

Wilcoxon Signed-Ranks Test for Paired Samples

Wilcoxon Signed-Ranks Test is a non-parametric test and is the non-parametric alternative to a paired samples t-test. When the requirements for the paired samples t-test are not satisfied, the Wilcoxon signed-rank tests for paired samples can often be used.

When to use a Wilcoxon Signed-Ranks Test:

- When data are not normally distributed
- When data are ranks (ordinal) instead of interval/ratio

Assumptions:

- Data are paired and members of pairs are from the same population

Underlying process:

- Similar to the paired samples t-test, but with ranks
- Difference scores equals to zero (0) are omitted from ranking and from analysis
- Ranks are assigned by ascending order of **absolute values**

Calculating Wilcoxon Signed-Ranks Test:

1. Compute the difference scores for each participant
2. Order the difference scores (ignoring the sign) from lowest to highest
3. Assign ranks to absolute values of difference scores
4. Attach the signs of observed differences to each rank
5. Calculate W^+ and W^-
6. Determine which is the smaller value
7. Compare against the Table of critical values of W
8. Indicate significance and acceptance or rejection of null hypothesis

Determining significance

The test statistic for the Wilcoxon Signed Rank Test is W , which is defined as the smaller of W^+ (the sum of all positive ranks) and W^- (the sum of all negative ranks).

If the **null hypothesis** is true, we expect to see more higher and positive ranks.

The sum of the ranks (ignoring the signs) will always equal $n(n+1)/2$.

When we calculate W^+ and W^- , we determine which is smaller and compare this **critical value** of W against the *Table of Critical Values of W* . If our W statistic is smaller than the Critical value, then we can reject the null hypothesis (and therefore imply that there is a significant change occurring).

Example to illustrate the process of Wilcoxon Signed-Ranks Test:

Let's assume that a researcher is looking to explore the effectiveness of a new drug that has been designed to reduce repetitive behaviours in children affected with autism. A total of 10 children with autism take part and the amount of time that each child engages in repetitive behaviour during a three-hour observation period is noted both before treatment and then again after taking the new medication for a period of one week. The data are shown below:

Participant	Before treatment	After 1 week of treatment
1	85	75
2	70	50
3	40	50
4	65	40
5	80	20
6	75	65
7	55	40
8	20	25
9	45	15
10	80	55

1. Compute the difference scores for each child

Participant	Before treatment	After 1 week of treatment	Difference (Before – After)
1	85	75	10
2	70	50	20
3	40	55	-15
4	65	40	25
5	80	20	60
6	75	65	10
7	55	40	15
8	20	25	-5
9	45	15	30
10	80	55	25

2. Order the difference scores (ignoring the sign) from lowest to highest

Difference (Before – After)	Ordered scores (absolute values)
10	-5
20	10
-15	10
25	-15
60	15
10	20
15	25
-5	25
30	30
25	60

3. Assign ranks to absolute values of difference scores

NB: where the values are the same, you assign the mean rank.

Difference (Before – After)	Ordered scores (absolute values)	Ranks
10	-5	1
20	10	2.5
-15	10	2.5
25	-15	4.5
60	15	4.5
10	20	6
15	25	7.5
-5	25	7.5
30	30	9
25	60	10

4. Attach the signs of observed differences to each rank

Difference (Before – After)	Ordered scores (absolute values)	Ranks	Signed Ranks
10	-5	1	-1
20	10	2.5	2.5
-15	10	2.5	2.5
25	-15	4.5	-4.5
60	15	4.5	4.5
10	20	6	6
15	25	7.5	7.5
-5	25	7.5	7.5
30	30	9	9
25	60	10	10

5. Calculate W+ and W-

W+	W-
2.5	-1
2.5	-4.5
4.5	
6	
7.5	
7.5	
9	
10	
Total = 49.5	Total = 5.5

(remember we can check here that the sum of the ranks is equal to $n(n+1)/2$)

$$10(11)/2 = 55$$

$$49.5 + 5.5 = 55$$

6. Determine which is the smaller value

W- is the smaller value (5.5) therefore $W = 5.5$

7. Compare against the Table of critical values of W

$N = 10$, $\alpha = .05$

Critical value = 8.

Critical Values of the Wilcoxon Signed Ranks Test

Our calculated $W = 5.5$ which is smaller than the critical value here (8) therefore we can determine that there is a **significant change occurring**.

n	Two-Tailed Test		One-Tailed Test	
	$\alpha = .05$	$\alpha = .01$	$\alpha = .05$	$\alpha = .01$
5	--	--	0	--
6	0	--	2	--
7	2	--	3	0
8	3	0	5	1
9	5	1	8	3
10	8	3	10	5
11	10	5	13	7
12	13	7	17	9
13	17	9	21	12
14	21	12	25	15

8. Indicate significance and acceptance or rejection of null hypothesis

The null hypothesis can be rejected; there is a significant change occurring in the behavioural symptoms of the children following the treatment; the beh