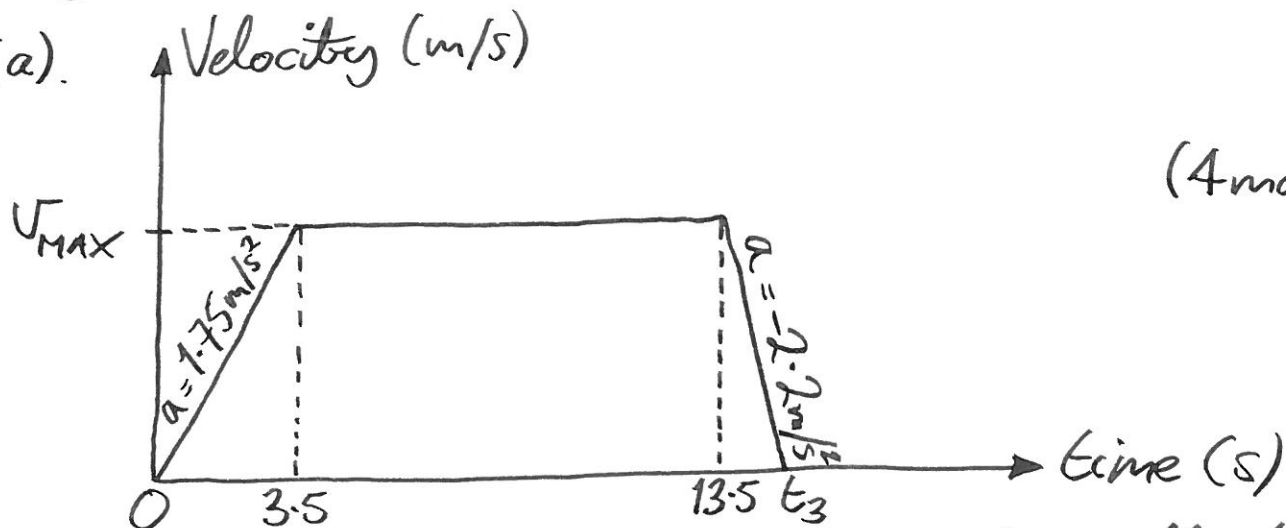


L5 ~ DYNAMICS - SOLUTIONS

①

Q1(a).



(4 marks)

(b). Need to find total distance travelled;
For accelerating phase;

$$S_1 = ut + \frac{1}{2}at^2$$
$$= 0 \cdot 3.5 + \frac{1}{2}(1.75)(3.5)^2$$
$$= 10.719 \text{ m}$$

For constant velocity phase;

$$S_2 = V_{MAX} \times t$$

where, $V_{MAX} = u + at$ (for acceleration).

$$= 0 + 1.75 \times 3.5$$

$$\text{so, } V_{MAX} = 6.125 \text{ m/s}$$

$$\therefore S_2 = 6.125 \times 10 = 61.25 \text{ m}$$

For deceleration phase;

$$S_3 = \frac{V_{\text{final}}^2 - u^2}{2a} = \frac{0^2 - (6.125)^2}{2(-2.2)}$$

$$\therefore S_3 = 8.526 \text{ m}$$

$$\therefore \text{height of building} = S_1 + S_2 + S_3$$

$$= 10.719 + 61.25 + 8.526$$

$$\text{Height} = \underline{80.495 \text{ m}} \quad (6 \text{ marks})$$

Q1(c). Need to find average Kinetic Energy by using average velocity;

$$V_{\text{AVG.}} = \frac{\text{Distance travelled}}{\text{Time Taken}} \quad \frac{(\text{m})}{(\text{s})}$$

First find t_3 ; use, $V = u + a.t$

$$\therefore t_3 = \frac{V - u}{a} = \frac{0 - 6.125}{-2.2}$$

$$\text{so, } t_3 = 2.784\text{s}$$

Hence, total time of travel is;

$$t_T = t_1 + t_2 + t_3 = 16.284\text{s}$$

$$\text{Now, } V_{\text{AVG.}} = \frac{80.495}{16.284} = 4.943\text{ m/s}$$

$$\text{So, } K.E._{\text{AVG}} = \frac{1}{2} m \cdot V_{\text{AVG}}^2$$

$$= 0.5 \times 150 \times 4.943^2$$

$$\therefore K.E._{\text{AVG.}} = 1832.64\text{ J} \quad (5\text{ marks})$$

(d). Find time for hoist to fall height of building.

Determine final velocity when impact;

Principle of Conservation of Energy

$$m \cdot g \cdot h = \frac{1}{2} m v^2$$

$$\text{So, } v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 80.495}$$

$$\therefore v = 39.741\text{ m/s}$$

Now, time for hoist to fall; $t = \frac{2s}{u+v}$

$$\therefore t = \frac{2 \times 80.495}{0 + 39.741}$$

$$= 4.051\text{s}$$

Time for sound to reach ground-level.

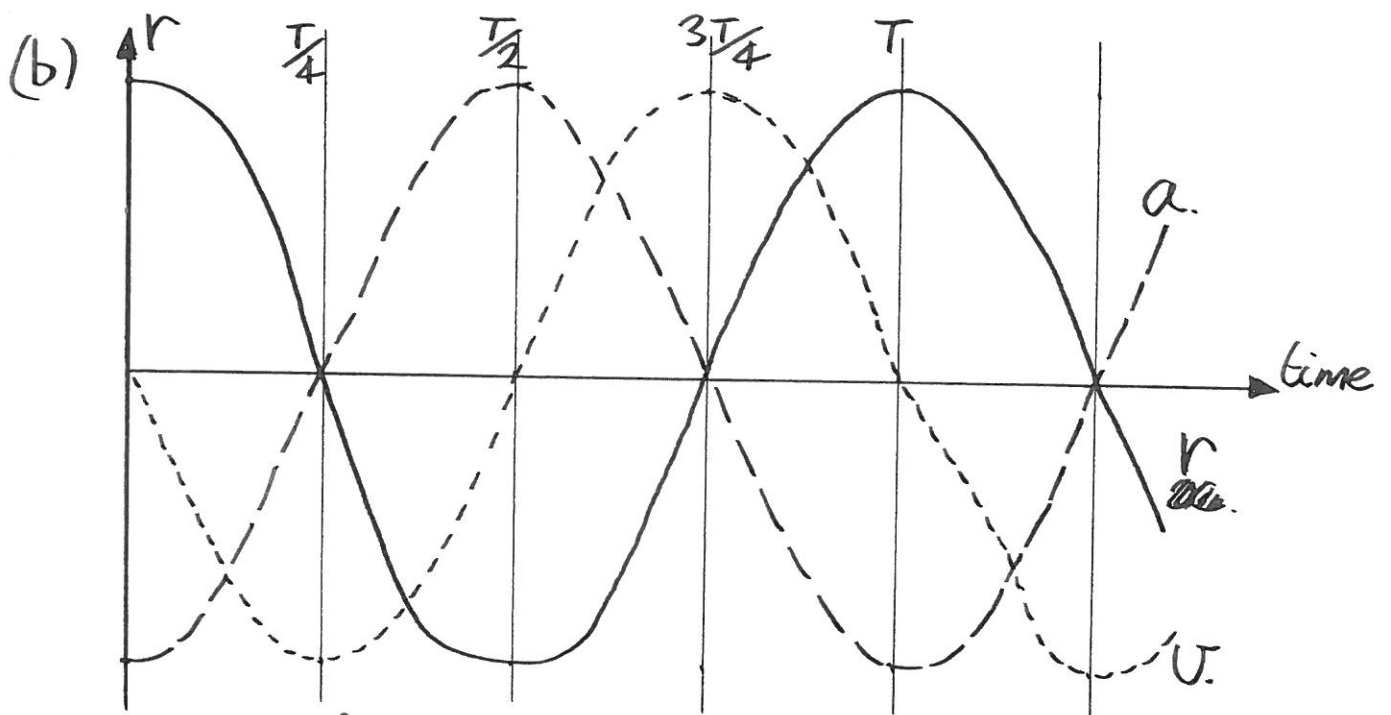
$$t = \frac{\text{Distance}}{\text{Speed of sound}} = \frac{80.495}{343} = 0.235\text{s}$$

$$\therefore \text{Warning time} = 4.051 - 0.235 = 3.816\text{s} \quad (5\text{ marks})$$

ASSUME:

No drag resistance on falling hoist, no extraneous noise

- Q2(a) • acceleration of body is always directed towards a fixed point in its path, and
 • magnitude of the acceleration of the body is proportional to its distance from the fixed point. (2 marks)



Acceleration, $a = \omega^2 r$

Velocity, $v = \omega r$

Displacement = r .

(4 marks)

- (c) Determine amplitude r by using;

$$v = \omega \sqrt{r^2 - x^2}$$

$$\therefore 6 = \omega \sqrt{r^2 - 0.125^2}$$

$$\text{and, } 3 = \omega \sqrt{r^2 - 0.200^2}$$

$$\text{So, } 2^2 = \frac{r^2 - 0.125^2}{r^2 - 0.200^2}$$

$$\text{now, } 4r^2 - 0.16 = r^2 - 0.015625$$

$$3r^2 = 0.144375$$

$$\text{Hence; amplitude, } r = 0.2194 \text{ m}$$

$$(r = 219.4 \text{ mm})$$

(6 marks)

Q2(d). Find maximum velocity from, $v = r\omega$ (4)
where r is amplitude & ω angular velocity.

$$\text{Now, } 3 = \omega \sqrt{(0.2194^2 - 0.2^2)}$$

$$\therefore \omega = 33.28 \text{ rad/s}$$

$$\text{So, } v_{\text{MAX}} = 0.2194 \times 33.28$$

$$\therefore \underline{v_{\text{MAX}} = 7.302 \text{ m/s}}$$

$$\text{Periodic time, } T = \frac{2\pi}{\omega} = \frac{2\pi}{33.28}$$

$$\therefore \underline{T = 0.189 \text{ s}}$$

(4 marks).

(e) Mass, $m = 0.45 \text{ kg}$.

$$\text{Maximum force, } F_{\text{MAX}} = m \times a_{\text{MAX}}$$

Find maximum acceleration from;

$$\begin{aligned} a_{\text{MAX}} &= \omega^2 r \\ &= (33.28)^2 \times 0.2194 \end{aligned}$$

$$\therefore a_{\text{MAX}} = 243 \text{ m/s}^2$$

$$\text{So, } F_{\text{MAX}} = 0.45 \times 243$$

$$\underline{F_{\text{MAX}} = 109.35 \text{ N}}$$

(4 marks).

Q3(a). To find accelerating torque, use;

$$T = I \cdot \alpha$$

where, for solid disc flywheel,

$$I = \frac{1}{2} m r^2 \quad (m = \text{mass of flywheel})$$

and, knowing $\omega_2 = \omega_1 + \alpha t$ it is possible to find α .

$$\begin{aligned} \text{Mass of flywheel, } m &= \text{Volume} \times \text{density} \\ &= \left(\left(\frac{\pi d^2}{4} \right) \times t \right) \times \rho \end{aligned}$$

$$m = \left(\frac{\pi \times 0.475^2 \times 0.075}{4} \right) \times 7800$$

$$\text{so, } m = 104.33 \text{ kg}$$

$$\text{Now, } \omega_1 = 0 \text{ rad/s}$$

$$\omega_2 = 45 \times \frac{2\pi}{60} = 4.7124 \text{ rad/s}$$

$$t = 12 \text{ s}$$

$$\therefore \alpha = \frac{\omega_2 - \omega_1}{t} = 0.3927 \text{ rad/s}^2$$

So, accelerating torque;

$$T_{\text{acc}} = \frac{1}{2} (104.33) (0.2375)^2 \times 0.3927$$

$$\therefore \underline{T_{\text{acc}} = 1.156 \text{ Nm}}$$

(8 marks)

Assume shaft is solid and same material as flywheel.

Q3 (b). 'New' torque will be, $T = T_{\text{acc.}} - T_{\text{friction}}$

$$\therefore T = 1.156 - 0.15$$

$$T = 1.006 \text{ Nm}$$

$$\text{Acceleration, } \alpha = \frac{T}{I} = \frac{1.006}{0.5(104.33)(0.2375^2)}$$

$$\therefore \underline{\alpha = 0.342 \text{ rad/s}^2}$$

(4 marks)

(c)(i) Upper speed is 4.7124 rad/s .

Retardation caused by frictional torque,

$$\text{So, } \alpha = \frac{T_{\text{friction}}}{I}$$

$$\therefore \alpha = \frac{0.15}{0.5(104.33)(0.2375^2)} = \underline{-0.051 \text{ rad/s}^2}$$

(negative sign due to retarding) (3 marks).

(ii) Time to come to rest,

$$t = \frac{\omega_2 - \omega_1}{\alpha}$$

$$\therefore t = \frac{0 - 4.7124}{-0.051} = \underline{92.4 \text{ s}}$$

(2 marks).

(iii) find angular distance travelled, θ ;

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta \Rightarrow \theta = \frac{\omega_2^2 - \omega_1^2}{2\alpha}$$

$$\text{So, } \theta = \frac{-4.7124^2}{2(-0.051)}$$

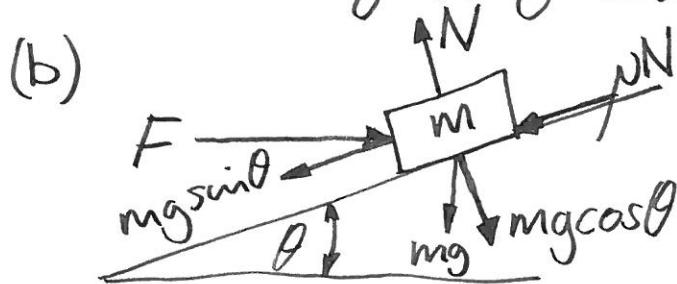
$$\therefore \theta = 217.713 \text{ rad}$$

$$\text{So, number of revolutions, } n = \frac{\theta}{2\pi}$$

$$\therefore \underline{n = 34.65 \text{ revs.}}$$

(3 marks).

- Q4(a). If a force applied to move a body over another, the opposing friction that arises is tangential to surface in contact and is just sufficient to balance the applied force. (7)
- Limiting force is proportional to the normal force between the surfaces, and is independent of area of contact.
 - Ratio of limiting frictional force (F) and normal reaction (R_N) is a constant and depends on nature of surfaces in contact, thus $F = \mu R_N$.



At point of just (3 marks) beginning to move, resultant force will be zero.

Resolve F into perpendicular ($F \sin \theta$) and parallel ($F \cos \theta$) components.

For components of forces perpendicular;

$$F \sin \theta + mg \cos \theta = N \quad \text{--- (1)}$$

For components parallel to plane;

$$F \cos \theta = mg \sin \theta + \mu N \quad \text{--- (2)}$$

Substituting equation (1) into (2) for N ;

$$F \cos \theta = mg \sin \theta + \mu (F \sin \theta + mg \cos \theta)$$

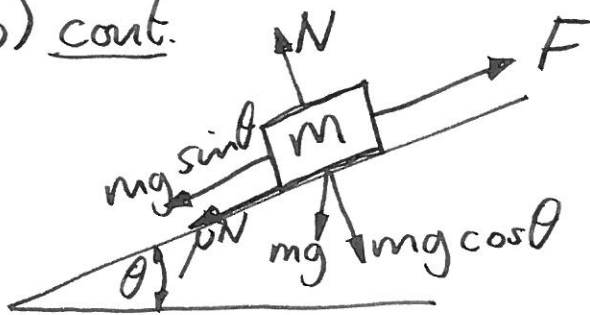
$$\therefore F = \frac{mg(\sin \theta + \mu \cos \theta)}{(\cos \theta - \mu \sin \theta)}$$

$$F = 100 \times 9.81 \frac{(\sin 10^\circ + 0.3 \cos 10^\circ)}{(\cos 10^\circ - 0.3 \sin 10^\circ)}$$

$$\text{So, } F = 493.38 \text{ N (horizontal push)}$$

Q4(b) cont.

(8)



Resolve forces along inclined plane;

$$F = mg \sin \theta + \mu N$$

where, $N = mg \cos \theta$ (perpendicular forces).

$$\text{so, } \mu N = \mu mg \cos \theta$$

$$\therefore F = mg \sin \theta + \mu mg \cos \theta$$

$$= 981 \sin 10^\circ + 0.3(981) \cos 10^\circ$$

$$\text{so, } F = 460.18 \text{ N (parallel pull)}$$

Hence, lower force required to PULL.
(10 marks)

(c) Required torque, $T = F \times r$

$$\text{where, } F = mg \sin \theta + \mu mg \cos \theta$$

$$\therefore F = 981 \sin 10^\circ + 0.2(981) \cos 10^\circ$$

$$F = 363.57 \text{ N}$$

$$\text{So, torque } T = 363.57 \times 0.125$$

$$\therefore \underline{T = 45.45 \text{ Nm}}$$

Assume:- no friction/losses in winding drum
- mass of cable ignored
- pulley is smooth. (7 marks)