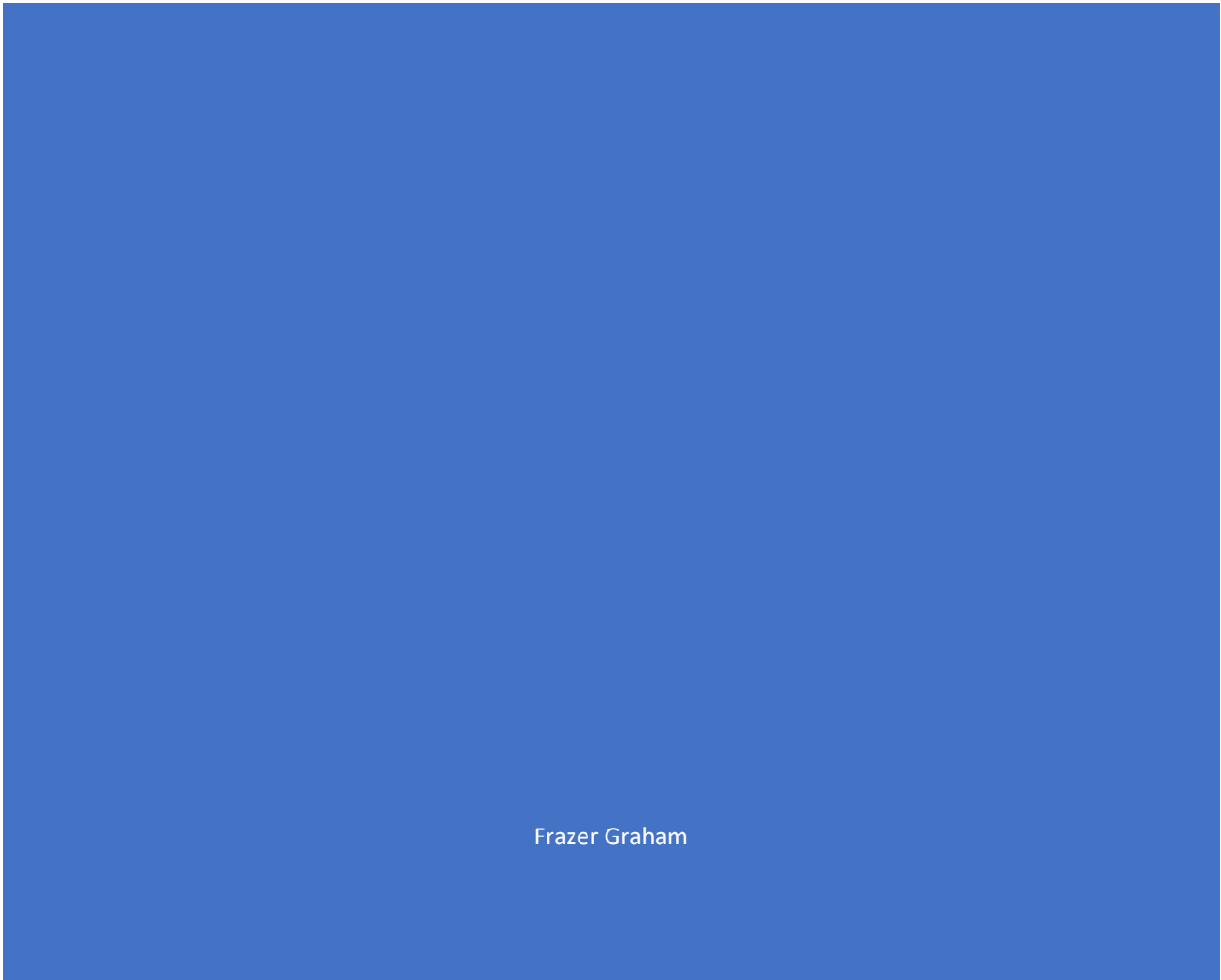




PHYSICS EXAM

2022 – 01 -13



Frazer Graham

Q1

(a)

$$\rho_1 U_1 = \rho_2 U_2$$

ρ_1 = Initial density
 U_1 = Initial velocity

ρ_2 = Density after shock
 U_2 = Velocity after shock

(1)

(b)

$$A = 1.45 \text{ km/s}$$

$$U_2 = 0.5 \text{ km/s}$$

$$P_1 = 0$$

$$B = 1.99$$

$$U_1 = 0$$

$$\rho_1 = 1000 \text{ kg/m}^3$$

$$D = a + bU$$

$$1.45 + (1.99 \times 0.5) = 2.445 \text{ km/s}$$

$$\rho_1 D = \rho_2 (D - U_2)$$

$$\frac{\rho_1 D}{(D - U_2)} = \rho_2$$

$$\frac{1000 \times 2.445}{(2.445 - 0.5)} = 1257.07 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_2 = 1257.07 \frac{\text{kg}}{\text{m}^3}$$

$$\frac{1}{\rho_2} = \frac{1}{1257.07} = 0$$

Spe

$$\text{Specific Volume} = \frac{1}{1257.07 \frac{\text{kg}}{\text{m}^3}}$$

unit to be on top

$$(P_2 - P_1) = \rho_1 D \cdot U_2$$

$$P_1 = 0$$

$$\therefore P_2 = \rho_1 D \cdot U_2$$

$$1000 \times 2.445 \times 0.5 = 1222.5 \text{ kPa}$$

$$\frac{\text{kg}}{\text{m}^3}$$

$$\frac{\text{km}}{\text{s}}$$

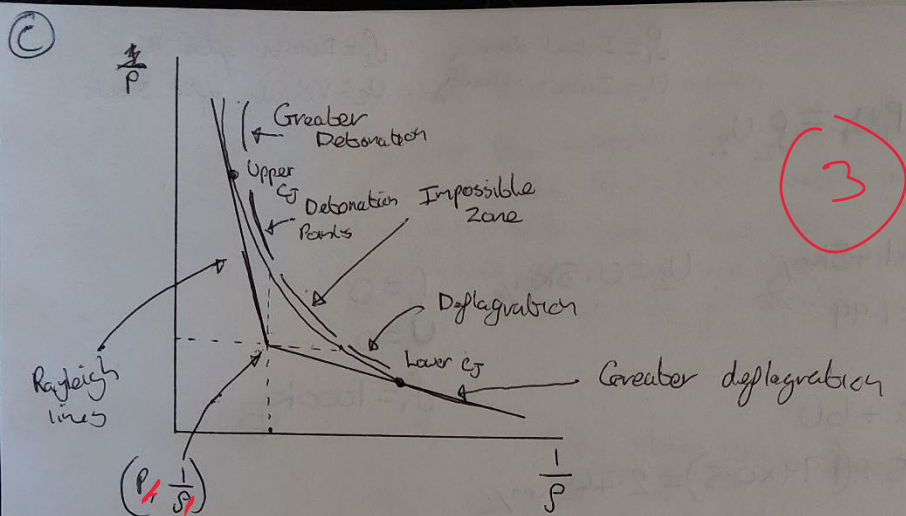
$$\frac{\text{km}}{\text{s}}$$

$$\frac{1}{\rho_a}$$

= Specific Pressure

$$\therefore \text{Specific pressure} = \frac{1}{1.22 \text{ MPa}}$$

not requested



④ Detonation is supersonic with velocities ranging ^{above} from 2000 m/s. As pressure increases from detonation this compresses the gas decreasing volume.

Deflagration is subsonic ($> 100 \text{ m/s}$) however as volume increases, the gas pressure decreases.

④

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Q2

(a) In a stress matrix there are 9 components

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

However the bottom 3 represent the top 3 reducing the stress/strain matrix to 6 values. With hooks law the 6 components of stress are linearly related to the 6 components of strain. (2)

(b) $E = 300 \text{ GPa}$ $\nu = 0.3$

Using stress-strain equation from formula sheet. (3)

$$\left[\frac{300 \times 10^9}{(1+0.3)(1-2 \times 0.3)} \right] \begin{bmatrix} 0.7 & 0.3 & 0.3 & 0 & 0 & 0 \\ 0.3 & 0.7 & 0.3 & 0 & 0 & 0 \\ 0.3 & 0.3 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.1 \\ 0.05 \\ 0.2 \\ 0.1 \\ 0.2 \end{bmatrix}$$

↓

$$576.92 \times 10^9$$

$$\begin{aligned} (576.92 \times 10^9) \times 0.7 &= 403.84 \times 10^9 \\ (576.92 \times 10^9) \times 0.3 &= 173.08 \times 10^9 \\ (576.92 \times 10^9) \times 0.4 &= 230.77 \times 10^9 \\ (576.92 \times 10^9) \times 0 &= 0 \end{aligned}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} 403.84 \times 10^9 & 173.08 \times 10^9 & 173.08 \times 10^9 & 0 & 0 & 0 \\ 173.08 \times 10^9 & 403.84 \times 10^9 & 173.08 \times 10^9 & 0 & 0 & 0 \\ 173.08 \times 10^9 & 173.08 \times 10^9 & 403.84 \times 10^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 230.77 \times 10^9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 230.77 \times 10^9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 230.77 \times 10^9 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.1 \\ 0.05 \\ 0.2 \\ 0.1 \\ 0.2 \end{bmatrix}$$

Strain Matrix would go here

6ii

$$\sigma_{xx} = ((403.84 \times 10^9) \times 0.05) + ((173.08 \times 10^9) \times 0.01) + ((173.08 \times 10^9) \times 0.05) + (0 \times 0.2) + (0 \times 0.1) + (0 \times 0.2)$$

$$\sigma_{xx} = 20.19 \times 10^9 + 1.73 \times 10^9 + 8.65 \times 10^9 + 0 + 0 + 0$$

$$\sigma_{xx} = 30.57 \text{ GPa}$$

6i Using the hydrostatic stress equation & the idea (5)
 $\frac{1}{3}(\sigma + \sigma + \sigma) = p$
in Q2 where values represent diagonally. By using the σ values this will provide hydrostatic stress.

Cii This is useful for materials like ~~polymers~~ ^{polymers or metals} which are anisotropic, having different values in each direction. (That has to be wrong else you cannot use the idea of reflecting diagonally. Therefore it must be for isotropic materials. However then there will be no difference.)

Final answer. Technique is useful for anisotropic which will help find deviation in stresses. (5)

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Q3)

(a) Ideal gas law
 $PV = nRT$

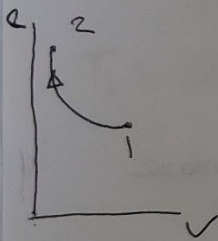
$$\begin{aligned} V &= 0.4 \text{ m}^3 \\ P &= 2.5 \times 10^6 \text{ Pa} \\ R &= 8.31 \text{ J/K mol} \\ T &= 40 + 273 = 313 \text{ K} \end{aligned}$$

$$\frac{PV}{RT} = n \quad \frac{(2.5 \times 10^6) \times 0.4}{8.31 \times 313} = 384.46$$

$$n = 384.46 \text{ mol}$$

2

(bi) Ideal gas Isothermal $\Delta T = 0$



$$\textcircled{1} \begin{aligned} P &= 5 \text{ MPa} \\ V &= 0.2 \text{ m}^3 \end{aligned}$$

$$\textcircled{2} \begin{aligned} P &= ? \\ V &= 0.15 \text{ m}^3 \end{aligned}$$

$$PV = C$$

$$P_1 V_1 = P_2 V_2$$

$$\frac{P_1 V_1}{V_2} = P_2$$

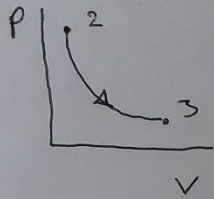
$$\frac{(5 \times 10^6) \times 0.2}{0.15} = 6.67 \text{ MPa}$$

3

$$P_2 = 6.67 \text{ MPa}$$

Volume decreases $\therefore$ Pressure increases
 $P_2 > P_1$

(biii) Adiabatic $\therefore \Delta Q = 0$



~~(3)~~ (3)

$$V = 0.2 \text{ m}^3 \\ P = ?$$

(2)

$$P = 6.67 \text{ MPa} \\ V = 0.15 \text{ m}^3$$

$$C_p = 919.81 \frac{\text{J}}{\text{kg K}} \quad C_v = 657 \frac{\text{J}}{\text{kg K}}$$

$$\gamma = \frac{C_p}{C_v} = \frac{919.81}{657} = 1.4$$

$$PV^\gamma = C \quad P_2 V_2^\gamma = P_3 V_3^\gamma \quad P_2 \left(\frac{V_2}{V_3} \right)^\gamma = P_3$$

$$(6.67 \times 10^6) \left(\frac{0.15}{0.2} \right)^{1.4} = 4.46 \text{ MPa}$$

(4)

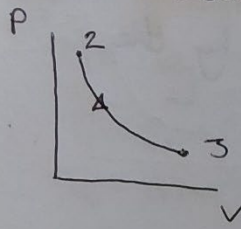
$$P_3 = 4.46 \text{ MPa}$$

Volume Increase $\therefore P$ decrease

$$~~P_2 < P_3~~ \quad P_2 > P_3$$

(biii)

Adiabatic



③

$$V = 0.2 \text{ m}^3$$

$$T = ?$$

$$\gamma = 1.4$$

②

$$V = 0.15 \text{ m}^3$$

$$T = 300 \text{ K}$$

$$\frac{T_3}{T_2} = \left(\frac{V_2}{V_3} \right)^{\gamma-1}$$

$$T_3 = \left(\frac{V_2}{V_3} \right)^{\gamma-1} \times T_2$$

$$300 \left(\frac{0.15}{0.2} \right)^{1.4-1} = 267.39 \text{ K}$$

$$T_3 = 267.39 \text{ K}$$

5

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Q4)

(a) $\Delta Q = \Delta U + \Delta W$ $\longleftrightarrow$ Work done by the system

Heat energy supplied to the system $\nearrow$ $\nwarrow$ change in internal energy.

3

(b) Adiabatic $\therefore \Delta Q = 0$

$M = 10g = 0.01$

$\Delta W = 250J$

$C = 500 J/kg K$

$\Delta Q = \Delta U + \Delta W$

~~$0 = \Delta U + 250$~~ $0 = \Delta U - 250$

~~$\Delta U = 250J$~~ $\Delta U = 250J$

~~$\Delta U = MC\Delta T$~~

$\frac{\Delta U}{MC} = \Delta T$

~~$250 = 0.01 \times 500 \times \Delta T$~~

~~$\Delta T = 50K$~~

$\frac{250}{0.01 \times 500} = 50K$

$\Delta T = 50K$

3

(c) $\Delta Q = 15J$
 $\Delta W = 7J$

$\Delta Q = \Delta U + \Delta W$

$15 = \Delta U + 7$

$15 - 7 = 8J$

$\Delta U = 8J$

3

Q4

①

$$M = 7 \text{ kg}$$

$$T_1 = 20 + 273 = 293 \text{ K}$$

$$T_2 = 80 + 273 = 353 \text{ K}$$

$$C = 4.2 \times 10^3 \text{ J/kg K}$$

$$\Delta S = MC \int_{T_1}^{T_2} \frac{\Delta Q}{T}$$

$$\Delta S = MC \ln\left(\frac{353}{293}\right)$$

$$7 \times (4.2 \times 10^3) \times \ln\left(\frac{353}{293}\right) = 5477.09 \frac{\text{J}}{\text{K}}$$

↑
net kJ

6

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