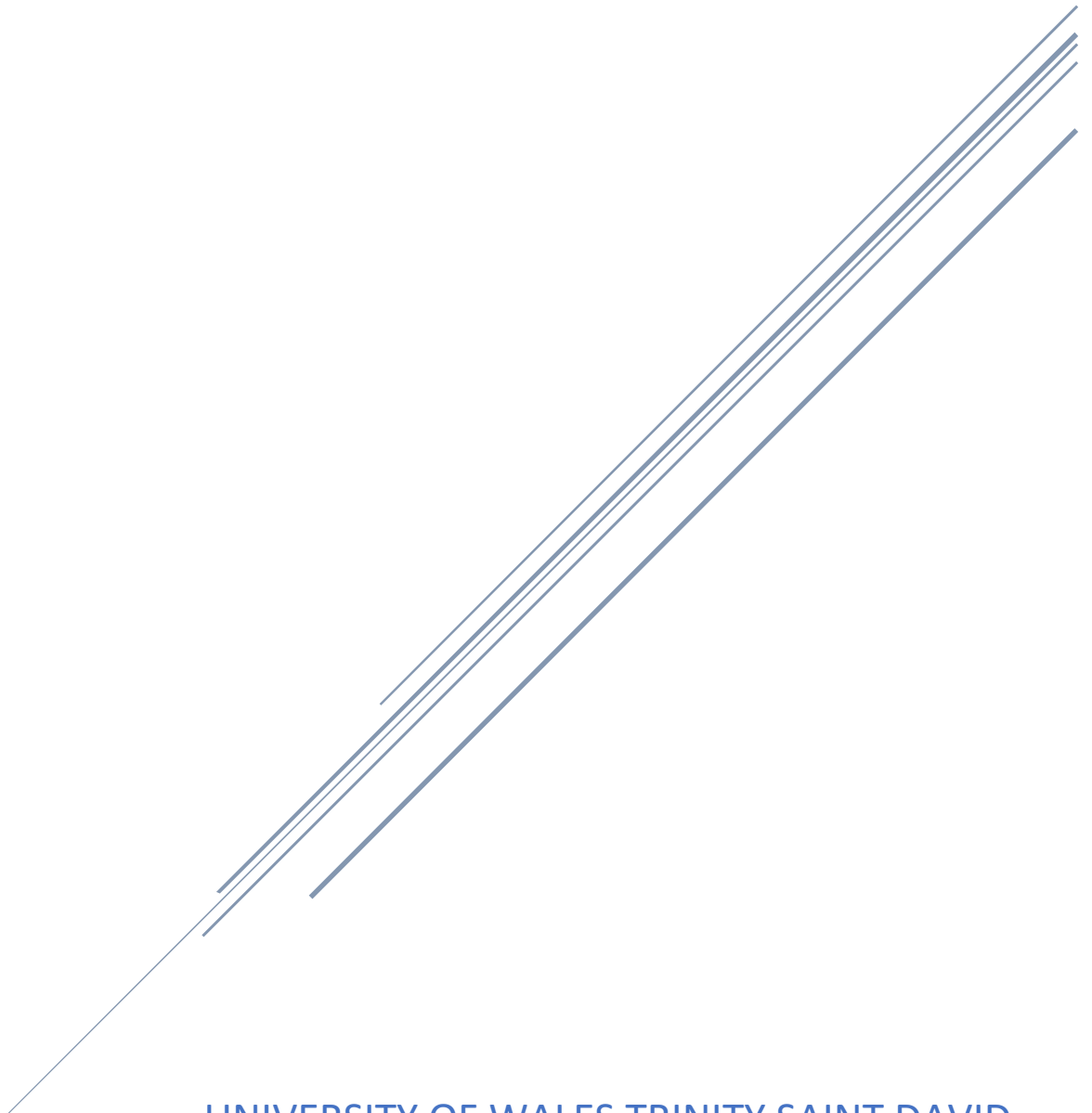


AC RESPONSE OF AN RLC CIRCUIT

ASSIGNMENT 1

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1 ABSTRACT

This document is an exploration into power factor correction and how it can be applied to systems to improve their efficiency.

A software package called Proteus is used to simulate an example circuit and to show how power factor correction can be applied to it to improve its power factor to ≥ 0.95 by adding a capacitor of a specific value in parallel to the resistive and inductive load. Calculations for selecting the correct value capacitor are shown.

A detailed analysis of how the PFC capacitor values change the power factor and an analysis of resonant frequency and how different frequencies affect the current and voltages at the load are explored using phasor diagrams included in this report.

2 INTRODUCTION

2.1 POWER FACTOR

Power factor is a value that shows the ratio of active and apparent power. Active power (P) is the energy used by the resistive elements in the circuit and apparent power (Q) is the energy used by the reactive elements such as inductors and capacitors. The total energy seen by the load is the Apparent power (S). The active power is the useful part of the system that does the work required (moving a motor for example) and the reactive part is undesirable.

The power factor can also be used to determine the phase difference between the supply voltage and supply current [1]. It ranges from 0 – 1. A power factor of 1 shows that the system is in unity and that the current and voltages are in phase with each other. When a system is in unity, there is no resultant reactance as the inductive and capacitive reactance negate each other's effect.

$$X_c = X_l$$

A low power factor indicates that the voltage and current have a considerable phase difference caused by an imbalance in the reactive parts of the circuit.

In industry it is possible that a motor could have a low undesirable power factor. This would be caused by the inductive elements in the motor, that would add a reactance to the circuit. The low power factor caused by an inductive load causes the current to lag the voltage. To bring the current back in phase, power factor correction can be applied.

2.2 POWER FACTOR CORRECTION

Power factor correction is the adjustment of the reactive elements of the circuit to improve the power factor. In the example of a motor, the reactance of the circuit is inductive, and so a capacitive reactance can be applied by connecting a capacitor in parallel. The effect of this capacitor is to reduce the overall reactance of the circuit and improve efficiency by reducing reactive power.

The circuit examined in the next section could be used to represent a motor as there are both resistive and inductive elements. Power factor correction is applied to the circuit with all calculations shown.

It can be useful to represent the power factor in a system in a power triangle, shown in figure 1. This is useful because if two of the sides of the power triangle are known then the third side can be calculated. The phase angle can also be calculated using basic trigonometry.

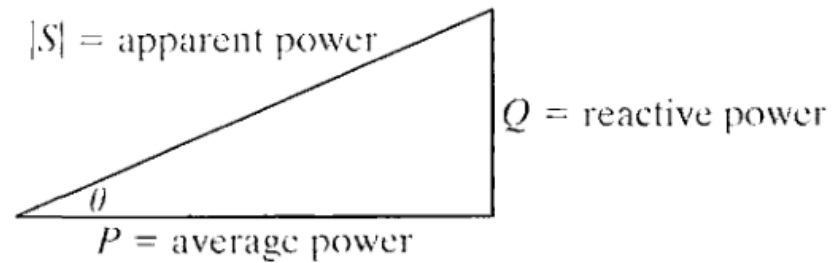


Figure 1 - Power Triangle [2]

The power can be represented in a complex form:

$$S = P + jQ$$

In this form the S is the sum of both the real and imaginary power components. By reducing the jQ value the apparent power in a system can be reduced. This is the purpose of power factor correction.

3 METHOD

3.1 SIMULATION ANALYSIS 1

This section will breakdown the analysis of the circuit in figure 2 and show all calculations required to determine the capacitor size that will be required to meet the PFC specification (≥ 0.95). The circuit is comprised of the following components:

- R1 $2.1\ \Omega$
- R2 $85\ \Omega$
- L1 800mH
- AC Supply (400V RMS, 50Hz)

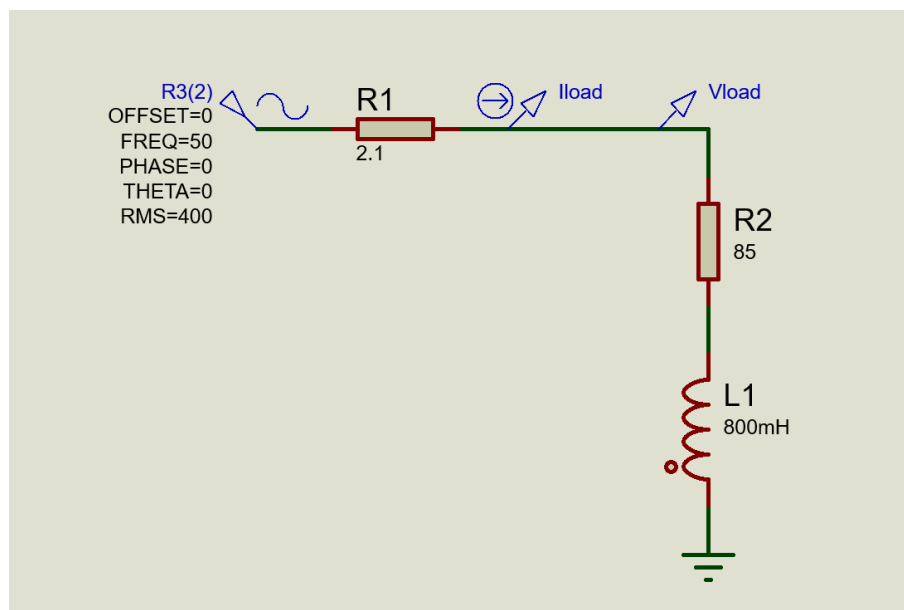


Figure 2 - RL Circuit Schematic in Proteus before PFC

The circuit in figure 1 was simulated in Proteus and the load voltage (green) and load current (red) were plotted over time. It is clear from figure 2 that the voltage and current are out of phase.

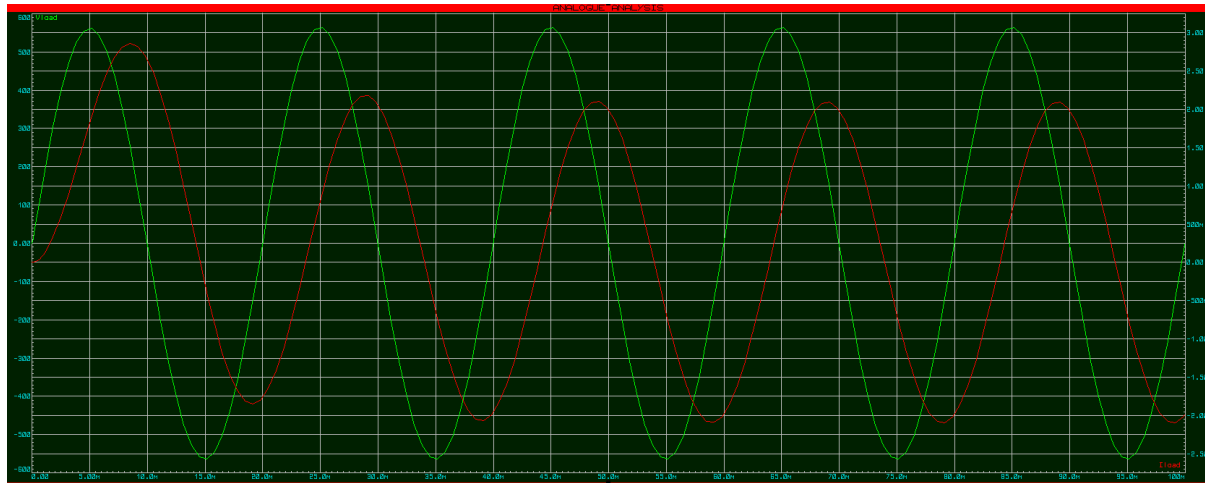


Figure 3 - RL Circuit, I and V Plot before PFC

The phase difference can be calculated using the following equation:

$$\phi = 360^\circ * f * \Delta t [3]$$

Where ϕ is the phase difference in degrees

f is the frequency of the waveform

Δt is the time difference between the two waveforms

The third peak times of the Vload and Iload were taken from the graph to determine the phase difference. The third peak was taken as the waveform seemed to stabilise after the first cycle.

$$V_{load\ peak} = 45ms$$

$$I_{load\ peak} = 49ms$$

$$\therefore \Delta t = 49ms - 45ms$$

$$\Delta t = 4ms$$

$$\phi = 360^\circ * 50Hz * 4ms$$

$$\phi = 72^\circ$$

$$PF = \cos(\phi)$$

$$PF = \cos(72)$$

$$PF = 0.309$$

3.2 POWER FACTOR CAPACITOR CALCULATIONS

The next step in the analysis is to identify the apparent, active and reactive power of the circuit. The following equation can be used to calculate apparent power (S):

$$S = I^2 * Z$$

Where I is the supply current

Z is the total impedance of the circuit.

Currently, both variables in the equation are unknown. However they can be calculated using the following equations:

$$Z = R + jX$$

Where R is the total resistance of the circuit

jX is the total reactance of the circuit.

There are two resistances in series $\therefore$

$$R = R1 + R2$$

$$R = 2.1\Omega + 85\Omega$$

$$R = 87.1\Omega$$

The reactance of the circuit made up of only inductive elements $\therefore$

$$jX = Xl$$

$$Xl = 2 * \pi * f * L$$

$$Xl = 2 * \pi * 50Hz * 800mH$$

$$Xl = 251.327 \Omega$$

$$\therefore$$

$$Z = 87.1\Omega + j251.327\Omega$$

The load current can be calculated using the formula:

$$I_{supply} = \frac{V}{Z}$$

Where I supply is the supply current

V is the supply voltage

Z is the total impedance of the circuit.

$$I_{supply} = \frac{400}{87.1\Omega + j251.327\Omega}$$

$$I_{supply} = 0.492425090366915 - 1.42089464646612i \text{ A}$$

$\therefore$

$$S = (0.492425090366915 - 1.42089464646612i)^2 * (87.1\Omega + j251.327\Omega)$$

$$S = 196.970036146766 - 568.357858586448i$$

The apparent power can be taken from the absolute value of the complex value above.

$$S = 601.5212802VA$$

The Reactive power (Q) is the absolute imaginary component of S

$$Q = 568.3578586Var$$

The Active power (P) is the real component of S .

$$P = 196.9700361W$$

The power factor can be calculated using the following equation:

$$PF = \frac{P}{S}$$

$$PF = \frac{196.9700361W}{601.5212802VA}$$

$$PF = 0.327453147$$

The power factor shows that the system is below the target specification of 0.95. Therefore, power factor correction must be done on the circuit by adding a capacitor in parallel with the RL part of the circuit.

The target phase angle must be identified first. This can be calculated from the following equation:

$$\phi = \cos^{-1}(PF)$$

Where ϕ is the target phase angle

PF is the target power factor.

$$\phi = \cos^{-1}(0.95)$$

$$\phi = 18.195^\circ$$

To calculate the new supply, current the following equation can be used:

$$I_{supply} = \frac{I_{active}}{\cos(\phi)}$$

Where I_{supply} is the new supply current,

I_{active} is the current through active elements in the circuit and $\cos(\phi)$ is the new phase angle.

This equation can be used because the active current will not change from the original as the resistances remain the same.

$$I_{\text{reactive}} = I_{\text{active}} * \sin(18.195^\circ)$$

Where I_{reactive} is the new current through the reactive part of the circuit at the specified power factor.

$\therefore$

$$I_{\text{supply}} = \frac{0.492425090366915\text{A}}{\cos(18.195^\circ)}$$

$$I_{\text{supply}} = 0.516\text{A}$$

$$I_{\text{reactive}} = 0.492425090366915\text{A} * \sin(18.195^\circ)$$

$$I_{\text{reactive}} = 0.162\text{A}$$

To calculate the new capacitor current:

$$I_c = I_{\text{supply}} - I_{lr}$$

Where I_c is the capacitor current,

I_{supply} is the new supply current,

And I_{lr} is the current through the inductor and resistor.

$$I_c = \text{Old } I_{\text{supply}} - I_{\text{reactive}}$$

$$I_c = 1.42089464646612\text{A} - 0.162\text{A}$$

$$I_c = 1.259\text{A}$$

Capacitor Reactance can be calculated using:

$$X_c = \frac{V_{\text{load}}}{I_c}$$

V_{load} does not equal 400V as there is a voltage drop across R1.

$$V_{\text{load}} = I_{\text{supply}} * V_{RLC}$$

$$Z_{\text{total}} = \frac{V_{\text{supply}}}{I_{\text{supply}}}$$

$$Z_{RLC} = Z_{total} - R1$$

$$Z_{total} = \frac{400V}{0.516A}$$

$$Z_{total} = 775.194\Omega$$

$$Z_{RLC} = 775.194\Omega - 2.1\Omega$$

$$Z_{RLC} = 773.094\Omega$$

$$V_{load} = 0.516A * 773.094\Omega$$

$$V_{load} = 398.9164\Omega$$

$$X_c = \frac{398.9164\Omega}{1.259A}$$

$$X_c = 316.852\Omega$$

To calculate the capacitor value, the following equation can be used:

$$C = \frac{1}{2 * \pi * f * X_c}$$

$$C = \frac{1}{2 * \pi * 50 * 316.852\Omega}$$

$$C = 10\mu F$$

The power factor capacitor value is $10\mu F$.

3.3 SIMULATION ANALYSIS 2

The circuit with a PFC capacitor can be seen in figure 4.

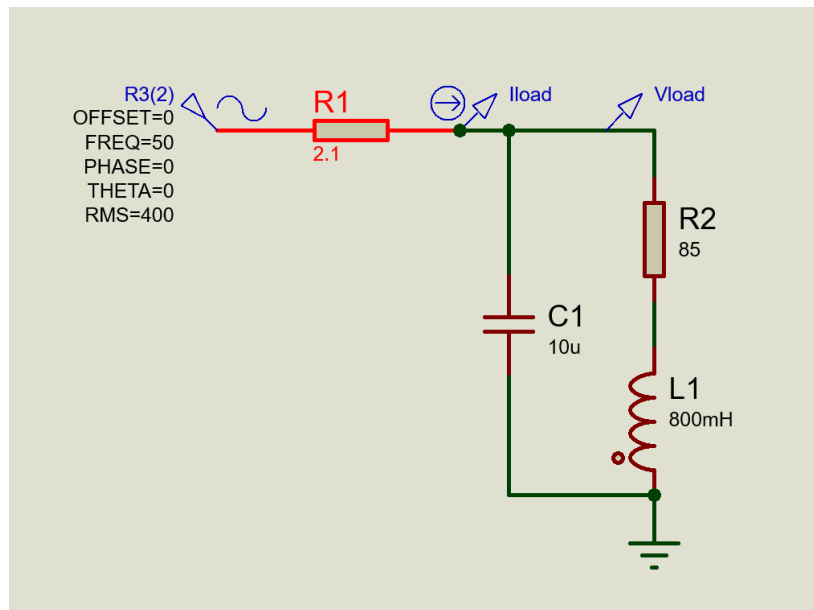


Figure 4 - RL Circuit Schematic in Proteus after PFC

It is clear from figure 5 that the current plot has moved more in phase that the one in figure 3.

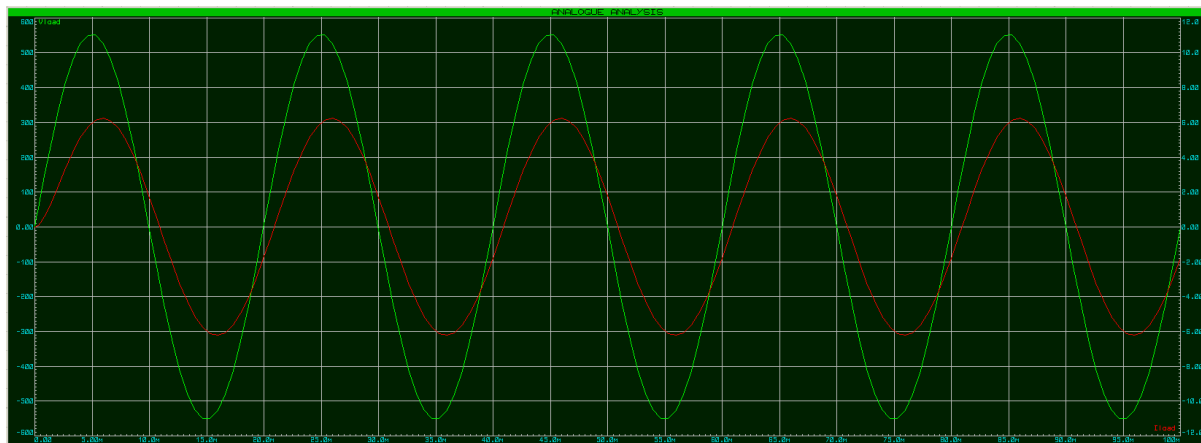


Figure 5 - RL Circuit, I and V Plot after PFC

Using the same calculation as previously shown, the phase angle and power factor can be calculated from the simulation results.

$$V_{load\ peak} = 45ms$$

$$I_{load\ peak} = 46ms$$

$$\phi = 18$$

$$PF = \cos(18)$$

$$PF = 0.951$$

The power factor has improved to 0.951 with the capacitor in parallel. This shows that the PFC was successful as the PF is within the ≥ 0.95 specification.

4 RESULTS

4.1 POWER FACTOR AGAINST CAPACITOR VALUE

This section of the report will show how the power factor is affected by changing the capacitor value. To graph this the following calculations were done for capacitor value ranging from 1 – 50uF.

Firstly, the capacitive reactance must be calculated using:

$$X_c = \frac{1}{2 * \pi * f * C}$$

Then the total impedance of C1, R2 and L1 is calculated:

$$Z_{LRC} = \frac{Z_{LR} * X_c}{Z_{LR} + X_c}$$

Where Z LR is the impedance of L1 and R2, and can be represented in the form:

$$Z_{LR} = R + X_l j$$

Where Xl is the inductive reactance and is calculated using:

$$X_l = 2 * \pi * f * L$$

The Vload must also be calculated using:

$$Z_{total} = Z_{LRC} + R_1$$

$$I_{supply} = \frac{V_{supply}}{Z_{total}}$$

$$V_{load} = \frac{I_{supply}}{Z_{LRC}}$$

The currents I supply, I RL and IC must be calculated:

$$I_{RL} = \frac{V_{load}}{Z_{RL}}$$

$$I_C = \frac{V_{load}}{X_c}$$

The phase angle is calculated using the imaginary part of the I supply.

Power factor is calculated using:

$$PF = \cos(\phi)$$

The graph in figure 5 shows how adjusting the capacitor value will affect the power factor for the circuit shown in figure 3. The peak of the graph in figure 5 shows the value required to achieve unity in the system.

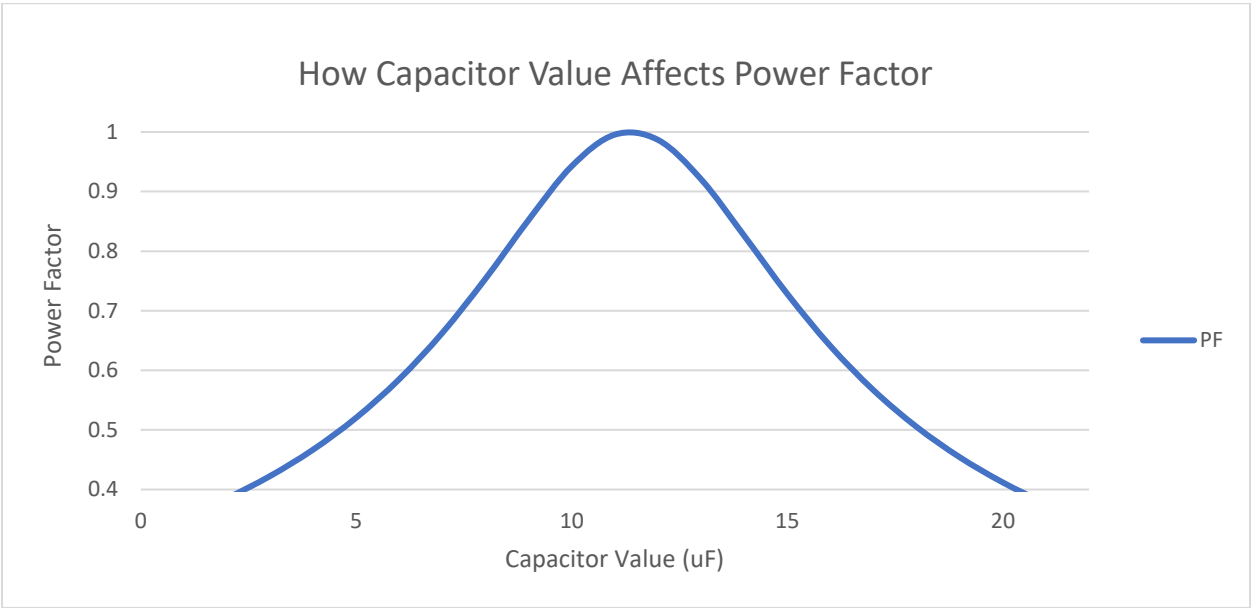


Figure 6 - How Capacitor Value Affects Power Factor

Figure 6 shows the same graph as in figure 5 however the active and reactive currents are also plotted to show how they are affected by the current value.

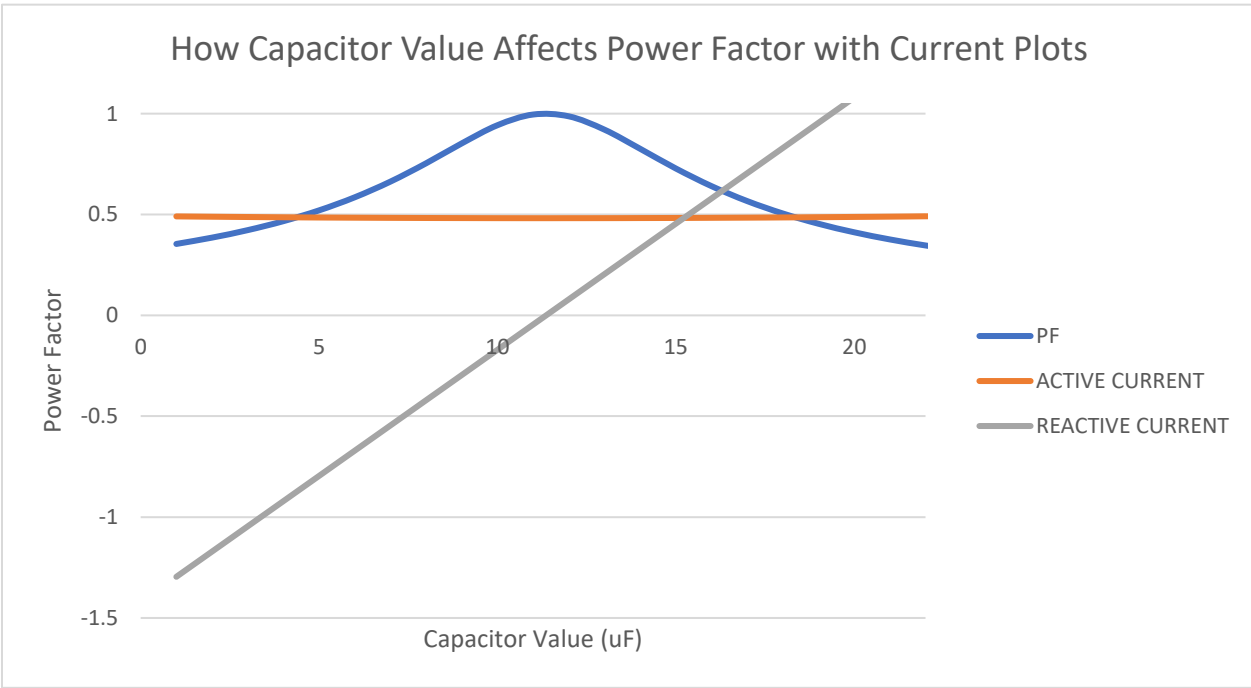


Figure 7 - How Capacitor Value Affects Power Factor with Current Plots

4.2 RESONANT FREQUENCY

Resonant frequency can be calculated using the following equation:

$$\omega = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Where ω is the resonant frequency,

L is the inductor value,

C is the capacitor value,

R is the resistor value.

Resonant frequency was calculated to be 53.66872176Hz.

4.3 PHASOR DIAGRAMS

Phasor diagrams have been created at the resonant frequency, resonance +25Hz, resonance -25Hz and 50Hz. The values plotted are IC, IRL, I supply, V supply and V load. Equations for these values can be seen previously.

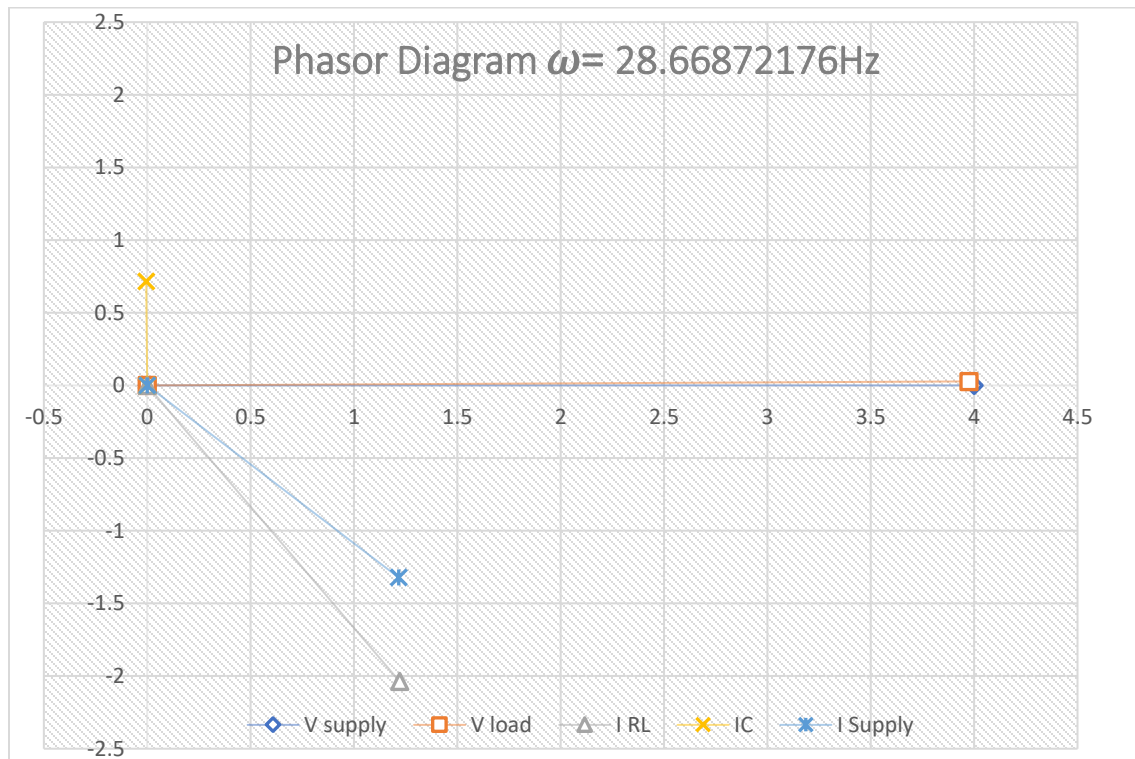


Figure 8 - Phasor Diagram $\omega = 28.66872176\text{Hz}$

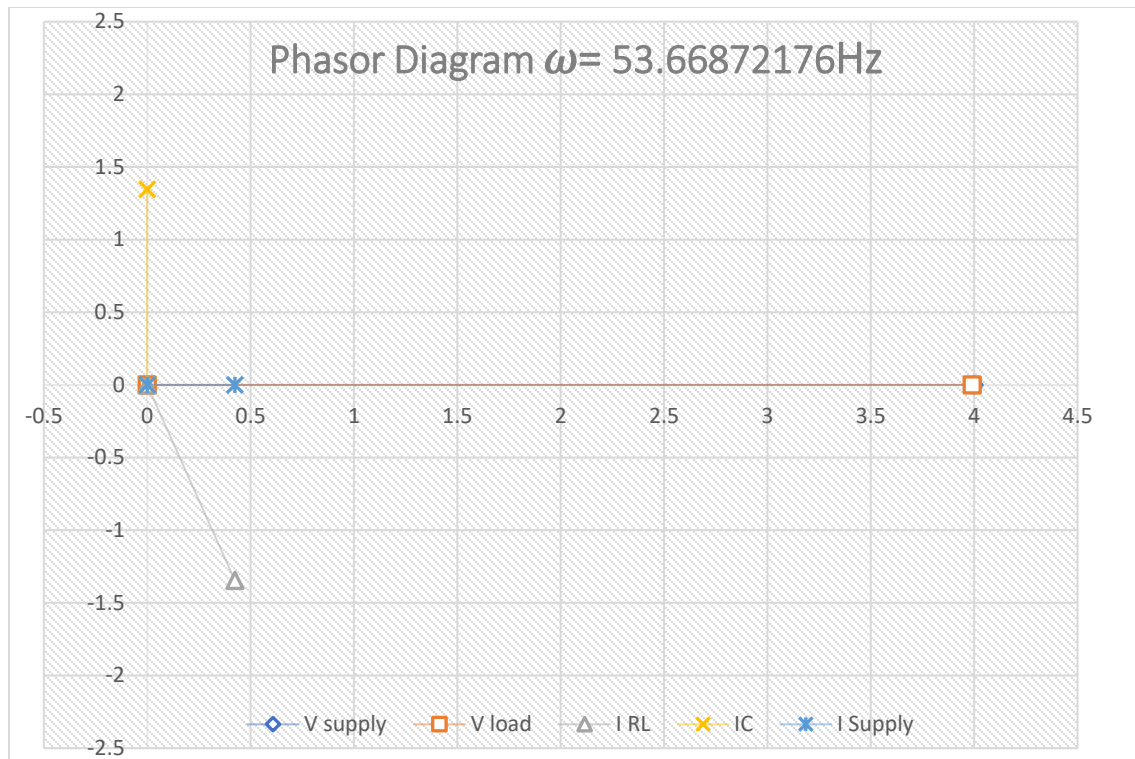


Figure 9 - Phasor Diagram $\omega = 53.66872176\text{Hz}$

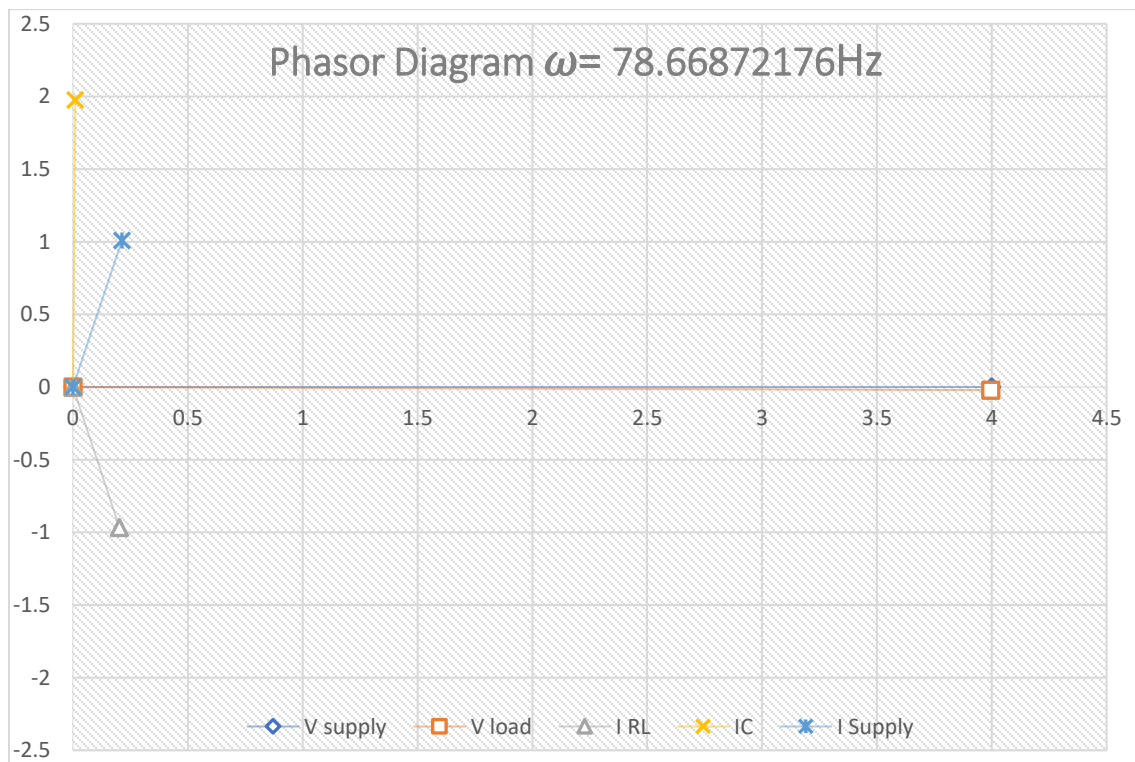


Figure 10 - Phasor Diagram $\omega = 78.66872176\text{Hz}$

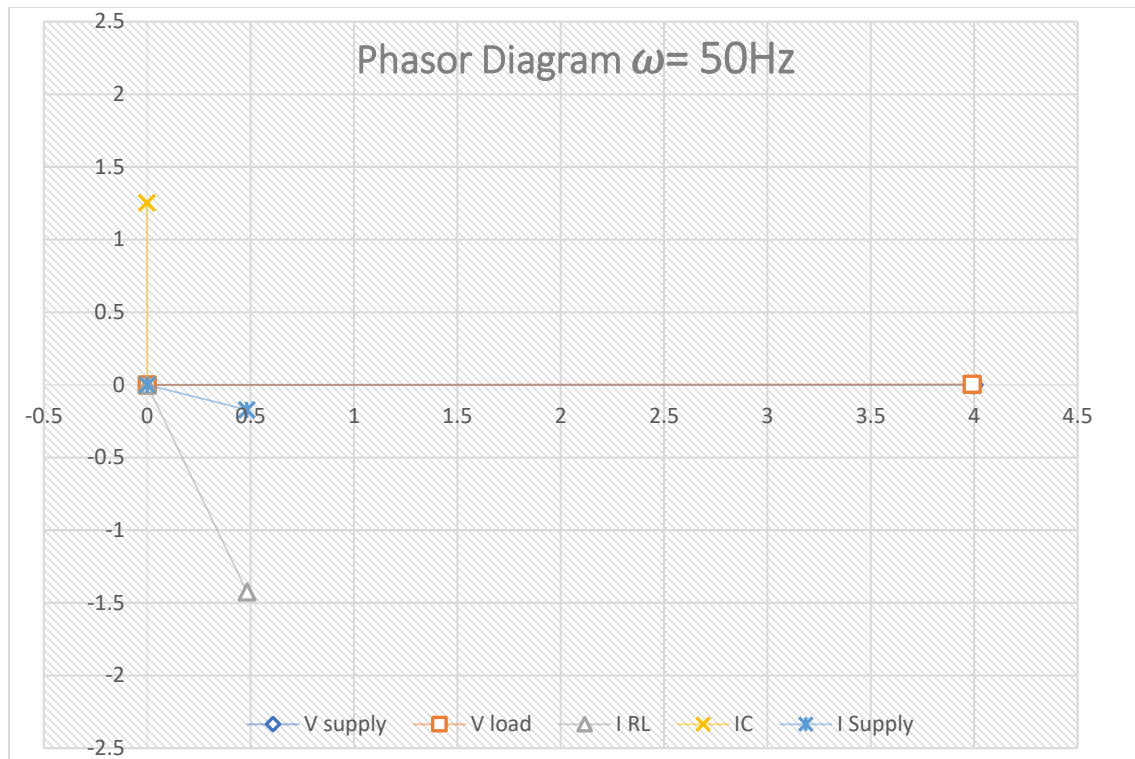


Figure 11 - Phasor Diagram $\omega = 50\text{Hz}$

5 DISCUSSION

The results of the phasor diagrams show that the frequency can be varied to adjust the phase angle. This occurs because capacitive and inductive reactance is affected by frequency. Increasing the frequency will increase the capacitive reactance and decreases the inductive reactance. Decreasing frequency has the opposite effect. The default circuit frequency value of 50Hz after power factor correction has a phase angle that showed the current was lagging the voltage.

If the circuits reactance is mostly inductive then the voltage will lead the current. If the reactance is mostly capacitive then the current will lead voltage. This has been shown in the phasor diagrams in figures 8 – 11.

5.1 RESONANCE

Resonance occurs when the voltage and current are in phase. This occurs when there is no resultant reactive elements in the circuit and is achieved when:

$$X_c = X_l$$

This can be best seen in the phasor diagram in figure 9, where the load current is in phase with the voltage.

5.2 MAXIMUM POWER TRANSFER

Maximum power transfer occurs when the largest amount of power is transferred through the load. In an AC system with reactive elements, this occurs when the load impedance is equal to the complex conjugate of the internal impedance of the source. [4]

$$Z_{internal} = R_1 + j0$$

$$Z_{load\ MPT} = R_1 - j0$$

The equations above show the complex form for the internal impedance of the source and the conjugate that would be required for maximum power transfer. This is not achievable in the current circuit because the resistance of the load is fixed at 85Ω. This means that the resistive component of the impedance cannot be changed. Only the reactive component (imaginary) can be changed by adding a capacitor in parallel with the load. The maximum power transfer of the circuit is therefore when $X = 0$. This occurs at resonance when the inductive and capacitive reactance's are balances.

6 CONCLUSION

The results of this investigation show that the efficiency of a system can be improved by applying power factor correction. This was successfully shown using the simulated circuit in proteus.

The power factor correction is applied by adding a capacitor in parallel with the inductive load. The capacitors value was specifically calculated to achieve a power factor ≥ 0.95 (method shown).

The results also show how adjusting the power factor capacitors value will change the power factor. The increase and decrease in power factor are caused by the change in reactance. When the capacitor value is low, the system is imbalanced as most of the reactance is capacitive. As the capacitor value increases, the capacitive reactance decreases. The power factor increases with the capacitor value, until the power factor hits resonance. Any increase in capacitance past this point will imbalance the system due to the higher inductive reactance in the system.

The methods shown could also be used in real work applications to improve the efficiency of inductive loads.

7 REFERENCES

- [1] J. Bird, "Electrical Circuit Theory and Technology - Fifth Edition," Routledge, 2014, p. 223.
- [2] S. A. R. James W. Nilsson, "Electric Circuits - 9th Edition," Pearson, 2011, p. 370.
- [3] "CALCULATING PHASE DIFFERENCE BETWEEN TWO WAVES," [Online]. Available: <https://voltage-disturbance.com/power-quality/calculating-phase-difference-between-two-waves/>. [Accessed 10 04 2022].
- [4] S. A. R. James W. Nilsson, "Electric Circuits - 9th Edition," Pearson, 2011, p. 380.