

Josh Brown

Analytical Methods exam 2

2004738

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Question 2 - Complex numbers

a) $Z_1 + Z_2$
 $(2+3j) + (1+5j)$
 $3 + 8j$

ii) $Z_1 + Z_3$
 $(2+3j) + (3-j)$
 $2+3j-3+j$
 $-1+4j$

iii) Z_1/Z_2
 $(2+3j)/(1+5j)$

Conjugate

$$\frac{2+3j}{1+5j} \times \frac{1-5j}{1-5j}$$

$$\frac{2+10j+3j-15j^2}{1+5j+8j-1}$$

$$\frac{2+10j+3j-14}{1-1}$$

~~$2+10j+3j-14$~~

$$\frac{2+13j-14}{2}$$

$$\frac{-12+13j}{2}$$

$$\frac{-12}{2} + \frac{13j}{2}$$

$$b) \quad \underset{a}{5}x^2 + \underset{b}{3}x + \underset{c}{12} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(3) \pm \sqrt{3^2 - 4(5)(12)}}{2(5)}$$

$$x = \frac{3 \pm \sqrt{9 - 24}}{10}$$

$$x = \frac{3 \pm \sqrt{249}}{10}$$

$$x = \frac{3 \pm 15.77}{10}$$

$$x = \frac{3 - 15.77}{10}$$

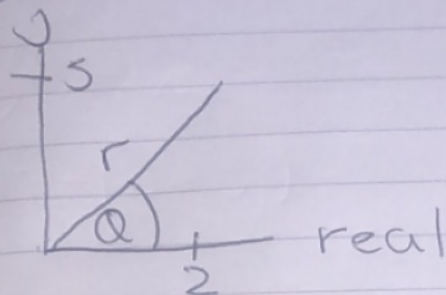
$$x = \frac{3 + 15.77}{10}$$

$$x = \frac{-12.77}{10}$$

$$x = \frac{18.77}{10}$$

$$c) (2 + 5j)^6$$

$$\text{theorem} = r^n (\cos Q + j \sin Q)$$



$$r^2 = 2^2 + 5^2$$

$$r = \sqrt{4 + 25}$$

$$r = \sqrt{29}$$

$$r = 5.38$$

$$\tan Q = \frac{5}{2}$$

$$Q = \tan^{-1} \left(\frac{5}{2} \right)$$

$$Q = 68.1985$$

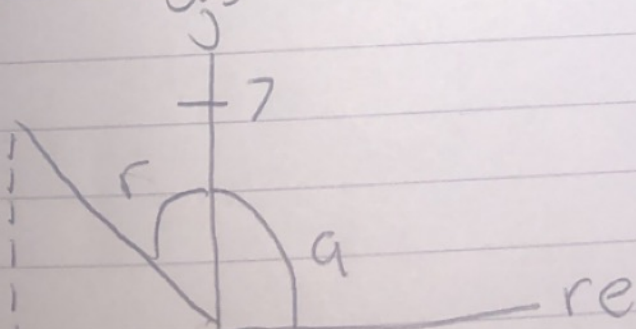
Put into equation

Polar Form

$$5.38^6 (\cos 6 \times 68.1985 + j \sin 6 \times 68.1985)$$

$$24248.93 (\cos 409.191 + j \sin 409.191)$$

$$ii) (-2 + 7j)^4$$



$$r^2 = 2^2 + 7^2$$

$$r^2 = \sqrt{4 + 49}$$

$$r^2 = 53$$

$$r = \sqrt{53}$$

$$r = 7.2801$$

$$\theta = \tan^{-1} \frac{7}{2}$$

$$\theta = \tan^{-1}(7/2)$$

$$\theta = 74.05^\circ$$

$$\theta = 180 - 74.05$$

$$\theta = 105.95^\circ$$

Put into equation

$$7.2801^4 (\cos 4 \times 105.95 + j \sin 4 \times 105.95)$$

$$2808.98 (\cos 423.8 + j \sin 423.8)$$

$$d) j^2 = -1$$

$$e) Z_1 Z_2 \bar{Z}_3$$

$$(5 + 4j) \times (1 + 5j) \times (2 + 3j)$$

$$(5 + 25j + 4j + 20j^2) (2 + 3j)$$

$$(5 + 48j) (2 + 3j)$$

$$10 + 15j + 96j + 144j^2$$

$$(10 + 254j)$$

$$f)$$

Question 3 - Integration

a) $\int (x^2 - x)(x^3 + 6x) dx$

$$\int x^5 + 6x^3 - x^4 + 6x^3 - 6x^2 dx$$

$$\frac{x^3(5x^3 + 6x^2 + 45x - 60)}{30} + C$$

ii) $\int (e^x + 5)(e^x + 6) dx$

$$\int e^{2x} + 6e^x + 5e^x + 30 dx$$

$$\hookrightarrow e^{2x} + 11e^x + 30x + C$$

iii) $\int_0^3 (x+6)(x-1) dx$

$$x^2 - x + 6x - 6$$

$$\int x^2 + 5x - 6$$

$$\hookrightarrow \frac{x^3}{3} + \frac{5}{2}x^2 - 6x + 6$$

Let $x=3$

$$\frac{(3)^3}{3} + \frac{5(3)^2}{2} - 6(3) = 13.5$$

iv) $\int_1^5 (2x+7)(x+3) dx$
 $= 2 \int x^2 dx + 13 \int x dx + 21 \int 1 dx$

$$\int x^2 dx$$

$$= \frac{x^3}{3}$$

$$\int 1 dx = x$$

$$b) \int 2x \cos x \, dx$$

Integrate by parts

$$= x \sin(x) - \int \sin(x) \, dx$$

$$= x \sin(x) + 2 \cos(x) + C$$

$$ii) \int_1^2 5x e^x \, dx$$

remove the constant

$$= 5 \int_1^2 x e^x \, dx$$

integration by parts

$$= 5 \left[e^x x - \int e^x \, dx \right]_1^2$$

$$= 5e^2$$

c)

$$iii) \int \frac{2x}{x^2} \, dx$$

$$= 2 \int 1/x \, dx$$

$$= 2 \ln x$$

$$= 2 \ln x + C$$

$$b) \int 2x \cos x \, dx$$

$$2 \int x \cos(x) \, dx$$

Integrate by parts

$$= \frac{x}{2} \sin(x) - \int \sin(x) \, dx$$

$$= \frac{x}{2} \sin(x) + 2 \cos(x)$$

$$= 2x \sin(x) + 2 \cos(x) + C$$

$$ii) \int_1^2 5x e^x \, dx$$

remove the constant

$$= 5 \int_1^2 x e^x \, dx$$

integration by parts

$$= 5 \left[e^x x - \int e^x \, dx \right]_1^2$$

$$= 5e^2$$

c)

$$iii) \int \frac{2x}{x^2} \, dx$$

$$= 2 \int \frac{1}{x} \, dx$$

$$= 2 \ln x$$

$$= 2 \ln x + C$$

$$ii) \int \frac{4}{7x+10} \, dx$$

$$= 4 \int \frac{1}{7x+10} \, dx$$

$$= 4 \int \frac{1}{u} \, du$$

$$= 4 \cdot \frac{1}{7} \int \frac{1}{u} \, du$$

$$= \frac{4}{7} \ln 7x+10$$

$$= \frac{4}{7} \ln u$$

$$= \frac{4}{7} \ln 7x+10$$

$$d) \int 5 \sin(x) \cos(x) dx$$

$$= 5 \int \sin(x) \cos(x) dx$$

$$= 5 \frac{u^{1+1}}{1+1}$$

$$= 5 \frac{\sin^{1+1}(x)}{1+1} = \frac{5}{2} \sin^2(x)$$

$$= \frac{5}{2} \sin^2(x) + C$$

$$e) \int 3x^2 \sqrt{x^3} dx \quad u = x^3$$

$$= 3 \int \sqrt{u} du$$

$$= 3^{1/3} \frac{u^{1/2+1}}{1/2+1} = \frac{3^{1/3} (x^3)^{3/2}}{3/2} = \frac{2}{3} x^{9/2}$$

$$= \frac{2}{3} x^{9/2} + C$$