 205 GPa.

(6 marks)

2. At a certain point in a material there is a strain in the 'x' direction of -0.31×10^{-6} and a strain in the 'y' direction of $+0.46 \times 10^{-6}$. Calculate the corresponding stresses in each direction and state whether they are compressive or tensile. Take Poisson's ratio to be 0.28 and the Modulus of Elasticity as 200 GPa.

(6 marks)

3. A cube of side length 0.32m is subjected to an external fluid pressure of 38MPa. If the bulk modulus of the cube material is 136GPa calculate the change in volume.

(8 marks)

4. A 4 m long simply supported beam carries a uniformly distributed load of 600N/m along its entire length. A point load of 1.0 kN also acts at the midpoint of the beams span. The beam has a flexural stiffness of 8.5MPa.

Calculate

- i) The corresponding deflection at the midpoint of the beam
- ii) How much the u.d.l may be increased before the deflection exceeds 1.0mm

(10 marks)

5. A thin walled cylinder acts as an air receiver. It has a diameter of 0.2m, is 0.6m in length and is made from sheet steel with a thickness of 3mm. The longitudinal and circumferential joint efficiencies are 70% and 48% respectively, and the maximum allowable stress is 100MPa. What is the corresponding maximum allowable air pressure?

(10 marks)

6. A hydraulic cylinder has an external diameter of 185 mm and an internal diameter of 165 mm. If it is pressurised internally to 175 Bar gauge, determine the radial and circumferential stresses at the inner and outer surfaces.

(10 marks)

Mechanical Principles Formula Sheet 1

Two-dimensional stress and strain equations:

$$\begin{aligned}\varepsilon_1 &= \frac{1}{E}(\sigma_1 - \nu\sigma_2) & \sigma_1 &= \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) \\ \varepsilon_2 &= \frac{1}{E}(\sigma_2 - \nu\sigma_1) & \sigma_2 &= \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)\end{aligned}$$

Three-dimensional strain equations:

$$\begin{aligned}\varepsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \\ \varepsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \\ \varepsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]\end{aligned}$$

Volumetric Strain:

$$\begin{aligned}\varepsilon_v &= \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z \\ \varepsilon_v &= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\nu)\end{aligned}$$

Relationships between elastic constants:

$$E = \frac{\sigma}{\varepsilon} \quad K = \frac{\sigma}{\varepsilon_v} = \frac{E}{3(1-2\nu)} \quad G = \frac{E}{2(1+\nu)} \quad K = \frac{2G(1+\nu)}{3(1-2\nu)}$$

Stress and strains in thin walled pressure vessels:

Cylinders		Spheres
$\sigma_c = \frac{pD}{2t\eta_L}$		$\sigma = \frac{pD}{4t\eta}$
$\varepsilon_c = \varepsilon_D = \frac{pD}{4tE}(2-\nu)$		
$\varepsilon_v = \frac{pD}{4tE}(5-4\nu)$		$\varepsilon_v = \frac{3pD}{4tE}(1-\nu)$

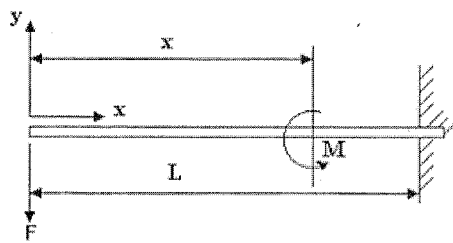
Stresses in thick walled cylinders: (Lame's Equations)

$$\sigma_R = a - \frac{b}{r^2} \quad \sigma_C = a + \frac{b}{r^2}$$

Where a and b are found
by applying boundary
conditions

Loaded beams : Standard Cases

Cantilever Beams:

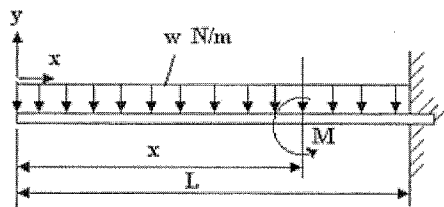


Max Slope

$$\frac{FL^2}{2EI}$$

Max Deflection

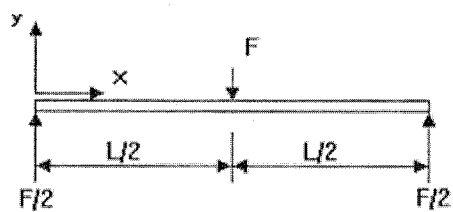
$$\frac{FL^3}{3EI}$$



$$\frac{wL^3}{6EI}$$

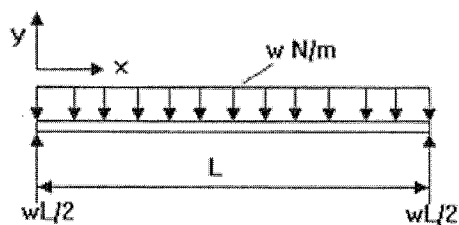
$$\frac{wL^4}{8EI}$$

Simply Supported Beams:



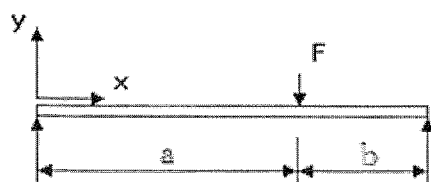
$$\frac{FL^2}{16EI}$$

$$\frac{FL^3}{48EI}$$



$$\frac{wL^3}{24EI}$$

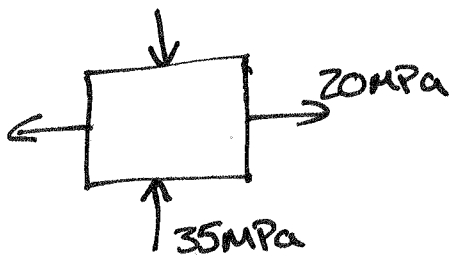
$$\frac{5wL^4}{384EI}$$



$$\frac{Fb}{6EIL}(2L^2 - 3Lb + b^2)$$

$$\frac{Fa^2b^2}{3EIL}$$

Q1/



$$\sigma_1 = 20 \times 10^6 \text{ Pa} \checkmark$$

$$\sigma_2 = -35 \times 10^6 \text{ Pa} \checkmark$$

$$E = 205 \times 10^9 \text{ Pa} \checkmark$$

$$\nu = 0.27$$

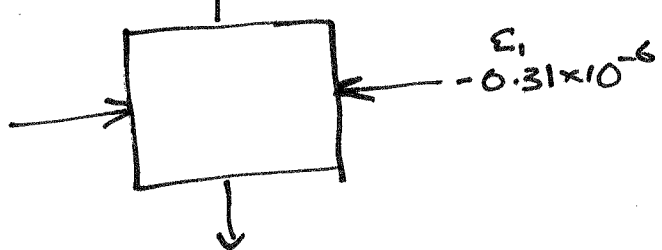
$$\epsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2) = \frac{1}{205 \times 10^9} \times (20 \times 10^6 - (0.27 \times -35 \times 10^6)) \checkmark$$

$$\epsilon_1 = 51.46 \times 10^{-6} \text{ (tensile)} \checkmark$$

$$\epsilon_2 = \frac{1}{E} (\sigma_2 - \nu \sigma_1) = \frac{1}{205 \times 10^9} \times (-35 \times 10^6 - (0.27 \times 20 \times 10^6)) \checkmark$$

$$\epsilon_2 = -1.98 \times 10^{-6} \text{ (compressive)} \checkmark$$

$$\epsilon_2 = 0.46 \times 10^{-6}$$



$$\epsilon_x = -0.31 \times 10^{-6}$$

$$\epsilon_y = 0.46 \times 10^{-6}$$

$$E = 200 \times 10^9 \text{ Pa} \checkmark$$

$$\nu = 0.28$$

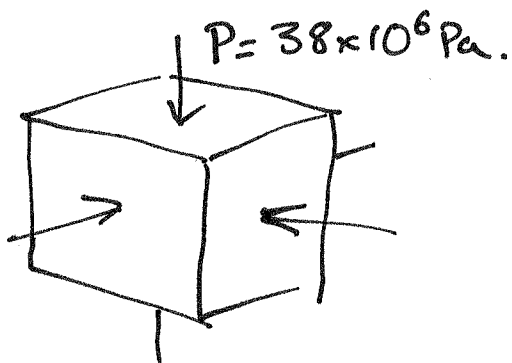
$$\sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = \frac{200 \times 10^9}{1-(0.28)^2} [-0.31 \times 10^{-6} + (0.28 \times 0.46 \times 10^{-6})] \checkmark$$

$$\sigma_1 = -39.32 \times 10^3 \text{ Pa} \text{ (compressive)} \checkmark$$

$$\sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) = \frac{200 \times 10^9}{1-(0.28)^2} [0.46 \times 10^{-6} + (0.28 \times -0.31 \times 10^{-6})] \checkmark$$

$$\sigma_2 = 80.99 \times 10^3 \text{ Pa} \text{ (Tensile)} \checkmark$$

Q3.



$$P = 38 \times 10^6 \text{ Pa}$$

$$L = 0.32 \text{ m}$$

$$K = 136 \times 10^9 \text{ Pa}$$

$$K = \frac{\sigma}{\epsilon_v}$$

$$\sigma = -P$$

$$\therefore K = \frac{-P}{\epsilon_v}$$

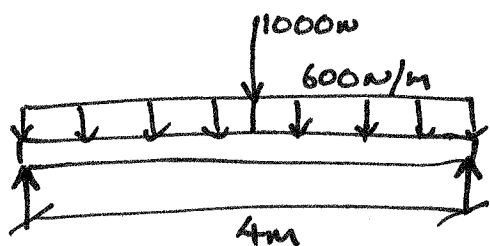
$$\epsilon_v = \frac{-P}{K} = \frac{-38 \times 10^6}{136 \times 10^9} = -279.41 \times 10^{-6}$$

$$\epsilon_v = \frac{\Delta V}{V} \quad \Delta V = \epsilon_v V = -279.41 \times 10^{-6} (0.32)^3$$

$$\Delta V = -9.156 \times 10^{-6} \text{ m}^3$$

$$\Delta V = -9.16 \times 10^{-6} \text{ m}^3$$

Q4.



$$EI = 8.5 \times 10^6 \text{ Pa}$$

$$y_{udl} = \frac{5\omega L^4}{384EI} = \frac{5 \times 600 \times 4^4}{384 \times 8.5 \times 10^6} = -0.235 \times 10^{-3} \text{ m}$$

$$y_{PL} = \frac{FL^3}{48EI} = \frac{-1000 \times 4^3}{48 \times 8.5 \times 10^6} = -0.157 \times 10^{-3} \text{ m}$$

$$y_{total} = y_{udl} + y_{PL} = 0.235 \times 10^{-3} + 0.157 \times 10^{-3} = -0.392 \times 10^{-3} \text{ m} = -0.392 \text{ mm}$$

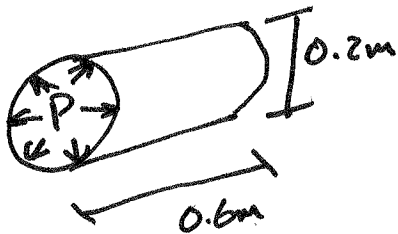
$$i) y_{udl \text{ increase}} = y_{max} - y_{udl} - y_{PL} = 1 - 0.392 = 0.608 \text{ mm}$$

$$y = \frac{5\omega L^4}{384EI}$$

$$\omega = \frac{384EI y}{5L^4} = \frac{384 \times 8.5 \times 10^6 \times 0.608 \times 10^{-3}}{5 \times 4^4}$$

$$udl \text{ increase} = 1550.4 \text{ N/m}$$

Q5



$$d = 0.2 \text{ m}$$

$$L = 0.6 \text{ m}$$

$$t = 3 \times 10^{-3} \text{ m}$$

$$\eta_L = 70\% = 0.7$$

$$\eta_c = 48\% = 0.48$$

$$\sigma_{\max} = 100 \times 10^6 \text{ Pa.}$$

$$\sigma_c = \frac{pd}{2t\eta_L} \checkmark$$

$$\sigma_L = \frac{pd}{4t\eta_c} \checkmark$$

$$P = \frac{2\sigma_c t \eta_L}{d} \checkmark$$

$$P = \frac{4\sigma_L t \eta_c}{d} \checkmark$$

$$P = \frac{2 \times 100 \times 10^6 \times 3 \times 10^{-3} \times 0.7}{0.2} \checkmark$$

$$P = \frac{4 \times 100 \times 10^6 \times 3 \times 10^{-3} \times 0.48}{0.2} \checkmark$$

$$P_{\max} = 2.1 \text{ MPa}$$

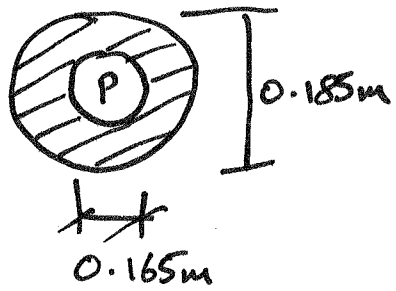
$$= 21 \text{ Bar} \checkmark$$

$$P = 2.88 \text{ MPa}$$

$$= 28.8 \text{ Bar} \checkmark$$

$$\therefore P_{\max} = \underline{\underline{21 \text{ Bar}}} \checkmark$$

Q6



$d_o = 0.185 \text{ m}$
 $d_i = 0.165 \text{ m}$
 $P = 175 \times 10^5 \text{ Pa}$
 r

@ inside surface $r_i = 0.0825$ $\sigma_r = -P = -175 \times 10^5 \text{ Pa}$ ✓

$$\sigma_r = a - \frac{b}{r^2} \checkmark$$

$$-175 \times 10^5 = a - \frac{b}{(0.0825)^2}$$

$$119.11 \times 10^3 = b - (0.0825)^2 a \quad (1) \checkmark$$

@ outside surface $r_o = 92.5 \times 10^{-3} = 0.0925 \text{ m}$ $\sigma_r = 0$ ✓

$$0 = a - \frac{b}{(0.0925)^2}$$

$$0 = b - (0.0925)^2 a \quad (2) \checkmark$$

①-② ✓

$$119.11 \times 10^3 = -(0.0825)^2 a + (0.0925)^2 a$$

$$a = \frac{119.11 \times 10^3}{(0.0925)^2 - (0.0825)^2} = 68.06 \times 10^6 \text{ Pa} \checkmark$$

$$b = (0.0925)^2 \times 68.06 \times 10^6 = 5.8236 \times 10^5 \text{ N} \checkmark$$

@ r_i

$$\sigma_c = a + \frac{b}{r^2} = 68.06 \times 10^6 + \frac{5.8236 \times 10^5}{(0.0825)^2} = 153.62 \text{ MPa} \checkmark$$

@ r_o

$$\sigma_c = a + \frac{b}{r^2} = 68.06 \times 10^6 + \frac{5.8236 \times 10^5}{(0.0925)^2} = 132.12 \text{ MPa} \checkmark$$

