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CW2: Heat Diffusion Model Report

Computational Methods ACMA6012

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1. Introduction

This report investigates and documents the analysis of heat diffusion within an aluminium plate, employing the Explicit Finite Difference Method (EFDM) as the primary computational tool. The phenomenon of heat diffusion is pivotal in various scientific and engineering fields, ranging from aerospace engineering to environmental studies. Understanding how heat propagates through materials not only enhances the design of thermal systems but also contributes to the optimization of energy usage and the development of new materials.

Initially, the report presents the theoretical background of the heat equation, followed by an in-depth discussion of the numerical methods used for its discretization. The core of the report revolves around the implementation of these methods in MATLAB, highlighting the step-by-step development of the code and its execution. Subsequent sections provide a detailed analysis of the code results, emphasizing sensitivity to various parameters. The report concludes with a critical examination of the model's limitations and potential avenues for future enhancements.

1.1. Aims and Objectives

The primary aim of this report is to explore the process of heat diffusion in a two-dimensional domain and to model this phenomenon using EFDM. Key objectives include:

1. Theoretical Exploration: To understand the governing principles of the heat equation, particularly in a two-dimensional context without a heat source.
2. Numerical Analysis: To implement the EFDM in MATLAB, effectively translating the theoretical heat equation into a computational model.
3. Model Evaluation: To assess the accuracy, efficiency, and stability of the EFDM in simulating heat diffusion.
4. Critical Analysis: To conduct a sensitivity analysis of the model, examining the impact of varying grid spacings, time steps, and thermal equilibrium tolerances on the simulation results.
5. Limitation Assessment and Improvement Suggestions: To identify the limitations inherent in the numerical method and the specific MATLAB implementation, proposing potential improvements and directions for future research.

2. The Governing Equation

2.1. Background Information

The heat equation, a cornerstone in the study of heat transfer, has its origins in the works of Joseph Fourier in the 1800s (Fourier, 1878). Fourier's work in thermodynamics laid the foundation for understanding how heat propagates through various substances.

Fourier's law describes the relationship between heat flux and temperature gradient, thereby facilitating the derivation of the heat equation, a pivotal mathematical construct that expresses heat distribution over time.

2.2. Importance of the Heat Equation

The heat equation is a fundamental principle within physics and engineering, providing an insight into how temperature changes within a material over time. It has significant applications across various fields, from designing thermal systems in aerospace engineering to predicting earth's subsurface temperatures in geothermal engineering. The equation is pivotal in solving climate change issues and material design. (Incropera & DeWitt, 2007)

2.3. Mathematical Definition for 2D Geometry Without a Heat Source

Conduction in solids is described by the following thermal diffusion equation (Carslaw & Jaeger, 1959),

$$\nabla^2 T = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t}$$

$$\therefore \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t}$$

This partial differential equation encapsulates the evolution of the temperature (T) in a planar geometry over time (t).

2.4. Definition of Terms

In this equation, ∇^2 is the Laplacian, defined by $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ in two dimensions,

t = time [s],

T = temperature [K or °C],

$\alpha = k/(\rho \cdot c)$ is the thermal diffusivity [$\text{m}^2 \cdot \text{s}^{-1}$], This quantifies the rate of heat conduction relative to the capacity of the material to store thermal energy, therefore influencing the rate of temperature change. It is a material-specific property derived from the thermal conductivity, density, and specific heat.

k = thermal conductivity [$\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$],

ρ = density [$\text{kg} \cdot \text{m}^{-3}$],

c = specific heat [$\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$],

x, y = the Cartesian coordinates on the plane of the material.

(Croft & Lilley, 1977)

3. Numerical Method for Discretisation

3.1. Starting Point: Taylor Series for Difference

Approximations

The discretization process begins with the Taylor Series expansion. It is the substitution of the derivatives in the differential thermal diffusion equation by forward, backward, and central differences. This transforms the partial differential equation into a series of linear equations which are resolvable to the initial and boundary conditions. The choice of the approximation is integral in the success of the model, with various stability, convergence, accuracy, and speed characteristics for each method (Croft & Lilley, 1977).

3.2. Derivation of Forward, Backward, and Central Differences from Taylor Series

These approximations are pivotal for converting the continuous mathematical representation of physical phenomena into a discrete form suitable for numerical computation. The Taylor Series expansion of a function provides the foundational mathematics for these approximations (Croft & Lilley, 1977).

3.2.1. Forward Difference Approximation

This method approximates the first derivative of a function at a point using the values at that point and the immediate next point on the grid. In a discretized domain, it is expressed as (Ozisik, 2017),

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

Where h is the distance from x_0 to the next grid point. This approximation is crucial in explicit finite difference methods for discretization purposes.

3.2.2. Backward Difference Approximation

This approximation uses the function value at a specific point and its preceding point along the x axis. It is expressed as (Ozisik, 2017),

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

This method is used for some specific numerical finite difference schemes such as IFDM.

3.2.3. Central Difference Approximation

This method offers a higher degree of accuracy by averaging the forward and backward differences. It is expressed as (Ozisik, 2017),

$$f''(x_0) \approx \frac{f(x_0 + h) - 2.f(x_0) + f(x_0 - h)}{h^2}$$

The central difference approximation is invaluable for spatial derivatives in numerical methods, providing a more symmetric and balanced representation of the derivative by incorporating information from both sides of the point of interest. According to (Chapra & Canale, 1988) “by truncating the [Taylor] series to a desired level of accuracy (typically the first or second derivative terms) and rearranging, it is possible to derive the forward, backward, and central difference approximations. The choice of approximation impacts the accuracy and stability of the numerical solution. Forward difference is simpler but less accurate, backward difference is useful for certain boundary conditions, and central difference, while computationally more intensive, provides a superior balance between accuracy and computational effort.”

3.3. Explicit Finite Difference Scheme

The explicit finite difference method (EFDM) is one possible technique for numerically solving partial differential equations, such as the 2D heat equation. This method replaces derivatives in the differential equation with finite difference approximations, leading to a set of linear equations that can be solved iteratively. The EFDM method uses Forward, and Central Difference approximations to accomplish this.

3.3.1. Temporal Discretization with Forward Difference

Approximation

The first derivative with respect to time (temporal derivative) in the heat equation is approximated using a forward difference. If $T(x,y,t)$ is the temperature at position (x,y) and

time t , and Δt is the time step, the forward difference approximation for the temporal derivative is (Croft & Lilley, 1977),

$$\frac{\partial T}{\partial t} \approx \frac{T(x, y, t + \Delta t) - T(x, y, t)}{\Delta t}$$

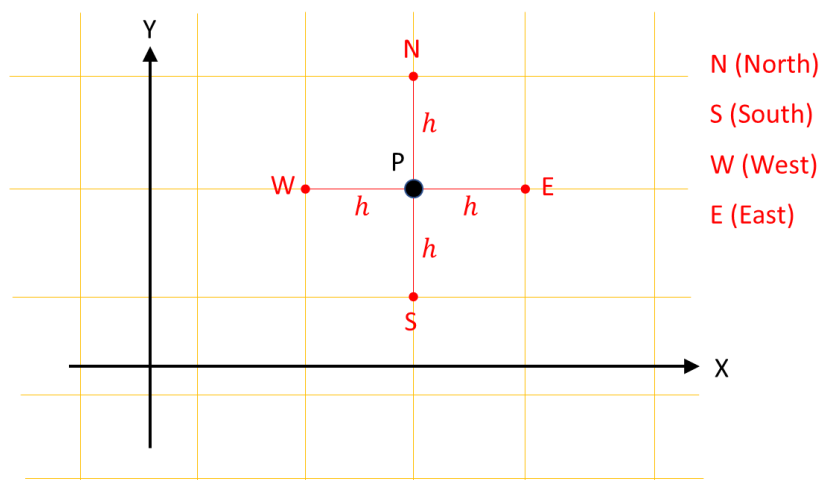
3.3.2. Spatial Discretization with Central Difference Approximation

The Derivative with respect to space (Spatial derivative) in the heat equation is approximated using a central difference. The central difference approximation for the Spatial derivative is (Croft & Lilley, 1977),

$$\frac{\partial T}{\partial x} \approx \frac{T(x - h, y, t) - 2T(x, y, t) + T(x + h, y, t)}{h^2}$$

3.4. Discretization of the Spatial Domain

The spatial domain is discretized into a grid of discrete points with uniform grid spacing, expressed as $h = \Delta x = \Delta y$.



Typical point P with its four neighbours in a 2D region with uniform grid spacing.

3.5. Discretization of the Temporal Domain

The temporal domain is divided into discrete time steps (Δt). The size of the time step is critical for the stability and accuracy of the solution. It must be selected in proportion to the grid spacing to balance between computational efficiency and the resolution of the changes in temperature over time.

3.6. Explicit Finite Difference Form of the Heat Equation and Fourier's Number

The differential thermal diffusion equation is represented in finite-difference form by using central difference form to discretize $\nabla^2 T$ and using forward difference form to discretise $\partial T / \partial t$ at the point P.

$$\begin{cases} \frac{\partial^2 T}{\partial x^2} = \frac{T_E^t - 2.T_P^t + T_W^t}{h^2} \\ \frac{\partial^2 T}{\partial y^2} = \frac{T_N^t - 2.T_P^t + T_S^t}{h^2} \end{cases}$$

and

$$\frac{\partial T}{\partial t} = \frac{T_P^{t+\Delta t} - T_P^t}{\Delta t}$$

Substituting into the thermal diffusion equation gives (Croft & Lilley, 1977) ,

$$T_P^{t+\Delta t} = F_0 \left(T_E^t + T_W^t + T_N^t + T_S^t - \left(4 - \frac{1}{F_0} \right) \cdot T_P^t \right)$$

In this form, the heat equation is expressed as a function of the discrete grid points and time steps, incorporating the Fourier number (F_0).

The Fourier number, F_0 is a dimensionless expression defined as (Croft & Lilley, 1977),

$$F_0 = \frac{\alpha \cdot \Delta t}{h^2}$$

Where the Fourier Number F_0 , relating the time step size to the grid size and thermal diffusivity, playing a crucial role in the stability and accuracy of the numerical solution.

This Finite Difference representation is called the explicit form because the unknown temperature $T_P^{t+\Delta t}$ can be explicitly determined knowing the temperatures at the previous time step.

The main disadvantage of this method is that once α and h are fixed, there is a maximum permissible time-step size Δt which should not exceed the value imposed by the stability criterion.

3.7. Stability Criterion

In the numerical solution of differential equations, the growth of round-off errors can cause a large deviation of the final answer from the exact solution. The stability criterion for the EFDM is crucial to ensure that the numerical solution does not exhibit unphysical oscillations or divergence over time.

A commonly used criterion for EFDM is the Courant-Friedrichs-Lewy (CFL) condition (Thomas, 2013), which for 2D heat transfer, translates to ensure the Fourier number (F_0) is between 0 and 0.25.

$$0 < F_0 \leq \frac{1}{4}$$

4. Initial and boundary conditions implemented

4.1. Initial Conditions

In the context of the heat diffusion model using the Explicit Finite Difference Method (EFDM), it is crucial to define the initial temperature distribution within the spatial domain at the beginning of the time interval. The initial condition sets the stage for how heat transfer evolves over time within the material.

This is expressed as:

$$T(x, y, 0) = 0 \text{ for } 0 < x < 6, 0 < y < 6$$

This condition implies that at the start of the simulation, the entire plate is at a uniform temperature (or homogenous state), which in this case is 0°C.

4.2. Boundary Conditions

Boundary conditions are essential to solve the heat equation, as they define how the system interacts with its surroundings.

4.3. South, East and West Boundaries (Constant Boundary Conditions)

The temperatures at the South ($y = 6$), East ($x = 0$) and West ($x = 6$) boundaries of the plate are held constant at 0 throughout the simulation. This is represented as:

$$\left. \begin{array}{l} T(0, y, t) = 0 \\ T(6, y, t) = 0 \end{array} \right\} \text{ for } 0 \leq y \leq 6 \text{ and } 0 \leq t \leq \text{thermal equilibrium}$$

4.4. North Boundary (Dirichlet condition with a linear temperature distribution)

The top edge of the plate (north boundary ($y=0$)) follows a Dirichlet condition with a linear temperature distribution described by the function $f(x)$. The equation for $f(x)$ varies linearly within two segments. This condition creates a varying temperature along the north boundary that changes linearly along the x -axis.

$$\left. \begin{array}{l} T(x, 0, t) = f(x) \\ T(x, 6, t) = 0 \end{array} \right\} \text{ for } 0 \leq x \leq 6 \text{ and } 0 \leq t \leq \text{thermal equilibrium}$$

Where,

$$f(x) = \begin{cases} 12.5x & \text{if } 0 \leq x \leq \left(\frac{2}{3} \times 6\right) \\ -25x + 150 & \text{if } \left(\frac{2}{3} \times 6\right) < x \leq 6 \end{cases}$$

4.5. Other Boundaries

For heat diffusion simulations, a variety of boundary conditions can be applied, each tailored to model specific physical scenarios such as convection, radiation, insulation etc.

4.5.1. Dirichlet Boundary condition

This condition specifies a constant temperature at the boundary. It is used when the temperature at the boundary is known and remains fixed throughout the simulation. (Holman, 2010)

4.5.2. Neumann Boundary Condition

This type of condition specifies the heat flux (rate of heat transfer per unit area) at the boundary. It is applicable in situations where the rate of heat transfer through the boundary is known, rather than the temperature itself. This condition is often used to model convective heat transfer or insulated boundaries. (Holman, 2010)

4.5.3. Adiabatic Boundary Condition

This type of boundary condition describes a scenario where no heat enters or leaves the material. This is equivalent to a Neumann boundary condition with a heat flux of zero. It is ideal for modelling insulated boundaries where the insulation is assumed to be perfect, and thus no heat transfer occurs across it. (Holman, 2010)

5. Code Details and Explanations

This section provides an illustration and explanation of each segment of the MATLAB code created for simulating the process of heat diffusion within a two-dimensional domain, specifically - a 6m x 6m aluminium plate. By employing the Explicit Finite Difference Method (EFDM), the code aims to provide a detailed and accurate representation of how heat propagates and reaches equilibrium within the material. The computational model, implemented in MATLAB, provides insight into the dynamic behaviour of heat transfer, accounting for various parameters like grid spacing, time steps, and thermal equilibrium tolerances.

5.1. Initialization Section

Prepares the MATLAB environment for running the simulation. It clears the Command Window (clc), removes all variables from the workspace (clear all), and closes all figures (close all), ensuring a clean slate for the simulation.

```
1  %% Initialization
2  % Initialise the program for Temperature Distribution in an Aluminium Plate as per assignment brief
3  clc % Clear the Command Window
4  clear all % Remove all variables from the workspace
5  close all % Close all figures (windows)
6
```

5.2. Material Properties Section

Physical properties of the aluminium plate are defined, including thermal conductivity (k), density (rho), and specific heat (c). These properties are used to calculate the thermal diffusivity (alpha), a parameter essential for the simulation.

```
7  %% Material Properties
8  % Define the material properties
9  k = 143; % Thermal conductivity in W/(m*C)
10 rho = 2800; % Density in kg/m^3
11 c = 795; % Specific heat in J/(kg*C)
12 alpha = k/(rho*c); % Calculate thermal diffusivity using the properties
13
```

5.3. Stability Criterion Section

The maximum Fourier number (Fo_{max}) for stability in the explicit method is specified, with a value that should not exceed 0.25 to maintain a stable solution in the 2D heat equation simulation.

```
14 %% Stability Criterion
15 Fo_max = 0.25; % Maximum Fourier number for stability in explicit method, must be <= 0.25 for a stable solution in a 2D heat equation
16
```

5.4. Plate Dimensions Section

This part sets the physical dimensions of the aluminium plate (L_x and L_y), which are necessary for defining the spatial domain of the simulation.

```
17 %% Plate Dimensions
18 % Define the plate dimensions
19 Lx = 6; % Length of the plate in the x-direction
20 Ly = 6; % Length of the plate in the y-direction
21
```

5.5. Grid Spacing Section

The user is prompted to enter the grid spacing (h), which defines the spatial resolution of the grid. The loop ensures that the grid spacing fits the plate dimensions correctly, and the number of rows (R) and columns (C) in the grid are calculated.

```
22 %% Grid Spacing
23 % User prompt for grid spacing (h)
24 while true % This loop will continue to prompt the user for a grid spacing value that fits the plate dimensions
25     h = input('\nEnter grid spacing (h) [m]: '); % User enters grid spacing
26     l = h; % Set grid spacing in y-direction equal to x-direction for a square grid
27     R = Ly/h+1; % Calculate the number of rows in the grid
28     C = Lx/l+1; % Calculate the number of columns in the grid
29
30     % Check if grid spacing is compliant with plate dimensions
31     if mod(Lx, h) <= 0 && mod(Ly, l) <= 0 && h>0 % Check if the entered grid spacing divides evenly into the plate dimensions, and is greater than zero
32         break; % Exit the loop if conditions are met
33     else
34         % If the grid spacing does not divide evenly, ask the user to enter a new value
35         fprintf('\nGrid spacing does not fit evenly into plate dimensions. Please enter a new value!\n');
36     end
37 end
38
39
```

5.6. Initialize Temperature Matrix Section

A temperature matrix (T) is initialized with zeros to represent the initial temperature distribution across the grid.

```
40 %% Initialize temperature matrix
41 % Create a matrix filled with zeros to represent the initial temperature distribution, where R is the number of rows and C is the number of columns
42 T = zeros(R, C);
43
44
```

5.7. Boundary Conditions Section

The boundary conditions for the simulation are established, specifically setting the temperature distribution along the north boundary of the plate with a prescribed variation.

```

45 %% Boundary conditions
46 % North boundary condition
47 for i = 1:R % This loop sets the temperature distribution along the north boundary of the plate according to the assignment specifications
48     x = (i-1)*h; % Calculate the distance from the left edge of the plate
49     if x <= 2/3*Lx
50         T(1,i) = 12.5*x; % Linear temperature increase for the first two-thirds of the plate
51     else
52         T(1,i) = -25*x + 150; % Linear temperature decrease for the last third of the plate
53     end
54 end
55 % Update temperature matrix
56 T;
57 T_new = T; % Copy the temperature matrix to a new matrix for the next time step
58

```

5.8. Time Step Input Section

This section prompts the user for the time step (dt) value and ensures that the chosen time step satisfies the stability criterion based on the Fourier number (Fo).

```

59 %% Time Step input
60 % Prompt for time step (dt) within a loop to ensure stability criterion is met
61 while true % This loop prompts the user for a time step that will satisfy the stability criterion for the simulation
62     dt = input('\nEnter time step (dt) [s]: '); % User enters the time step
63     Fo = (alpha*dt/h^2); % Calculate the Fourier number based on the entered time step and grid spacing
64
65     % Stability Check
66     % Check stability criterion (assuming 2D heat conduction)
67
68     if Fo <= Fo_max && Fo>0 % Check if the Fourier number is within the stable range
69         fprintf('\nStability criterion has been met at %.2f\n',Fo);
70         break
71     else % If the Fourier number is not within the stable range, ask the user to enter a new time step
72         fprintf('\nStability criterion is not met, \n\nCalculated Fo value is: %.2f , it needs to be between %.2f and 0. \n\nPlease choose a smaller
73     end
74 end
75

```

5.9. Time-Stepping Loop Section

A loop is entered for the time-stepping process, the main part of the heat propagation simulation, where the temperature matrix is updated at each time step using the Explicit Finite Difference Method (EFDM). A check for thermal equilibrium is performed by comparing the change in temperature between time steps against a set accuracy value (TEV). If thermal equilibrium is reached, the loop breaks.

```

76 %% Time-stepping loop
77
78 tic % Start a timer to measure the duration of the simulation process
79
80 t = 0; % Initialize the simulation time
81 while true % Enter an infinite loop to perform time stepping until the break condition (thermal equilibrium) is met
82     % Iterate through interior grid points to update temperatures using the Explicit Finite Difference Method (EFDM)
83     for i = 2:R-1 % This is the matrix for calculating the interior points, it ignores the south boundary row, effectively setting the South boundar
84
85         for j = 2:C-1 % this also sets the East and West boundary condition to 0 as per the assignment brief
86
87             T_new(i,j) = Fo*(T(i,j+1)+T(i,j-1)+T(i+1,j)+T(i-1,j))-(4-1/Fo)*T(i,j);
88
89         end
90     end
91
92     %Thermal equilibrium accuracy value
93     TEV = 0.01;
94
95     % Check for thermal equilibrium
96     if max(max(abs(T_new - T))) < TEV
97         fprintf('\nThermal equilibrium reached at %.2f seconds\n\nThe Equilibrium Tolerance value is set to %.5f degrees[c]\n', t, TEV);
98         TTE=t;
99         break;
100     end
101
102     % Update time
103     t = t + dt;
104
105     % Update temperature matrix
106     T = T_new;
107

```

5.10. Plotting Section 1

A contour plot of the temperature distribution at the current time step is generated, showing the simulation's progress.

```

108 %% Plotting section 1
109
110 % Colour map of EFDM
111 figure(1);
112 contourf(flip(T));
113 colormap("jet") % This setting changes the color bar to make cold=blue, hot=red
114 colorbar;
115 title(sprintf('Colour Map of Temperature Distribution at %.2f[s] using Finite Difference Analysis\nh = %.2f, dt = %.2f, TEV = %.6f', t,h,dt,TEV));
116 xlabel('Column number');
117 ylabel('Row number');
118 axis equal tight;
119

```

5.11. Calculate Analytical Solution Section

The analytical solution to the temperature distribution simulation is calculated for comparison with the numerical solution.

```

121 %% Calculate analytical solution
122
123 T_check = zeros(R,C);
124 T_check(1,:)=T(1,:);
125
126 for i = 2:R-1
127     for j = 2:C-1
128         for n = 1:200
129             T_check(i,j) = T_check(i,j) + (sin(2*n*pi/3) / (n^2 * sinh(n*pi))) * sin(n*pi*(i-1)*h/6) * sinh(n*pi*(j-1)*h/6);
130         end
131     end
132 end
133 T_check(2:R-1,2:C-1)=rot90(T_check(2:R-1,2:C-1)*(450/pi^2));
134 format shortG
135 T_check;

```

5.12. Plotting Section 2

A contour plot of the temperature distribution from the analytical solution at thermal equilibrium is generated, allowing for visual comparison with the numerical solution.

```

136 %% Plotting section 2
137
138 % Colour map of analytical solution at thermal equilibrium
139 figure(2);
140 contourf(flip(T_check));
141 colormap("jet") % This setting changes the color bar to make cold=blue, hot=red
142 colorbar;
143 title(sprintf('Colour Map of Temperature Distribution at %.2f[s] using Analytical Solution\nh = %.2f, dt = %.2f, TEV = %.6f', t,h,dt,TEV));
144 xlabel('Column number');
145 ylabel('Row number');
146 axis equal tight;
147

```

5.13. Values at Thermal Equilibrium Section

The temperature matrices from both the numerical and analytical solutions at thermal equilibrium are displayed or stored for further analysis.

```

148 %% Values at thermal Equilibrium
149 T;
150 T_check;

```

5.14. Check Analytical Solution Against Explicit Method Section

The difference between the numerical and analytical solutions is computed, and a check is performed to determine if the numerical solution is within an acceptable tolerance of the analytical solution. The runtime of the simulation is also calculated and displayed, indicating the efficiency of the simulation process.

```

151 %% Check analytical solution against explicit method
152 Diff_T_check = abs(T-T_check);
153 Ndiff_T_check = max(max(abs(T-T_check)));
154 Explicit_Analytical_Tolerance = 10;
155
156 if Ndiff_T_check<Explicit_Analytical_Tolerance
157     fprintf('\n\nThe explicit method has produced results that are within %f degrees[c] of the analytical solution. \n\nThis is within the tolerance \
158 else
159     fprintf('\n\nThe explicit method has produced results that are %f degrees[c] different to the analytical solution. \n\nThis is outside the tolerar
160 end
161
162 Simulation_Runtime = toc;
163 fprintf('\n\nThe time taken to complete the simulation is %f seconds\n',Simulation_Runtime)
164
165 %% End of the program

```

5.15. Comparative Graph Plotting Section

This section of the code is designed for comparative graphical analysis, overlaying contour plots of temperature distributions from both the Explicit Finite Difference Method (EFDM)

and the Analytical Solution. It involves creating a figure, plotting the EFDM results (T) in blue, and plotting the Analytical Solution (T_check) in a red dashed line for contrast.

```

164 %% Comparative Graph Plotting Section
165 % Create a new figure with a white background
166 figure('Color', 'w');
167
168 % Plot the first contour plot
169 contour(flip(T), 'LineWidth', 1.0, 'LineColor', 'b'); % Blue line for T
170 colormap("jet"); % Set colormap to 'jet'
171 colorbar;
172 xlabel('Column number');
173 ylabel('Row number');
174 axis equal tight;
175 hold on; % Hold on to overlay the next plot
176
177 % Overlay the second contour plot with a different line style or color
178 contour(flip(T_check), 'LineWidth', 1.0, 'LineStyle', '--', 'LineColor', 'r'); % Red dashed line for T_check
179
180 % Add labels and title
181 title(sprintf('Temperature Distribution of EFDM and Analytical Solution\n t = %.2f[s], h = %.2f, dt = %.2f, TEV = %.6f', t,h,dt,TEV));
182 xlabel('Column number');
183 ylabel('Row number');
184
185 % Create dummy plots for the legend
186 h1_legend = plot(NaN, NaN, 'b-', 'LineWidth', 1.0); % Blue solid line for T
187 h2_legend = plot(NaN, NaN, 'r--', 'LineWidth', 1.0); % Red dashed line for T_check
188
189 % Add legend
190 legend([h1_legend, h2_legend], {'T', 'T_check'}, 'Location', 'best');
191
192 hold off; % Release the plot
193
194 % End of the program

```

6. Validation of the code

6.1. Temperature Map Comparison

Using the parameters $h=1$, $dt = 3600$, $TEV = 0.01$, the temperature maps produced for both methods as shown in Figure 1 and Figure 2, appear to be similar to one another visually. This suggests that the model is performing correctly, and that the EFDM approximation used is an acceptable alternative to the analytical solution. This method of analysis, known as qualitative analysis, is useful for a quick or initial assessment of the model performance. It is also possible to overlay each graph to allow for a comparative assessment of the two methods, as shown in Figure 3. This graph shows that the EFDM has propagated deeper into the material than the Analytical solution, but the profile of propagation is very similar for both.

However, to provide a more accurate analysis, a quantitative analysis such as a matrix comparison would be more effective.

Map of Temperature Distribution at 140400.00[s] using Finite Difference Analysis

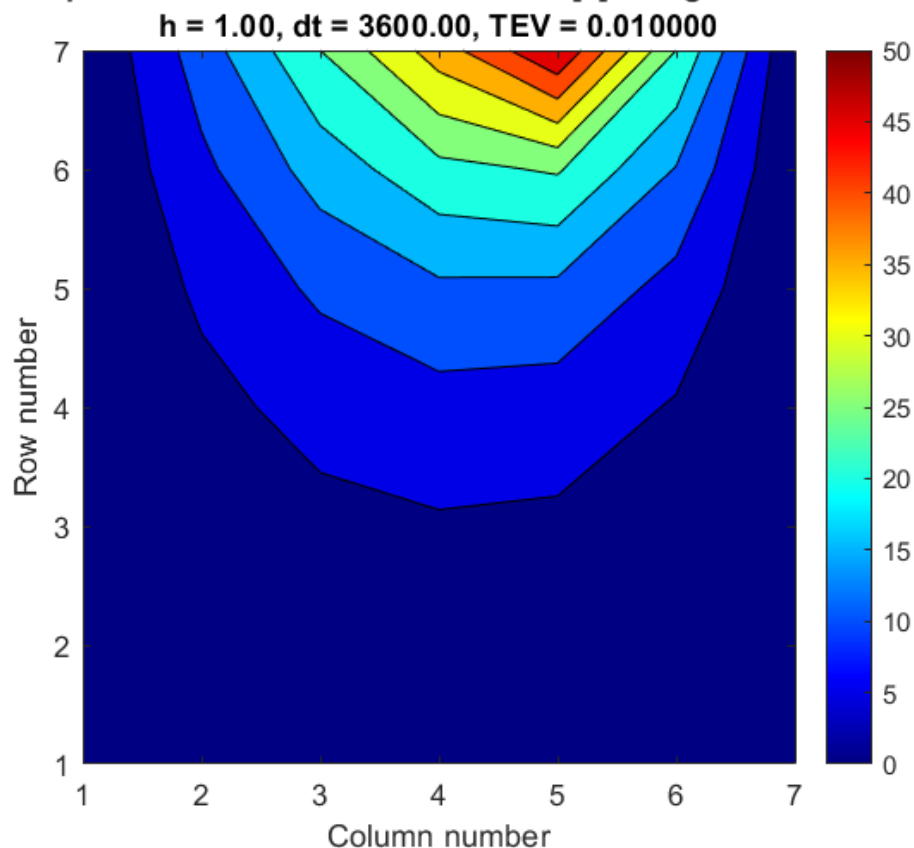


Figure 1 – EFDM Heat Map

our Map of Temperature Distribution at 140400.00[s] using Analytical Solution

$h = 1.00$, $dt = 3600.00$, $TEV = 0.010000$

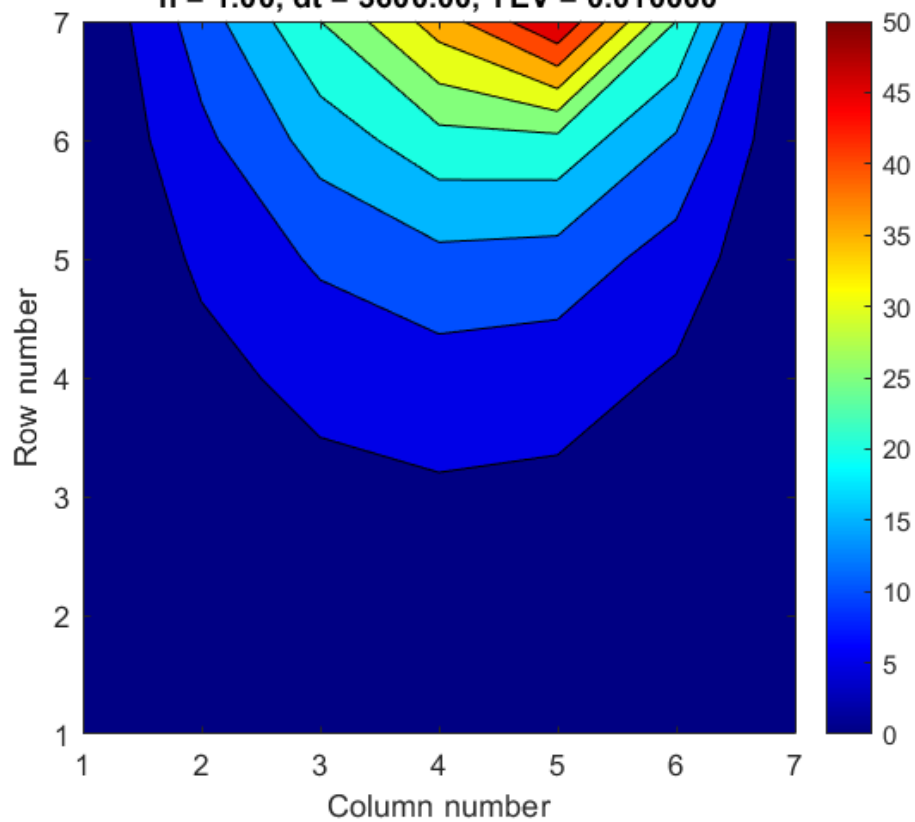


Figure 2 - Analytical Solution Heat Map

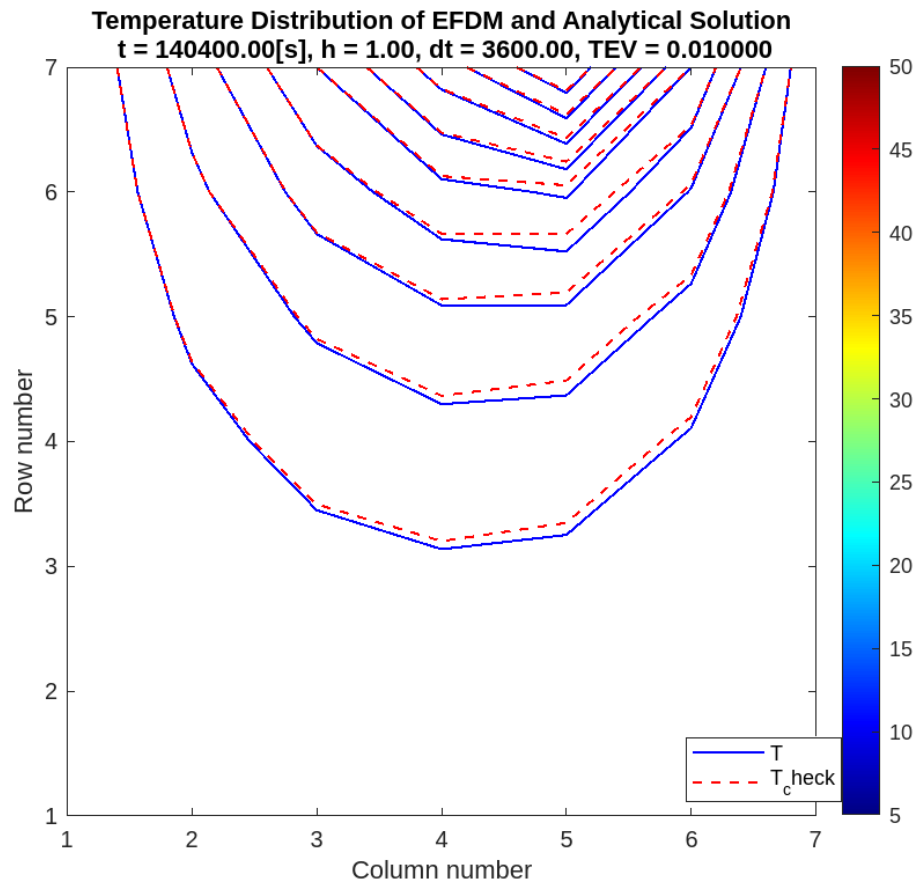


Figure 3 – Comparative Graph

6.2. Matrix Comparison

Using the parameters $h=1$, $dt = 3600$, $TEV = 0.01$, the EFDM matrix at thermal equilibrium ' T ' is compared to the analytical solution matrix at thermal equilibrium ' T_{check} '. The difference is represented as a matrix ' $Diff_T_Tcheck$ '.

From these matrices it is reasonable to conclude that the parameters used have produced EFDM results that are within a very close approximation of the Analytical solution with the maximum difference between the two methods being $1.9623^{\circ}C$ at a single point of the matrix, and the standard deviation of the difference between the two matrixes being $0.403^{\circ}C$.

T - Explicit finite difference method (EFDM) at thermal equilibrium						
0	12.5	25	37.5	50	25	0
0	8.8514	17.075	23.56	25.541	14.708	0
0	5.84	10.907	14.142	13.914	8.3021	0
0	3.6182	6.5998	8.2211	7.6983	4.603	0
0	2.0521	3.6864	4.483	4.0889	2.4309	0
0	0.92057	1.6398	1.969	1.7724	1.0484	0
0	0	0	0	0	0	0

SUM	MIN	MAX	Standard Deviation
347.67237	0.92057	25.541	6.702612518

T_check - Analytical Solution (AS) at thermal equilibrium						
0	12.5	25	37.5	50	25	0
0	8.8572	17.033	23.175	23.579	14.318	0
0	5.8115	10.763	13.663	12.933	7.8515	0
0	3.5691	6.442	7.8677	7.1874	4.2986	0
0	2.0086	3.5713	4.2716	3.8276	2.263	0
0	0.89674	1.5822	1.8731	1.6622	0.97641	0
0	0	0	0	0	0	0
SUM	MIN	MAX	Standard Deviation			
340.28175	0.89674	23.579	6.434775935			

Diff_T_TCheck - Difference between EFDM and AS						
0	0	0	0	0	0	0
0	0.005728	0.042782	0.38447	1.9623	0.3902	0
0	0.028527	0.14425	0.47917	0.98074	0.45065	0
0	0.049059	0.15787	0.35346	0.51088	0.3044	0
0	0.043478	0.11519	0.21137	0.26132	0.16789	0
0	0.023826	0.057569	0.095827	0.11024	0.072001	0
0	0	0	0	0	0	0
SUM	MIN	MAX	Standard Deviation			
7.403197	0.005728	1.9623	0.40343829			

7. Sensitivity Analysis

Sensitivity analysis of the dataset reveals significant insights into the impact of varying grid spacing (h), time step (dt), and thermal equilibrium tolerance on the code's performance. This analysis is crucial for understanding the behaviour of the numerical model in thermal simulation. To streamline the sensitivity analysis of the code and automate the process, I restructured the code, enabling systematic variation of key parameters such as grid spacing, time step, and thermal equilibrium tolerance. This was accomplished by encompassing the simulation logic within a function, which was then called in the main script iteratively with different sets of variables. The outcome of each simulation was recorded in a table, then converted, and saved as a .csv file for analysis in Microsoft Excel. To ensure the reliability of the results, the code is designed using 'if' statements, to flag errors whenever the stability criterion is not met or the grid spacing is incompatible before moving onto the next variable. I also included the 'tic-toc' timer function to time each iteration of the simulation. This would enable me to determine the computational demand for each variable and then evaluate the trade-off between sensitivity and computational load.

This new code and function are shown in Appendix 2 and Appendix 3.

7.1. Grid Spacing (h)

Grid spacing plays a crucial role in the numerical simulation as it determines the resolution of the spatial domain. As the grid spacing increases, the model's ability to capture detailed spatial variations decreases, leading to less accurate results. However, larger grid sizes can significantly reduce computation time. The challenge is to find an optimal grid size that balances accuracy /resolution with computational efficiency. This is demonstrated in the dataset below.

Grid Spacing (h)	Time Step (dt)	Thermal Equilibrium Tolerance (°C)	Time to Equilibrium (s)	Explicit/ Analytical Difference (°C)	Time to compute (s)
0.1	150	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	150	0.001	118950	0.384930344	0.0243637

0.25	150	0.001	118950	0.481079373	0.0114896
0.5	150	0.001	119400	0.961313377	0.0031387
0.75	150	0.001	119550	0.255570185	0.0017075
1	150	0.001	120900	1.909349408	0.0012588
1.2	150	0.001	117900	0.45169555	0.0011384
1.5	150	0.001	121350	0.674299787	0.0007387
2	150	0.001	119250	3.52928171	0.000704
3	150	0.001	129150	1.274443948	0.0007238

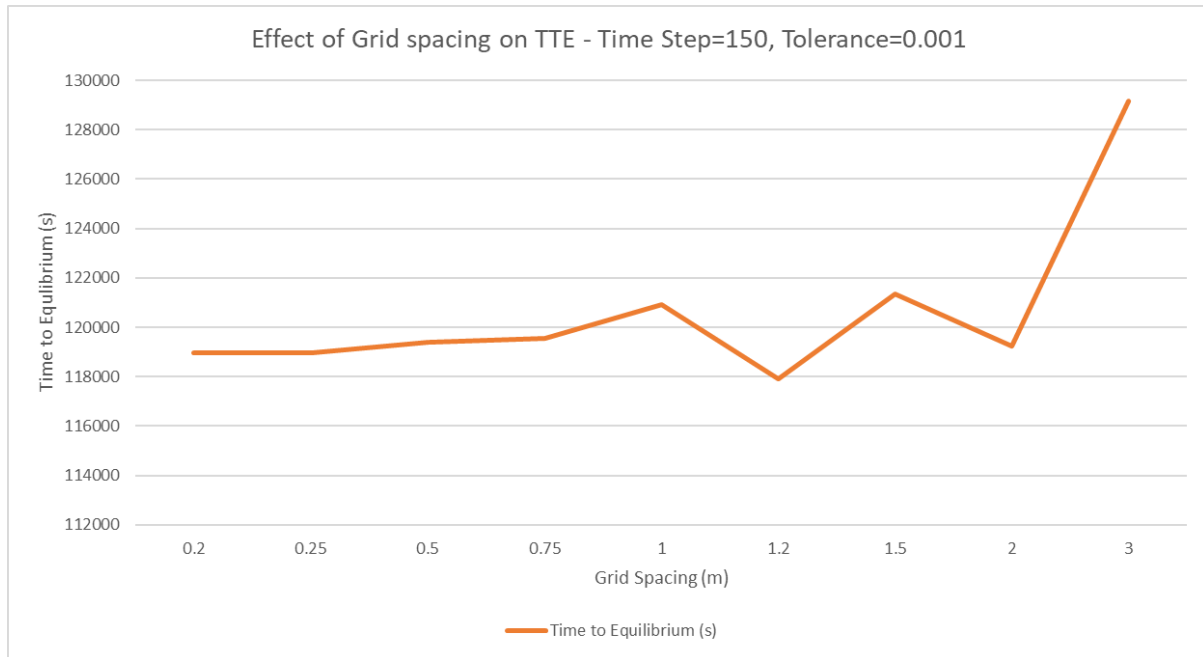


Figure 4 - Grid spacing and TTE Graph

The graph in Figure 4 illustrates the relationship between Grid Spacing (h) in metres and Time to reach Thermal Equilibrium (TTE) in seconds. The values for smaller grid spacings, 0.2 m to 1 m, exhibit less variation compared to larger grid spacings. This indicates a convergence of solution, where the model becomes less sensitive to changes in grid spacing. Therefore, at smaller grid spacings, it is capturing the physical behaviour of the system more accurately, and further decreases in grid spacing do not significantly alter the time to thermal equilibrium. However, the reduction in grid size does increase the computational load. The increased time to thermal equilibrium at larger grid spacings may reflect stability issues within the model. As grid spacing increases, the spatial resolution decreases, which can affect the accuracy of the simulated heat diffusion process. This is consistent with the numerical analysis principle that

a finer grid can represent the continuous system with greater accuracy. (Burden & Faires, 2005)

The graph in Figure 5 displays the relationship between the grid spacing and the explicit/analytical difference in temperature. It appears that as the grid spacing increases, the difference between the numerical and analytical solutions also increases, indicating a loss in accuracy. This increase is not always uniform, suggesting that the relationship between grid spacing and accuracy might be influenced by other factors as well, such as boundary conditions or the nature of the physical problem being modelled. The peak at a grid spacing of 2 m indicates a particularly significant deviation from the analytical solution at that point, the cause of which potentially includes numerical instability, error accumulation, or an inappropriate choice of grid spacing relative to the problem scale. The graph shows lower differences for smaller grid spacings, implying better accuracy of the numerical model when using a finer mesh. This is as expected, due to the finer mesh capturing the temperature distribution with higher resolution, thus, providing results that are closer to the true continuous phenomenon being modelled. The convergence of results at smaller grid spacings suggests that the numerical method becomes more reliable as it approximates the continuous nature of the problem more discretely. However, it is important to balance the desire for accuracy with the computational cost, as a finer grid requires more computational resource. To achieve results that are accurate enough for the application being modelled, and computationally acceptable, it would be worth accepting the largest grid spacing that yields an error that is within an allowable relative tolerance. For example, if modelling temperature changes within a human body, accuracy within 0.5°C might be necessary, whereas modelling temperature changes on the surface of an aeroplane might only require an accuracy within 10°C .

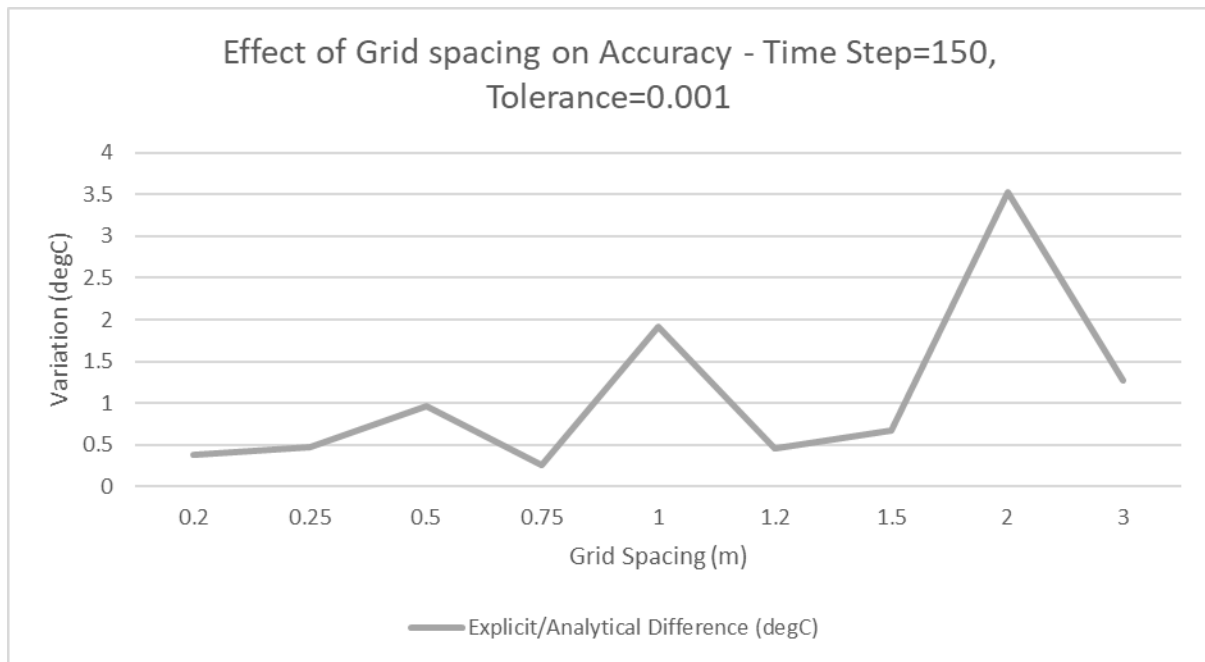


Figure 5 - Grid Spacing and Accuracy Graph

The trade-off between accuracy and computational efficiency in the choice of grid spacing is often discussed in literature and this is evident from looking at the graph in Figure 6. (Cengel, 2007) has “observed that the computational time seems to decrease with increasing grid spacing, which is typical behaviour due to the reduction in the number of computational cells.”

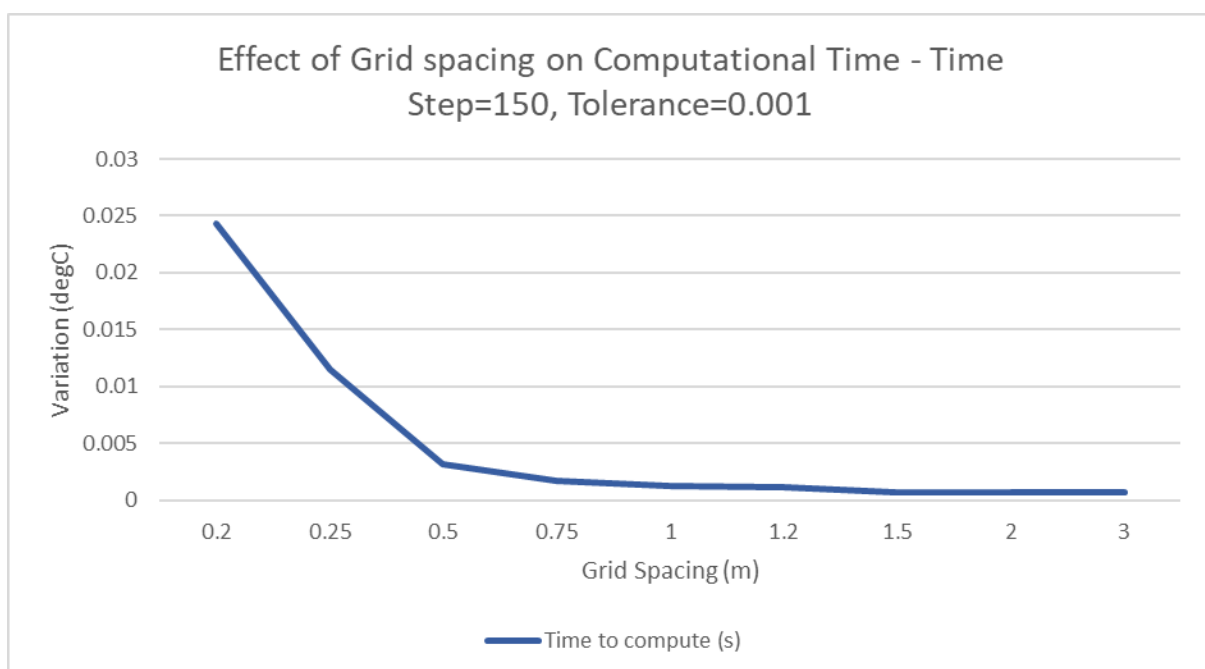


Figure 6 - Grid Spacing and Computational time Graph

7.2. Time Step (dt)

The time step is another critical parameter, especially within dynamic simulations. A smaller time step leads to more accurate results as it captures the system's transient behaviour more precisely at the cost of increased computational time. Alternatively, larger time steps can significantly reduce the computation time as shown in the graph Figure 9, but at the risk of missing important transient behaviours or potentially leading to stability issues, as shown in the dataset below, where "Stability criterion not met" is present.

Grid Spacing (h)	Time Step (dt)	Thermal Equilibrium Tolerance (°C)	Time to Equilibrium (s)	Explicit/Analytical Difference (°C)	Time to compute (s)
0.75	0.1	0.001	0	28.4093	0.0008018
0.75	0.5	0.001	4116.5	15.534132	0.0092001
0.75	1	0.001	7879	12.4981	0.0088566
0.75	6	0.001	31824	4.09937843	0.008578
0.75	30	0.001	73500	0.846599988	0.0033068
0.75	60	0.001	93360	0.384984014	0.0026779
0.75	100	0.001	108000	0.221255126	0.0018707
0.75	150	0.001	119550	0.255570185	0.0017075
0.75	200	0.001	127800	0.27280876	0.0014769
0.75	360	0.001	144360	0.295486192	0.0012051
0.75	400	0.001	147200	0.298229328	0.0012062
0.75	500	0.001	153500	0.303423323	0.0011359
0.75	600	0.001	158400	0.306757555	0.0011301
0.75	800	0.001	165600	0.310762421	0.0010371
0.75	1000	0.001	172000	0.313549473	0.0009891
0.75	1500	0.001	181500	0.316785013	0.0009622
0.75	2000	0.001	188000	0.318488692	0.0009035
0.75	3600	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	6000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met

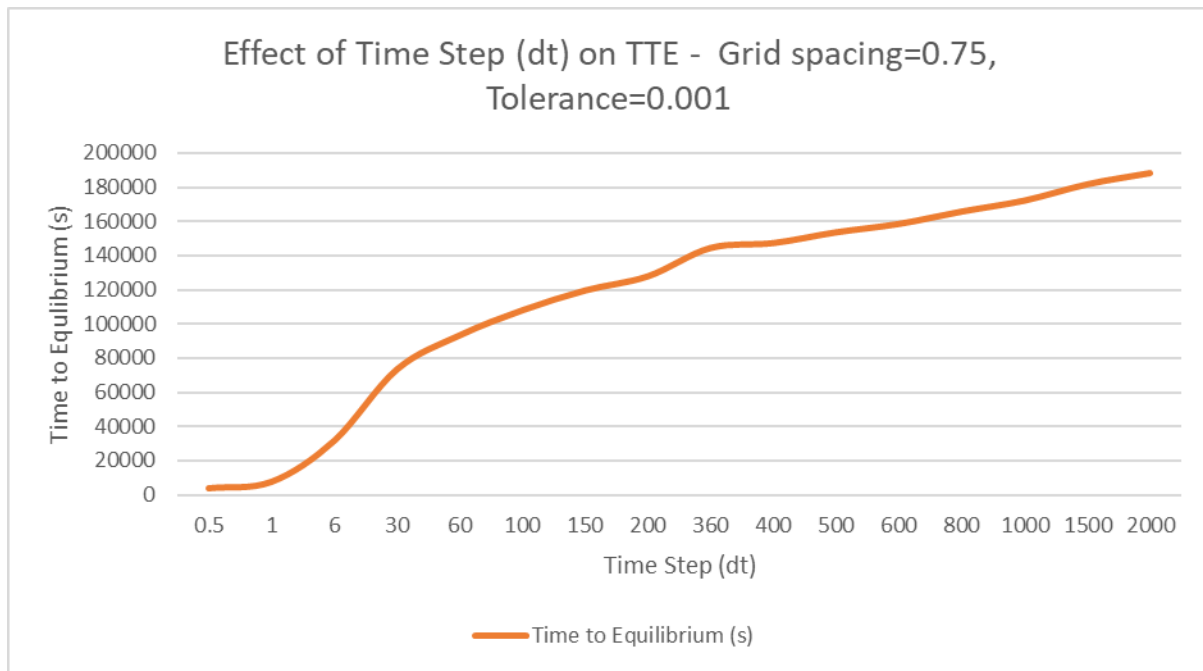


Figure 7 - Time Step and TTE Graph

The graph in Figure 7 illustrates the relationship between the time step (dt) and the time required to reach thermal equilibrium. From the graph, it is shown how altering the time step affects the time it takes for the system to reach thermal equilibrium. It clearly shows that as the time step increases, the time to equilibrium increases, indicating a slower approach to equilibrium. (Ferziger & Peric, 2012) suggests this is “due to the system overshooting the state of thermal equilibrium and therefore requiring additional iterations to dampen out the resulting oscillations, leading to an apparent slower approach to equilibrium.”

The zero time to equilibrium for a time step of 0.1 seconds is likely due to the time step being too short to capture the dynamics of the system accurately; this suggests the rate of temperature change in the material is slower than the selected time step and therefore the system recognises no significant change (less than the tolerance value) between the two timestep iterations of calculation. Thus, the model reaching thermal equilibrium. This can be overcome by using a more robust method for recognizing thermal equilibrium. These results reinforce the significance of correct discretization parameters to ensure stability and accuracy when modelling using EFD. The time step must be chosen carefully to balance the accuracy of the simulation with computational efficiency. If the time step is too large, it may skip over critical thermal dynamics; if it is too small, it may not increase accuracy but will increase

computational load. This trade-off would need to be made whilst considering the real-world implications of the system being modelled.

In the context of numerical simulations, this behaviour is expected as the time step is inversely proportional to the number of iterations required to reach equilibrium.

This is in line with the Courant-Friedrichs-Lewy (CFL) condition, which is a stability condition that must be satisfied when solving certain partial differential equations numerically by methods such as the finite difference method. According to (Thomas, 2013) “the time step must be small enough to maintain stability in the simulation but making it too small may lead to unnecessary computations without proportional gains in the precision of the solution”.

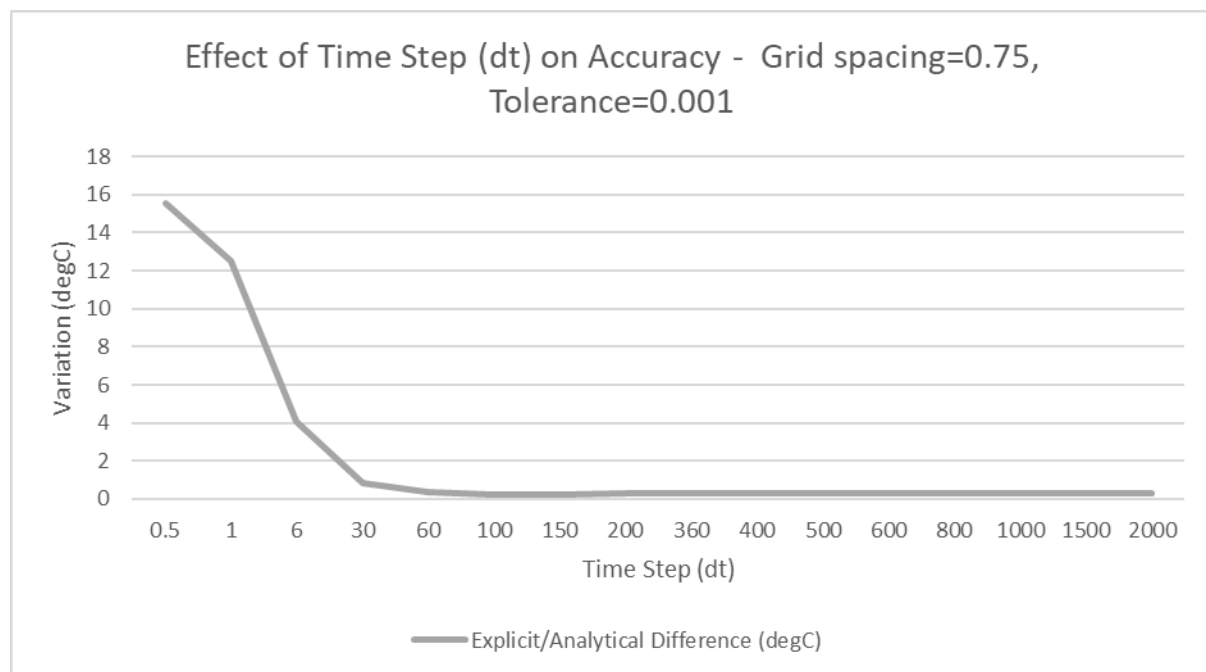


Figure 8 - Time Step and Accuracy Graph

By plotting the difference between the explicit and analytical solutions as a function of the time step size, Figure 8 shows there is a dramatic increase in the accuracy as the time step size decreases, especially noticeable below a time step size of 100. This suggests that larger time steps significantly improve the accuracy of the model in comparison to the analytical solution, which is un-expected since smaller time steps would typically provide better resolution of the thermal gradients and, therefore, a closer approximation to the continuous

nature of the phenomenon being modelled. The reason for this should be investigated further.

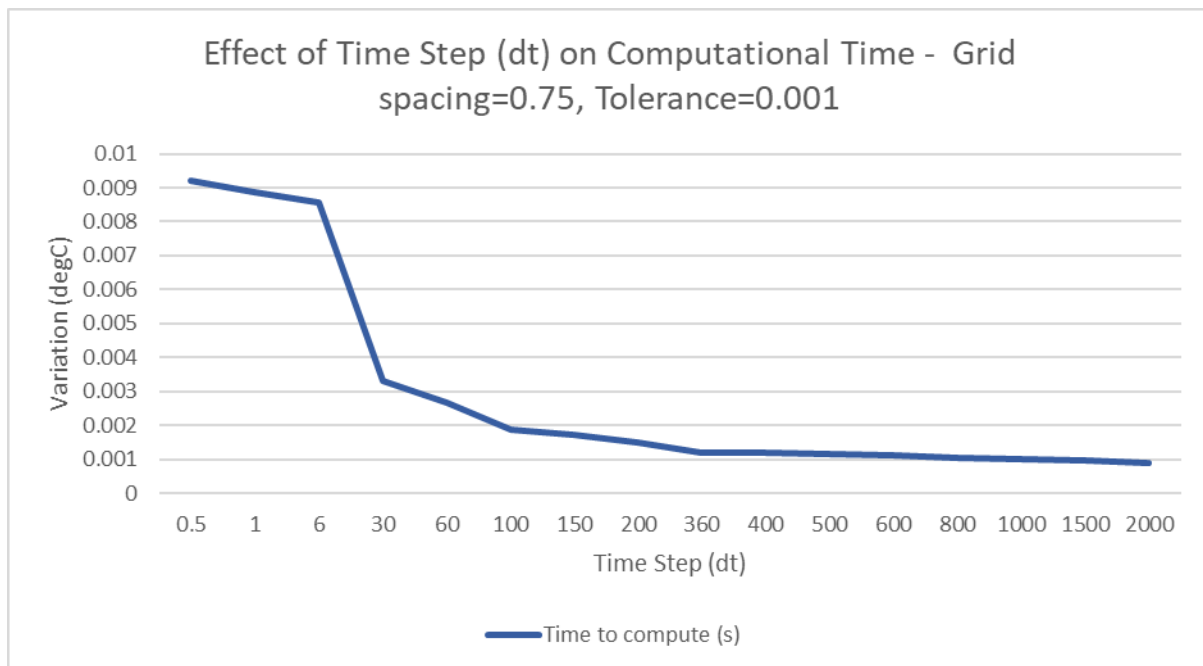


Figure 9 - Time Step and Computational Time

7.3. Thermal Equilibrium Tolerance

In this model, thermal equilibrium is determined by comparing the matrix values of the current time step with the matrix values of the previous time step. The thermal equilibrium tolerance value (TEV) is the threshold value for determining the minimum difference between the two matrices and therefore thermal equilibrium. A smaller tolerance value means the system must reach a closer steady state, to be considered at equilibrium.

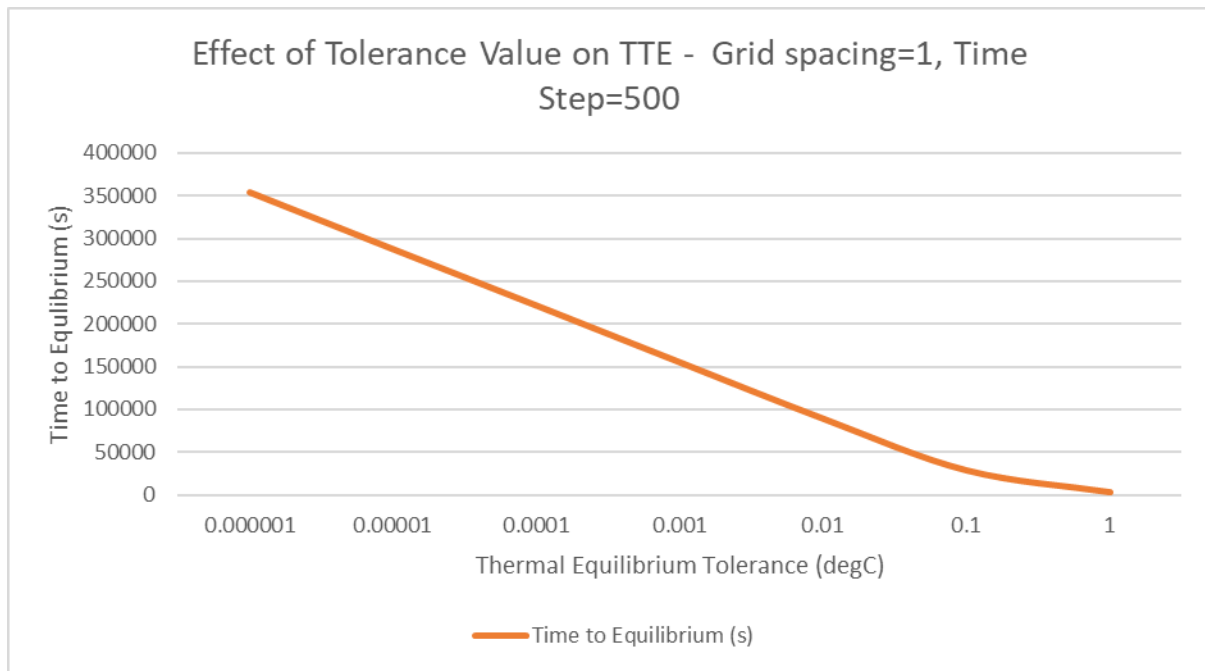


Figure 10 - TEV and TTE Graph

Figure 10 shows that as TEV decreases (becomes stricter), the time required to reach equilibrium increases significantly. This line suggests a logarithmic relationship; as tolerance requirements are made ten times smaller, the time to equilibrium increases in a near-linear fashion on the scale. (Patankar, 2018) states, this is because “the code is iterating more times to achieve a more precise equilibrium state, which is computationally more demanding.”

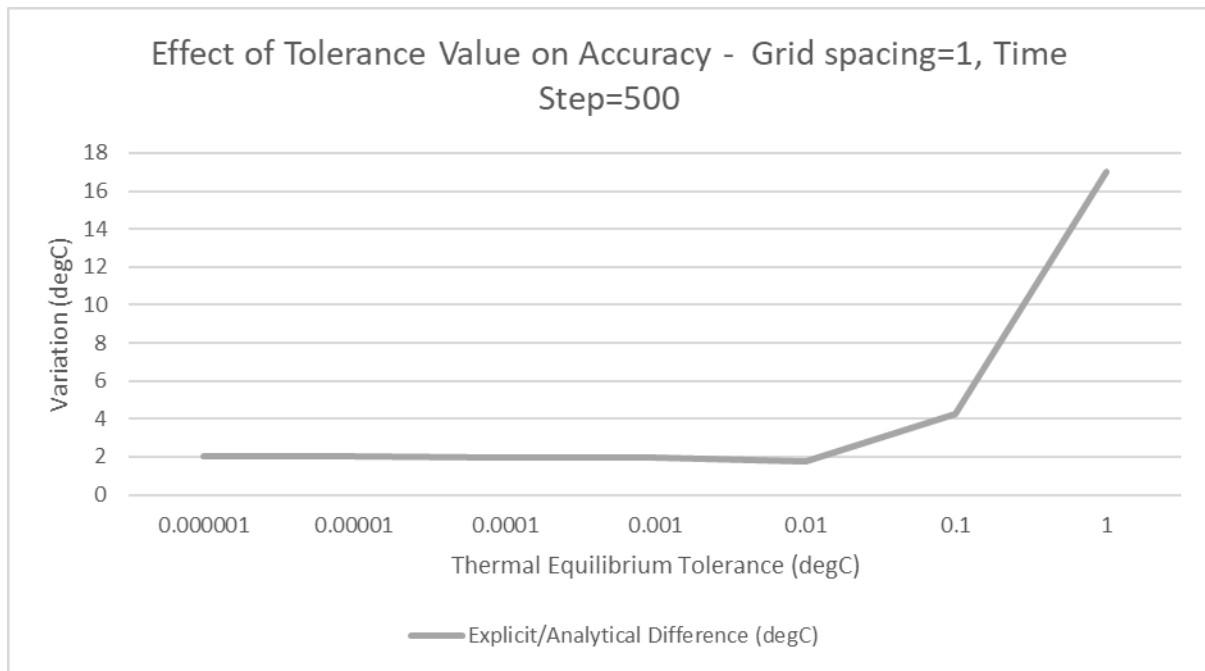


Figure 11 - TEV and Accuracy Graph

In Figure 11, it is shown that the explicit/analytical temperature difference remains constant for a wide range of tolerances, with a sharp increase between 0.01 and 1. This could be attributed to the numerical method used reaching its precision limit or an accumulation of truncation errors.

Below certain tolerance thresholds, the numerical solution can approximate the analytical solution with a reasonable level of accuracy. However, as the tolerance is relaxed, the numerical solution becomes considerably-less accurate. (Chapra, 2008)

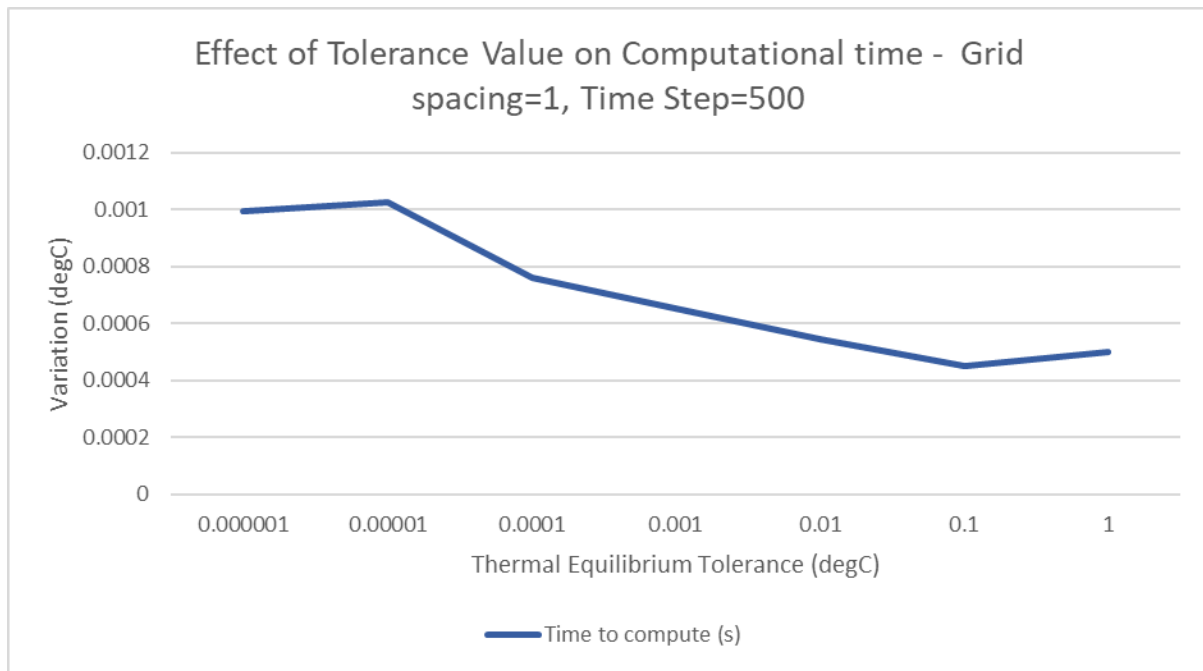


Figure 12 - TEV and Computational Time Graph

Figure 12 shows that as the tolerance value is relaxed (increases), the computational time decreases, this can be a crucial factor depending on the system being modelled. There is an interesting behaviour shown in the dataset below. Where thermal equilibrium is reached after 0 seconds when a tolerance value of 10°C is used, this is due to the rate of change between time steps being lower than the tolerance value used.

Grid Spacing (h)	Time Step (dt)	Thermal Equilibrium Tolerance (°C)	Time to Equilibrium (s)	Explicit/Analytical Difference (°C)	Time to compute (s)
1	500	0.000001	354000	1.993521987	0.000994
1	500	0.00001	287500	1.993295883	0.0010244
1	500	0.0001	221500	1.991063039	0.0007608
1	500	0.001	155000	1.968522579	0.0006539
1	500	0.01	89000	1.74153833	0.0005427
1	500	0.1	28500	4.24128192	0.0004486
1	500	1	3000	17.012217	0.0004992
1	500	10	0	23.57901	0.0004243

7.4. Overall Insights

7.4.1. Model Stability

The sensitivity analysis indicates that larger grid spacings coupled with intermediate time steps are beneficial to model stability. This stability is paramount, as it ensures that the simulation will not produce erratic or divergent results over time, which could compromise the validity of the simulation. However, it is important to note that excessively large grid spacings can miss the finer details of the thermal diffusion process, leading to a loss of resolution in the simulation's output.

7.4.2. Computational Efficiency

As the grid spacing and time step sizes are reduced, the model tends to offer a finer resolution, which can enhance the accuracy of the simulation by capturing more detailed thermal behaviours. Nevertheless, this increase in accuracy comes at a cost. The computational demand rises exponentially, as the number of calculations required per unit time increases. Furthermore, there exists a threshold beyond which further reductions in grid spacing or time step sizes do not yield significant improvements in accuracy but instead lead to numerical instability. This instability can manifest as non-physical oscillations or numerical errors that deteriorate the simulation's quality.

7.4.3. Accuracy vs Efficiency

The balance between accuracy and computational efficiency is a central theme in numerical modelling. Finer grid spacings and smaller time steps lead to more accurate simulations by more closely approximating the continuous nature of the physical problem. However, this increased accuracy must be weighed against the exponential growth in computational time and resource demand. There is also a risk of encountering stability problems, particularly when the ratio of the time step size to grid spacing does not satisfy the Courant–Friedrichs–Lewy (CFL) stability criteria.

7.5. Conclusion

In conclusion, each of the parameters trade-off accuracy for stability and computational demand. Whilst the model shows some sensitivity to thermal equilibrium tolerance, it is less critical compared to grid spacing and time step.

Smaller tolerances do not necessarily improve the model's accuracy significantly but do increase computational demands.

The ideal values for each parameter depend on the specific requirements of the simulation, such as the desired level of accuracy and available computational resources. (Anderson, 1995) states “A balance must be struck to ensure that the model is both accurate and efficient enough for each application.” This analysis underlines the importance of sensitivity analysis in computational modelling.

7.6. Recommendations

From the sensitivity analysis, it is possible to suggest optimal settings for specific goals.

7.6.1. Optimal Parameters for quickest time to equilibrium (>5°C accuracy)

Grid Spacing (h)	Time Step (dt)	Thermal Equilibrium Tolerance (°C)	Time to Equilibrium (s)	Explicit/ Analytical Difference (°C)	Time to compute (s)
3	6	0.001	16596	4.329392	0.00195
3	60	0.01	16620	4.323258	0.000226
3	600	0.1	16800	4.271663	0.000137

7.6.2. Optimal Parameters for quickest computing time (>5°C accuracy)

Grid Spacing (h)	Time Step (dt)	Thermal Equilibrium Tolerance (°C)	Time to Equilibrium (s)	Explicit/ Analytical Difference (°C)	Time to compute (s)
3	3600	0.1	75600	0.54615	0.0000556
3	800	0.1	26400	2.866114	0.0000607
3	1500	0.1	48000	0.802959	0.0000624

7.6.3. Optimal Parameters for Accuracy

Grid Spacing (h)	Time Step (dt)	Thermal Equilibrium Tolerance (°C)	Time to Equilibrium (s)	Explicit/ Analytical Difference (°C)	Time to compute (s)
3	2000	0.1	58000	0.196557191	0.0000646
0.1	30	0.0001	138750	0.19678218	0.2039075
0.1	0.5	0.000001	153326.5	0.198501463	9.32449

8. Limitations of the Model

There are inherent limitations of the numerical method and the coding used in the MATLAB model for simulating heat diffusion in an aluminium plate. Understanding these limitations is crucial for interpreting the results and for future model improvements.

8.1. Limitations Due to the Numerical Method

8.1.1. Round-off Error

Round-off errors arise due to the finite precision of numerical representation in computers. When dealing with extremely small or extremely large numbers, or in operations involving many calculations, (Quarteroni, et al., 2017) states “the accumulation of these errors can affect the accuracy of the results.”

8.1.2. Truncation Error

Truncation errors occur when an infinite process (like a Taylor series) is approximated by a finite one. (Burden & Faires, 2005) suggests that in the case of the heat equation, this error stems from “approximating derivatives using finite differences”.

8.1.3. Complex Geometry Handling

The current model assumes a simple 2D rectangular geometry. However, real-world problems often involve more complex geometries, which can be challenging to discretize and simulate accurately with the methods used. (Anderson, 1995)

8.1.4. Stability

Stability is a significant concern in explicit finite difference methods. If the time step is not chosen correctly (adhering to the stability criterion), the method can produce unphysical results, such as oscillations or divergence of the solution. (Ames, et al., 2014)

8.2. Limitations Due to the Coding

8.2.1. Modelling Thermal Equilibrium

In the model, thermal equilibrium is defined as a difference between successive temperature distributions. This approach, while computationally straightforward and less demanding, may

not accurately represent the physics of reaching thermal equilibrium. When talking about using EFDM to model thermal equilibrium (Cengel, 2007), Mentions that true equilibrium will involve more complex thresholds not captured by this simplistic approach.

8.3. General Considerations

The limitations highlighted above highlight the necessity for careful interpretation of the simulation results and the importance of validating the model against analytical solutions. They also open avenues for future improvements, such as incorporating more sophisticated numerical methods, enhancing the stability and accuracy of the simulations, or adopting more advanced approaches to determining thermal equilibrium.

9. Suggested Improvements and Future Developments

There are potential improvements and future developments that could be made to the heat diffusion model, focusing on enhancing accuracy, stability, and the ability to simulate complex geometries. Additionally, exploring more robust methods for modelling and recognising thermal equilibrium. These enhancements could make the model more versatile and applicable to a broader range of real-world problems in heat transfer.

9.1. Implementing ADI or Implicit Finite Difference Method

9.1.1. Alternating Direction Implicit (ADI) Method

ADI, a variant of the implicit method, offers better stability, especially for two-dimensional problems. It could allow larger time steps without sacrificing accuracy or stability. Implementing ADI would involve modifying the current explicit scheme to an implicit one, where the solution at a future time step depends on both the current and future states, thus requiring the solution of a system of linear equations at each step. (Patankar, 2018)

9.1.2. General Implicit Finite Difference Method

Unlike explicit methods, implicit methods are unconditionally stable for linear problems, allowing for larger time steps. Transitioning to an implicit scheme would significantly increase the computational effort per time step, as it involves solving a linear or non-linear system of equations, but it could enhance stability and potentially accuracy. (Chapra & Canale, 1988)

9.2. Numerical Alternatives for Complex Geometries

9.2.1. Finite Element Method (FEM)

FEM is particularly adept at handling complex geometries and boundary conditions. It offers flexibility in mesh generation and is more accurate in capturing the physical phenomena in irregular domains. Implementing FEM would require the development of a mesh of the domain, formulation of element equations, and assembly of a global system of equations. (Versteeg & Malalasekera, 2007)

9.2.2. Finite Volume Method (FVM)

FVM is widely used in computational fluid dynamics and heat transfer for its conservation properties and ability to model complex geometries. It involves dividing the physical domain into control volumes and applying the conservation laws to the volumes. (Versteeg & Malalasekera, 2007)

9.2.3. Boundary Element Method (BEM)

BEM is advantageous for problems with infinite or semi-infinite domains and reduces the dimensionality of the problem. It requires discretizing only the boundaries of the domain, making it computationally efficient for certain types of problems. (Brebbia & Walker, 2016)

9.3. More Robust Modelling of Thermal Equilibrium

There are other mechanisms for determining thermal equilibrium, by integrating one or more of these approaches, the model could not only determine equilibrium based on temperature but also account for the physical processes that drive the system towards this state. This would result in a more accurate and reliable simulation, which is particularly important for systems with complex thermal behaviours or where precise control of temperature is critical, such as in the use of light therapy for cancer treatment. These methods can also be calibrated based on experimental data or validated against well-established benchmarks to ensure their accuracy and reliability. (Cengel, 2007)

9.3.1. Dynamic Thresholds and Rate of Change

Instead of a static temperature threshold, the use of dynamic thresholds that adapt based on the rate of temperature change. This method would involve calculating the derivative of temperature with respect to time and setting a threshold for thermal equilibrium when this rate falls below a certain value that is physically justifiable, such as a percentage of the initial temperature change rate. (Zienkiewicz, et al., 2005)

9.3.2. Thermal Expansion

Ensuring spatial consistency in temperature distribution by implementing a check for uniformity across the spatial domain could prevent false positives for equilibrium in cases where parts of the system have not yet reached steady-state conditions. This is based on the

principle that materials expand and contract based on their temperature, also known as thermal expansion. (Callister & Rethwisch, 2018)

9.3.3. Energy Equilibrium

In addition to monitoring temperature, verifying that the energy within the system is at balance could be beneficial. This means the heat entering, exiting, and being generated within the system equals the heat stored, accounting for transient changes and ensuring a more accurate determination of equilibrium. (Reddy, 2006)

9.3.4. Artificial Intelligence

The training of machine learning algorithms to recognize patterns of thermal equilibrium from vast amounts of simulation data, could potentially offer a sophisticated way to predict when equilibrium is reached based on the unique characteristics of each simulation. (Goodfellow, et al., 2016)

9.3.5. Multi-Physics Models

For systems where thermal equilibrium is influenced by other physical processes (like fluid flow or phase change), using thermal analysis with other physics-based models can provide a more comprehensive view of equilibrium. (Bower, 2009)

9.3.6. Material-Specific Thermal Responses

Incorporating material-specific properties such as thermal inertia and time constants into the equilibrium criteria enables the model to account for the inherent thermal response characteristics of specific materials. (Incropera & DeWitt, 2007)

9.4. Adaptive Step-size Control

Another potential improvement that could be made to the model is Adaptive Step-size Control. Typically used in Runge-Kutta integration, it may be possible to apply this method to the thermal modelling simulation and adjust the integration step size dynamically in response to the changing behaviour of the system. This technique allows the program to take larger steps when the solution varies slowly, thus saving computational effort, and smaller steps when the solution exhibits rapid changes, thereby maintaining accuracy.

In the context of modelling thermal equilibrium, adaptive step size control could be a significant improvement over a fixed-step method. For instance, in regions where the temperature field is changing gradually, a larger time step could be used without sacrificing accuracy, which would lead to a reduction in the total number of computational steps required to reach equilibrium. Conversely, in areas with steep temperature gradients or near boundaries where conditions may change rapidly, the adaptive programming would reduce the step size, ensuring that the model captures the essential physics without introducing numerical instability. (Shampine, 2005)

10. Conclusion

In conclusion, this report has investigated the complexities of modelling heat diffusion in an aluminium plate, using the Explicit Finite Difference Method (EFDM). The report has documented the comparison of EFDM results against the Analytical Solution, establishing its accuracy in simulating thermal diffusion up to thermal equilibrium.

The sensitivity analysis revealed a trend of convergence with finer meshing. This observation emphasised the trade-off that comes with increased mesh resolution: while it offers a more detailed view of temperature distribution, it also demands higher computational resources. This finding resonates with core principles of numerical analysis, where the pursuit of accuracy inevitably leads to increased computational costs and potential challenges in numerical stability.

The study of time step variations has highlighted the critical relationship between time step size and the rate of reaching thermal equilibrium. Larger time steps were found to slow down the process of achieving equilibrium, whereas smaller steps enhanced the resolution of thermal dynamics. This discovery highlights the crucial need for careful selection of time step size, balancing between capturing essential thermal dynamics and avoiding superfluous computational efforts.

In examining thermal equilibrium tolerance conditions, the investigation revealed the complexity in defining equilibrium within a numerical model. The observed variance in time to reach equilibrium across different tolerance thresholds demands a more sophisticated and comprehensive definition of thermal equilibrium, one that extends beyond simple rate of change models, to better simulate the complex physics of heat transfer.

Reflecting on the initial objectives, this report has achieved its goals. It has not only conducted a theoretical examination of the heat equation but also implemented EFDM in MATLAB with precision, conducted a thorough evaluation of the model, and provided insightful critical analysis. This investigation has shed light on the intricate balance between accuracy, efficiency, and stability in thermal modelling, offering valuable insights for both academic and practical applications in engineering and scientific research.

Regarding the EFDM methodology used to model thermal equilibrium, the analysis suggests limitations in scalability, particularly under less stringent conditions. This poses potential challenges in complex or large-scale applications, where computational demands are high, and tolerances are less strict. In such scenarios, the accumulation of errors may render EFDM less suitable when a high degree of accuracy is required.

Extensive background research reveals the pivotal role of numerical modelling in addressing a wide array of challenges, from climate change and material development to innovative medical treatments. The field is continuously evolving, with ongoing advancements in threshold definition and discretization parameters promising enhanced accuracy and reduced computational load for increasingly complex problems.

Future enhancements to the model's code are outlined, focusing on resolving encountered issues, enhancing readability and maintainability through improved commenting, and refining error handling for user inputs. These improvements aim to bolster the model's robustness and user-friendliness, with the aim to aid the use of numerical modelling in solving scientific and engineering issues.

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12. Appendices

12.1. Appendix 1: DA_Comp_Methods_CW1.m MATLAB

code

```

%% Initialization
% Initialise the program for Temperature Distribution in an Aluminium Plate as per
assignment brief
clc % Clear the Command Window
clear all % Remove all variables from the workspace
close all % Close all figures (windows)

%% Material Properties
% Define the material properties
k = 143; % Thermal conductivity in W/(m*C)
rho = 2800; % Density in kg/m^3
c = 795; % Specific heat in J/(kg*C)
alpha = k/(rho*c); % Calculate thermal diffusivity using the properties

%% Stability Criterion
Fo_max = 0.25; % Maximum Fourier number for stability in explicit method, must be
<= 0.25 for a stable solution in a 2D heat equation

%% Plate Dimensions
% Define the plate dimensions
Lx = 6; % Length of the plate in the x-direction
Ly = 6; % Length of the plate in the y-direction

%% Grid Spacing
% User prompt for grid spacing (h)
while true % This loop will continue to prompt the user for a grid spacing value
that fits the plate dimensions
    h = input('\nEnter grid spacing (h) [m]: '); % User enters grid spacing
    l = h; % Set grid spacing in y-direction equal to x-direction for a square
grid
    R = Ly/h+1; % Calculate the number of rows in the grid
    C = Lx/l+1; % Calculate the number of columns in the grid

    % Check if grid spacing is compliant with plate dimensions
    if mod(Lx, h) <= 0 && mod(Ly, l) <= 0 && h>0 % Check if the entered grid
spacing divides evenly into the plate dimensions, and is greater than zero
        break; % Exit the loop if conditions are met
    else
        % If the grid spacing does not divide evenly, ask the user to enter a new
value
        fprintf('\nGrid spacing does not fit evenly into plate dimensions. Please
enter a new value!\n');
    end
end

%% Initialize temperature matrix
% Create a matrix filled with zeros to represent the initial temperature
distribution, where R is the number of rows and C is the number of columns
T = zeros(R, C);

```

```

%% Boundary conditions
% North boundary condition
for i = 1:R % This loop sets the temperature distribution along the north boundary
of the plate according to the assignment specifications
    x = (i-1)*h; % Calculate the distance from the left edge of the plate
    if x <= 2/3*Lx
        T(1,i) = 12.5*x; % Linear temperature increase for the first two-thirds of
the plate
    else
        T(1,i) = -25*x + 150; % Linear temperature decrease for the last third of
the plate
    end
end
% Update temperature matrix
T;
T_new = T; % Copy the temperature matrix to a new matrix for the next time step

%% Time Step input
% Prompt for time step (dt) within a loop to ensure stability criterion is met
while true % This loop prompts the user for a time step that will satisfy the
stability criterion for the simulation
    dt = input('\nEnter time step (dt) [s]: '); % User enters the time step
    Fo = (alpha*dt/h^2); % Calculate the Fourier number based on the entered time
step and grid spacing

    % Stability Check
    % Check stability criterion (assuming 2D heat conduction)

    if Fo <= Fo_max && Fo>0 % Check if the Fourier number is within the stable
range
        fprintf('\nStability criterion has been met at %.2f\n',Fo);
        break
    else % If the Fourier number is not within the stable range, ask the user to
enter a new time step
        fprintf('\nStability criterion is not met, \n\nCalculated Fo value is:
%.2f , it needs to be between %.2f and 0. \n\nPlease choose a smaller time
step!\n\n',Fo,Fo_max);
    end
end

%% Time-stepping loop

tic % Start a timer to measure the duration of the simulation process

t = 0; % Initialize the simulation time
while true % Enter an infinite loop to perform time stepping until the break
condition (thermal equilibrium) is met
    % Iterate through interior grid points to update temperatures using the
Explicit Finite Difference Method (EFDM)
    for i = 2:R-1 % This is the matrix for calculating the interior points, it
ignores the south boundary row, effectively setting the South boundary condition
to 0 as per the assignment brief

        for j = 2:C-1 % this also sets the East and West boundary conditon to 0 as
per the assignment brief

            T_new(i,j) = Fo*(T(i,j+1)+T(i,j-1)+T(i+1,j)+T(i-1,j)-(4-1/Fo)*T(i,j));

```

```

        end
    end

    %Thermal equilibrium accuracy value
    TEV = 0.01;

    % Check for thermal equilibrium
    if max(max(abs(T_new - T))) < TEV
        fprintf('\nThermal equilibrium reached at %.2f seconds\n\nThe Equilibrium
Tolerance value is set to %.5f degrees[c]\n', t, TEV);
        TTE=t;
        break;
    end

    % Update time
    t = t + dt;

    % Update temperature matrix
    T = T_new;

    %% Plotting section 1

    % Colour map of EFDM
    figure(1);
    contourf(flip(T));
    colormap("jet") % This setting changes the colour bar to make cold=blue,
hot=red
    colorbar;
    title(sprintf('Colour Map of Temperature Distribution at %.2f[s] using Finite
Difference Analysis\nh = %.2f, dt = %.2f, TEV = %.6f', t,h,dt,TEV));
    xlabel('Column number');
    ylabel('Row number');
    axis equal tight;

end
%% Calculate analytical solution

T_check = zeros(R,C);
T_check(1,:)=T(1,:);

for i = 2:R-1
    for j = 2:C-1
        for n = 1:200
            T_check(i,j) = T_check(i,j) + (sin(2*n*pi/3) / (n^2 * sinh(n*pi))) *
sin(n*pi*(i-1)*h/6) * sinh(n*pi*(j-1)*h/6);
        end
    end
end

T_check(2:R-1,2:C-1)=rot90(T_check(2:R-1,2:C-1)*(450/pi^2));
format shortG
T_check;
%% Plotting section 2

% Colour map of analytical solution at thermal equilibrium
figure(2);
contourf(flip(T_check));
colormap("jet") % This setting changes the colour bar to make cold=blue, hot=red
colorbar;

```

```

title(sprintf('Colour Map of Temperature Distribution at %.2f[s] using Analytical
Solution\nh = %.2f, dt = %.2f, TEV = %.6f', t,h,dt,TEV));
xlabel('Column number');
ylabel('Row number');
axis equal tight;

%% Values at thermal Equilibrium
T;
T_check;
%% Check analytical solution against explicit method
Diff_T_Tcheck = abs(T-T_check);
Ndiff_T_Tcheck = max(max(abs(T-T_check)));
Explicit_Analytical_Tolerance = 10;

if Ndiff_T_Tcheck<Explicit_Analytical_Tolerance
    fprintf('\nThe explicit method has produced results that are within %f
degrees[c] of the analytical solution. \n\nThis is within the tolerance value of
%.2f degrees[c]\n',Ndiff_T_Tcheck,Explicit_Analytical_Tolerance);
else
    fprintf('\nThe explicit method has produced results that are %f degrees[c]
different to the analytical solution. \n\nThis is outside the tolerance value of
%.2f degrees[c]\n',Ndiff_T_Tcheck,Explicit_Analytical_Tolerance);
end

Simulation_Runtime = toc;
fprintf('\nThe time taken to complete the simulation is %f
seconds\n',Simulation_Runtime)

%% Comparative Graph Plotting Section
% Create a new figure with a white background
figure('Color', 'w');

% Plot the first contour plot
contour(flip(T), 'LineWidth', 1.0, 'LineColor', 'b'); % Blue line for T
colormap("jet"); % Set colormap to 'jet'
colorbar;
xlabel('Column number');
ylabel('Row number');
axis equal tight;
hold on; % Hold on to overlay the next plot

% Overlay the second contour plot with a different line style or colour
contour(flip(T_check), 'LineWidth', 1.0, 'LineStyle', '--', 'LineColor', 'r'); %
Red dashed line for T_check

% Add labels and title
title(sprintf('Temperature Distribution of EFDM and Analytical Solution\n t =
%.2f[s], h = %.2f, dt = %.2f, TEV = %.6f', t,h,dt,TEV));
xlabel('Column number');
ylabel('Row number');

% Create dummy plots for the legend
h1_legend = plot(NaN, NaN, 'b-', 'LineWidth', 1.0); % Blue solid line for T
h2_legend = plot(NaN, NaN, 'r--', 'LineWidth', 1.0); % Red dashed line for T_check

% Add legend
legend([h1_legend, h2_legend], {'T', 'T_check'}, 'Location', 'best');

hold off; % Release the plot

```

```
% % End of the program
```

12.2. Appendix 2: runSimulations.m MATLAB code

```

%% Initialise the program
clc
clear all
close all

%% Start total simulation timer
tic;
fprintf('The simulation has started...\n')

%% Define parameter ranges
h_values = [-1,0.1,0.2,0.25,0.5, 0.75, 1.0,1.2,1.5,2,3];
dt_values = [6000,3600,2000,1500,1000,800,600,500,400,360,200,150,100,60,30,6,1,0.5,0.1];
TEV_values = [10,1,0.1,0.01,0.001,0.0001,0.00001,0.000001];

%% Initialize results array
results = [];

%% Run simulations with different parameters
for h = h_values
    for dt = dt_values
        for TEV = TEV_values
            [time_to_equilibrium, max_diff, sim_step_timer] =
simulateTemperatureDistribution(h, dt, TEV);
            results = [results; h, dt, TEV, time_to_equilibrium, max_diff,
sim_step_timer];
        end
    end
end

%% Convert results to table and write to CSV
resultsTable = array2table(results, 'VariableNames', {'Grid_Spacing_(h)',
'Time_Step_(dt)', 'Thermal_Equilibrium_Tolerance_(degC)',
'Time_to_Equilibrium_(s)', 'Explicit/Analytical_Difference_(degC)', 'Time_to_compute_(s)'});

% Get the current date and time
currentDateTime = datetime('now');

% Format the date and time as a string suitable for a filename
formattedDateTime = datestr(currentDateTime, 'dd_mm_yyyy_HH_MM_SS');

% Write the filename
filename = ['simulation_results_' formattedDateTime '.csv'];
writetable(resultsTable, filename);

%% End the total simulation timer
total_sim_time = toc

```

12.3. Appendix 3: simulateTemperatureDistribution.m

MATLAB function code

```
function [time_to_equilibrium, max_diff, sim_step_timer] =
simulateTemperatureDistribution(h, dt, TEV)
    %Start Timer
    tic;
    % Define the material properties and plate dimensions
    k = 143; rho = 2800; c = 795; alpha = k / (rho * c); Fo_max = 0.25;
    Lx = 6; Ly = 6;

    % Calculate grid dimensions
    R = Ly / h + 1; C = Lx / h + 1;

    % Check if grid spacing fits into plate dimensions
    if mod(Lx, h) ~= 0 || mod(Ly, h) ~= 0 || h < 0
        % Return NaN values to indicate an error
        time_to_equilibrium = "Grid";
        max_diff = "Grid";
        sim_step_timer = "Grid";
        return;
    end

    % Initialize temperature matrix
    T = zeros(R, C);

    % Apply boundary conditions
    for i = 1:R
        x = (i-1) * h;
        if x <= 2/3 * Lx
            T(1, i) = 12.5 * x;
        else
            T(1, i) = -25 * x + 150;
        end
    end

    % Check stability criterion
    Fo = alpha * dt / h^2;
    if Fo > Fo_max || Fo <= 0
        % Return NaN values to indicate an error
        time_to_equilibrium = "Stability";
        max_diff = "Stability";
        sim_step_timer = "Stability";
        return;
    end

    T_new = T; t = 0;

    % Time-stepping loop
    while true
        for i = 2:R-1
            for j = 2:C-1
                T_new(i, j) = Fo * (T(i, j+1) + T(i, j-1) + T(i+1, j) + T(i-1, j)
- (4 - 1/Fo) * T(i, j));
            end
        end
    end
```

```

        end

        if max(max(abs(T_new - T))) < TEV
            time_to_equilibrium = t;
            break;
        end

        t = t + dt;
        T = T_new;
    end

%% Calculate analytical solution

T_check = zeros(R,C);
T_check(1,:)=T(1,:);

for i = 2:R-1
    for j = 2:C-1
        for n = 1:200
            T_check(i,j) = T_check(i,j) + (sin(2*n*pi/3) / (n^2 * sinh(n*pi))) *
sin(n*pi*(i-1)*h/6) * sinh(n*pi*(j-1)*h/6);
        end
    end
end
T_check(2:R-1,2:C-1)=rot90(T_check(2:R-1,2:C-1)*(450/pi^2));
format shortG
T_check;

% Check analytical solution against explicit method
Diff_T_Tcheck = abs(T-T_check);
Ndiff_T_Tcheck = max(max(abs(T-T_check)));
max_diff = max(max(abs(T - T_check)));

%% End Timer
sim_step_timer = toc;
end

```

12.4. Appendix 4: Simulation Results Data Set

Grid Spacing (h)	Time Step (dt)	Thermal Equilibrium Tolerance (°C)	Time to Equilibrium (s)	Explicit/Analytical Difference (°C)	Time to compute (s)
-1	0.1	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.1	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.1	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.1	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.1	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.1	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.1	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.1	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.5	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.5	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.5	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.5	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.5	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.5	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.5	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	0.5	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size

-1	1	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	30	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	30	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	30	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	30	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	30	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	30	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	30	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	30	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	60	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	60	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	60	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	60	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	60	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	60	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	60	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size

-1	60	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	100	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	100	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	100	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	100	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	100	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	100	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	100	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	100	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	150	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	150	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	150	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	150	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	150	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	150	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	150	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	150	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	200	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	200	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	200	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	200	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	200	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	200	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	200	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	200	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size

-1	360	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	360	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	360	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	360	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	360	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	360	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	360	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	360	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	400	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	400	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	400	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	400	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	400	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	400	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	400	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	400	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	500	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	500	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	500	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	500	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	500	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	500	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	500	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	500	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	600	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size

-1	600	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	600	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	600	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	600	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	600	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	600	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	600	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	800	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	800	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	800	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	800	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	800	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	800	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	800	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	800	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1000	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1000	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1000	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1000	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1000	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1000	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1000	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1000	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1500	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1500	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size

-1	1500	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1500	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1500	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1500	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1500	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	1500	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	2000	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	2000	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	2000	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	2000	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	2000	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	2000	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	2000	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	2000	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	3600	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	3600	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	3600	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	3600	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	3600	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	3600	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	3600	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	3600	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6000	0.000001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6000	0.00001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6000	0.0001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size

-1	6000	0.001	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6000	0.01	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6000	0.1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6000	1	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
-1	6000	10	Incompatible Grid size	Incompatible Grid size	Incompatible Grid size
0.1	0.1	0.000001	107625.3	0.279608409	32.3813161
0.1	0.1	0.00001	43997.2	2.69329699	12.9773207
0.1	0.1	0.0001	8603.9	12.525975	2.6293515
0.1	0.1	0.001	1072.1	26.593886	0.4285307
0.1	0.1	0.01	105.6	37.21661	0.0874191
0.1	0.1	0.1	0	44.7063	0.0549775
0.1	0.1	1	0	44.7063	0.0558903
0.1	0.1	10	0	44.7063	0.0655034
0.1	0.5	0.000001	153326.5	0.198501463	9.32449
0.1	0.5	0.00001	87960	0.56320837	5.2016767
0.1	0.5	0.0001	28888	4.75739493	1.7481781
0.1	0.5	0.001	4751.5	16.724174	0.3312664
0.1	0.5	0.01	540.5	30.47656	0.0834091
0.1	0.5	0.1	38	40.1437	0.055131
0.1	0.5	1	0	44.7063	0.0523777
0.1	0.5	10	0	44.7063	0.0522176
0.1	1	0.000001	173008	0.199789606	5.1286707
0.1	1	0.00001	107624	0.279604354	3.2017764
0.1	1	0.0001	43997	2.69324375	1.3361629
0.1	1	0.001	8604	12.525642	0.3040567
0.1	1	0.01	1072	26.594964	0.0834952
0.1	1	0.1	105	37.23666	0.0562062
0.1	1	1	0	44.7063	0.0558698
0.1	1	10	0	44.7063	0.0625036
0.1	6	0.000001	223866	0.200862339	1.1536488
0.1	6	0.00001	158490	0.198931086	0.8354314
0.1	6	0.0001	93120	0.468751583	0.5089436
0.1	6	0.001	32472	4.13586471	0.2119629
0.1	6	0.01	5562	15.615304	0.081263
0.1	6	0.1	660	29.32089	0.057964
0.1	6	1	54	39.3678	0.0566394
0.1	6	10	0	44.7063	0.0576472
0.1	30	0.000001	269460	0.201033963	0.3429018
0.1	30	0.00001	204120	0.200648139	0.2631126
0.1	30	0.0001	138750	0.19678218	0.2039075
0.1	30	0.001	73500	0.938943865	0.1288003
0.1	30	0.01	20310	6.798311	0.0739952

0.1	30	0.1	3000	19.924909	0.0584294
0.1	30	1	330	32.9703	0.0685312
0.1	30	10	0	44.7063	0.085298
0.1	60	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	60	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	60	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	60	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	60	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	60	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	60	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	60	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	100	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	100	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	100	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	100	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	100	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	100	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	100	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	100	10	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.1	150	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	150	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	150	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	150	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	150	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	150	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	150	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	150	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	200	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	200	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	200	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	200	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	200	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	200	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	200	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	200	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	360	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.1	360	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	360	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	360	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	360	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	360	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	360	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	360	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	400	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	400	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	400	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	400	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	400	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	400	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	400	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	400	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	500	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	500	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.1	500	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	500	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	500	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	500	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	500	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	500	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	600	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	600	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	600	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	600	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	600	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	600	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	600	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	600	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	800	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	800	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	800	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.1	800	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	800	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	800	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	800	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	800	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1500	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1500	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1500	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1500	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.1	1500	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1500	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1500	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	1500	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	2000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	2000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	2000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	2000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	2000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	2000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	2000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	2000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	3600	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	3600	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	3600	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	3600	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	3600	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.1	3600	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	3600	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	3600	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	6000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	6000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	6000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	6000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	6000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	6000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	6000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.1	6000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	0.1	0.000001	107679.6	0.37621916	7.7211919
0.2	0.1	0.00001	43984.5	2.68050418	3.2517207
0.2	0.1	0.0001	8598.9	12.47946	0.6393087
0.2	0.1	0.001	1035.1	26.363386	0.0918842
0.2	0.1	0.01	0	41.0326	0.0132165
0.2	0.1	0.1	0	41.0326	0.0133974
0.2	0.1	1	0	41.0326	0.013132
0.2	0.1	10	0	41.0326	0.013042
0.2	0.5	0.000001	153412	0.396945657	2.3352385
0.2	0.5	0.00001	87982.5	0.550905739	1.3315768
0.2	0.5	0.0001	28869.5	4.74426549	0.440678
0.2	0.5	0.001	4733.5	16.698734	0.0816045
0.2	0.5	0.01	509	29.77659	0.0206714
0.2	0.5	0.1	0	41.0326	0.0130683
0.2	0.5	1	0	41.0326	0.0130265
0.2	0.5	10	0	41.0326	0.0160764

0.2	1	0.000001	173107	0.399520226	1.2891472
0.2	1	0.00001	107678	0.376219291	0.8126457
0.2	1	0.0001	43984	2.68048272	0.3278266
0.2	1	0.001	8599	12.479212	0.0733572
0.2	1	0.01	1035	26.363661	0.0200857
0.2	1	0.1	0	41.0326	0.0129852
0.2	1	1	0	41.0326	0.0131377
0.2	1	10	0	41.0326	0.0127955
0.2	6	0.000001	224004	0.401664304	0.2825043
0.2	6	0.00001	158580	0.397804477	0.1997771
0.2	6	0.0001	93156	0.456157375	0.1242839
0.2	6	0.001	32484	4.11366419	0.052663
0.2	6	0.01	5556	15.50965	0.0195843
0.2	6	0.1	600	29.25542	0.0135851
0.2	6	1	0	41.0326	0.013049
0.2	6	10	0	41.0326	0.0208896
0.2	30	0.000001	269640	0.4020073	0.0772272
0.2	30	0.00001	204240	0.401236119	0.0608899
0.2	30	0.0001	138840	0.393513701	0.0456737
0.2	30	0.001	73530	0.926405692	0.0307233
0.2	30	0.01	20310	6.77582813	0.0177907
0.2	30	0.1	2970	19.880487	0.0135868
0.2	30	1	300	32.61742	0.0128982
0.2	30	10	0	41.0326	0.0129808
0.2	60	0.000001	289200	0.402050174	0.0481118
0.2	60	0.00001	223800	0.401664439	0.0404547
0.2	60	0.0001	158460	0.397809157	0.0321498
0.2	60	0.001	93120	0.455281376	0.0240686
0.2	60	0.01	32460	4.11164695	0.0167403
0.2	60	0.1	5520	15.543179	0.01346
0.2	60	1	600	29.34775	0.0130685
0.2	60	10	0	41.0326	0.0129259
0.2	100	0.000001	303500	0.402067304	0.0345417
0.2	100	0.00001	238200	0.401836467	0.0302548
0.2	100	0.0001	172900	0.399528825	0.0259144
0.2	100	0.001	107500	0.376231891	0.0206047
0.2	100	0.01	43900	2.68092429	0.015874
0.2	100	0.1	8600	12.459135	0.0134068
0.2	100	1	1000	26.55879	0.0137795
0.2	100	10	0	41.0326	0.0129191
0.2	150	0.000001	314700	0.402075844	0.0283977
0.2	150	0.00001	249450	0.401921781	0.0244685
0.2	150	0.0001	184200	0.400381356	0.0215418
0.2	150	0.001	118950	0.384930344	0.0189583
0.2	150	0.01	54300	1.83791898	0.016324
0.2	150	0.1	12000	10.1750833	0.0134224

0.2	150	1	1500	24.361781	0.0129524
0.2	150	10	150	35.63962	0.0128992
0.2	200	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	200	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	200	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	200	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	200	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	200	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	200	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	200	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	360	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	360	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	360	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	360	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	360	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	360	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	360	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	360	10	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.2	400	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	400	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	400	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	400	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	400	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	400	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	400	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	400	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	500	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	500	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	500	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	500	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	500	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	500	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	500	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	500	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	600	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.2	600	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	600	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	600	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	600	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	600	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	600	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	600	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	800	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	800	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	800	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	800	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	800	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	800	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	800	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	800	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.2	1000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1500	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1500	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1500	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1500	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1500	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1500	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1500	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	1500	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	2000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	2000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	2000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.2	2000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	2000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	2000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	2000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	2000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	3600	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	3600	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	3600	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	3600	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	3600	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	3600	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	3600	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	3600	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	6000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	6000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	6000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	6000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.2	6000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	6000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	6000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.2	6000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	0.1	0.000001	107720.4	0.470230716	5.1203175
0.25	0.1	0.00001	43944.7	2.67822066	2.0925483
0.25	0.1	0.0001	8594.5	12.479599	0.4131285
0.25	0.1	0.001	973.1	26.32446	0.0545798
0.25	0.1	0.01	0	39.4354	0.0082612
0.25	0.1	0.1	0	39.4354	0.0084964
0.25	0.1	1	0	39.4354	0.0085353
0.25	0.1	10	0	39.4354	0.009089
0.25	0.5	0.000001	153476.5	0.496126943	1.4687965
0.25	0.5	0.00001	88013	0.541411624	0.841041
0.25	0.5	0.0001	28869	4.72828588	0.2814981
0.25	0.5	0.001	4726	16.640482	0.0517642
0.25	0.5	0.01	487	30.14202	0.0127066
0.25	0.5	0.1	0	39.4354	0.0080271
0.25	0.5	1	0	39.4354	0.0081248
0.25	0.5	10	0	39.4354	0.0080478
0.25	1	0.000001	173181	0.499343434	0.8280632
0.25	1	0.00001	107719	0.470231108	0.5230226
0.25	1	0.0001	43944	2.67821766	0.2171618
0.25	1	0.001	8594	12.479777	0.0482391
0.25	1	0.01	973	26.32563	0.0127828
0.25	1	0.1	0	39.4354	0.0082817
0.25	1	1	0	39.4354	0.008174
0.25	1	10	0	39.4354	0.0082207
0.25	6	0.000001	224106	0.502022232	0.1826328
0.25	6	0.00001	158646	0.497199701	0.1359866
0.25	6	0.0001	93192	0.44832662	0.0823455
0.25	6	0.001	32478	4.11220849	0.035063
0.25	6	0.01	5502	15.559047	0.0130984
0.25	6	0.1	600	29.05824	0.0094471
0.25	6	1	0	39.4354	0.0093115
0.25	6	10	0	39.4354	0.0086484
0.25	30	0.000001	269760	0.502450714	0.0503992
0.25	30	0.00001	204330	0.501487232	0.0408798
0.25	30	0.0001	138900	0.491840264	0.0306606

0.25	30	0.001	73530	0.9178731	0.0199214
0.25	30	0.01	20250	6.7597222	0.0125115
0.25	30	0.1	2910	19.981055	0.0084965
0.25	30	1	210	32.57233	0.0082132
0.25	30	10	0	39.4354	0.0085523
0.25	60	0.000001	289320	0.502504268	0.030755
0.25	60	0.00001	223920	0.50202274	0.0257325
0.25	60	0.0001	158520	0.497204412	0.0214007
0.25	60	0.001	93120	0.448366634	0.0155391
0.25	60	0.01	32460	4.10941077	0.010624
0.25	60	0.1	5460	15.597445	0.0094284
0.25	60	1	600	29.05448	0.0085701
0.25	60	10	0	39.4354	0.0081453
0.25	100	0.000001	303600	0.502525649	0.022071
0.25	100	0.00001	238300	0.502237207	0.0202886
0.25	100	0.0001	172900	0.499345803	0.0169535
0.25	100	0.001	107600	0.470314512	0.0134325
0.25	100	0.01	43900	2.67465385	0.0100969
0.25	100	0.1	8600	12.448262	0.0084123
0.25	100	1	900	26.71458	0.008231
0.25	100	10	0	39.4354	0.0080681
0.25	150	0.000001	314850	0.502536368	0.0178576
0.25	150	0.00001	249600	0.502344157	0.0162482
0.25	150	0.0001	184350	0.500424587	0.0139901
0.25	150	0.001	118950	0.481079373	0.0125662
0.25	150	0.01	54300	1.82732516	0.0096493
0.25	150	0.1	12000	10.1624876	0.0084587
0.25	150	1	1500	24.135267	0.0081825
0.25	150	10	0	39.43541	0.0084992
0.25	200	0.000001	322800	0.50254176	0.0157876
0.25	200	0.00001	257600	0.502397955	0.0140976
0.25	200	0.0001	192400	0.500961477	0.0127034
0.25	200	0.001	127000	0.486469699	0.0116282
0.25	200	0.01	62200	1.37156959	0.0096358
0.25	200	0.1	14800	8.73488196	0.0086008
0.25	200	1	2000	22.200223	0.009322
0.25	200	10	200	33.3347	0.0089491
0.25	360	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	360	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	360	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.25	360	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	360	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	360	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	360	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	360	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	400	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	400	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	400	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	400	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	400	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	400	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	400	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	400	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	500	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	500	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	500	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	500	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.25	500	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	500	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	500	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	500	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	600	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	600	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	600	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	600	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	600	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	600	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	600	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	600	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	800	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	800	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	800	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	800	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	800	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.25	800	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	800	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	800	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1500	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1500	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1500	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1500	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1500	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1500	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.25	1500	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	1500	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	2000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	2000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	2000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	2000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	2000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	2000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	2000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	2000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	3600	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	3600	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	3600	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	3600	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	3600	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	3600	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	3600	1	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.25	3600	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	6000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	6000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	6000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	6000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	6000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	6000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	6000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.25	6000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	0.1	0.000001	108060.2	0.939603383	1.7184283
0.5	0.1	0.00001	44010.4	2.57779006	0.6996999
0.5	0.1	0.0001	8528.9	12.243954	0.1365629
0.5	0.1	0.001	426	28.49935	0.009059
0.5	0.1	0.01	0	32.8056	0.0018131
0.5	0.1	0.1	0	32.8056	0.0018442
0.5	0.1	1	0	32.8056	0.001811
0.5	0.1	10	0	32.8056	0.0018259
0.5	0.5	0.000001	154012.5	0.99119796	0.4846604
0.5	0.5	0.00001	88268	0.873774008	0.2812084
0.5	0.5	0.0001	28777	4.67085402	0.0920263
0.5	0.5	0.001	4251	16.73112	0.0156752
0.5	0.5	0.01	0	32.8056	0.0018954
0.5	0.5	0.1	0	32.8056	0.0019096
0.5	0.5	1	0	32.8056	0.0019622
0.5	0.5	10	0	32.8056	0.0019424
0.5	1	0.000001	173802	0.997604082	0.2842698
0.5	1	0.00001	108059	0.939604615	0.1718225
0.5	1	0.0001	44010	2.57776455	0.070974
0.5	1	0.001	8529	12.243651	0.0153335
0.5	1	0.01	426	28.49849	0.0026701
0.5	1	0.1	0	32.8056	0.0018492

0.5	1	1	0	32.8056	0.0018509
0.5	1	10	0	32.8056	0.0019034
0.5	6	0.000001	224946	1.002938837	0.0606702
0.5	6	0.00001	159204	0.993334442	0.0446533
0.5	6	0.0001	93468	0.895930696	0.0293756
0.5	6	0.001	32466	4.04098786	0.0103481
0.5	6	0.01	4998	15.441976	0.0031615
0.5	6	0.1	0	32.8056	0.0018453
0.5	6	1	0	32.8056	0.0019917
0.5	6	10	0	32.8056	0.0019029
0.5	30	0.000001	270780	1.003791997	0.0159654
0.5	30	0.00001	205080	1.001873018	0.01264
0.5	30	0.0001	139380	0.982664388	0.0090571
0.5	30	0.001	73650	0.843028327	0.0056457
0.5	30	0.01	20220	6.68021678	0.0028971
0.5	30	0.1	2760	19.737131	0.0020044
0.5	30	1	0	32.8056	0.0018508
0.5	30	10	0	32.8056	0.0019057
0.5	60	0.000001	290460	1.003898838	0.009368
0.5	60	0.00001	224760	1.00293984	0.0076802
0.5	60	0.0001	159060	0.993337013	0.0058368
0.5	60	0.001	93420	0.896102627	0.0045076
0.5	60	0.01	32460	4.03636845	0.0026932
0.5	60	0.1	5040	15.403614	0.001982
0.5	60	1	0	32.8056	0.0018375
0.5	60	10	0	32.8056	0.001879
0.5	100	0.000001	304800	1.003941383	0.0064861
0.5	100	0.00001	239200	1.003366876	0.0058022
0.5	100	0.0001	173600	0.997626447	0.0044283
0.5	100	0.001	107900	0.939678962	0.0035662
0.5	100	0.01	44000	2.57176672	0.0024803
0.5	100	0.1	8500	12.243066	0.0021525
0.5	100	1	500	27.72927	0.0018124
0.5	100	10	0	32.8056	0.0019412
0.5	150	0.000001	316050	1.003962642	0.0052801
0.5	150	0.00001	250500	1.003579136	0.004522
0.5	150	0.0001	184950	0.99974654	0.0038283
0.5	150	0.001	119400	0.961313377	0.0033508
0.5	150	0.01	54450	1.74814888	0.0025432
0.5	150	0.1	11550	10.0624608	0.0020269
0.5	150	1	1200	23.137619	0.0018205
0.5	150	10	0	32.80559	0.0018169
0.5	200	0.000001	324000	1.003973352	0.0043203
0.5	200	0.00001	258600	1.003687189	0.003766
0.5	200	0.0001	193000	1.000814437	0.0035971
0.5	200	0.001	127600	0.972158881	0.0027736

0.5	200	0.01	62200	1.3034062	0.002282
0.5	200	0.1	14800	8.59570079	0.0021554
0.5	200	1	1800	21.882201	0.001868
0.5	200	10	0	32.80559	0.001806
0.5	360	0.000001	339840	1.003987595	0.0033944
0.5	360	0.00001	274680	1.003829393	0.0030997
0.5	360	0.0001	209160	1.002234401	0.0027634
0.5	360	0.001	144000	0.986387322	0.0024221
0.5	360	0.01	78480	0.822113553	0.0021464
0.5	360	0.1	22680	5.90866201	0.0021581
0.5	360	1	3240	18.592266	0.0021065
0.5	360	10	0	32.80559	0.001846
0.5	400	0.000001	342800	1.003989484	0.0031795
0.5	400	0.00001	277200	1.003845452	0.0029873
0.5	400	0.0001	212000	1.002411549	0.002734
0.5	400	0.001	146800	0.98810344	0.0024481
0.5	400	0.01	81600	0.842058022	0.0022568
0.5	400	0.1	24400	5.50589534	0.0019872
0.5	400	1	3200	18.663828	0.00186
0.5	400	10	0	32.80559	0.0018473
0.5	500	0.000001	348500	1.003992641	0.002934
0.5	500	0.00001	283000	1.003877312	0.002752
0.5	500	0.0001	218000	1.00273303	0.002583
0.5	500	0.001	153000	0.991351617	0.0023635
0.5	500	0.01	87500	0.874019395	0.0021217
0.5	500	0.1	28500	4.66912871	0.0019368
0.5	500	1	4000	16.927535	0.0019665
0.5	500	10	0	32.80559	0.0018493
0.5	600	0.000001	352800	1.003994663	0.0027687
0.5	600	0.00001	288000	1.003899963	0.0026958
0.5	600	0.0001	222600	1.002938935	0.0024699
0.5	600	0.001	157800	0.993432486	0.0022611
0.5	600	0.01	93000	0.898057095	0.0020926
0.5	600	0.1	32400	3.98938735	0.0019364
0.5	600	1	5400	15.04544	0.0018636
0.5	600	10	0	32.80559	0.0018931
0.5	800	0.000001	360000	1.003997421	0.0025984
0.5	800	0.00001	295200	1.003926658	0.002514
0.5	800	0.0001	230400	1.003219498	0.0023803
0.5	800	0.001	165600	0.996151087	0.0076678
0.5	800	0.01	100800	0.924982507	0.0045147
0.5	800	0.1	38400	3.1281677	0.0034116
0.5	800	1	7200	13.219629	0.0027495
0.5	800	10	800	24.47594	0.0027038
0.5	1000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.5	1000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1500	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1500	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1500	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1500	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1500	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1500	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1500	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	1500	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	2000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	2000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.5	2000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	2000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	2000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	2000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	2000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	2000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	3600	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	3600	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	3600	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	3600	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	3600	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	3600	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	3600	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	3600	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	6000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	6000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	6000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.5	6000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	6000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	6000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	6000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.5	6000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	0.1	0.000001	108145.5	0.221086315	1.0818897
0.75	0.1	0.00001	43838.3	2.57332547	0.429856
0.75	0.1	0.0001	7879.2	12.498093	0.0790287
0.75	0.1	0.001	0	28.4093	0.0007861
0.75	0.1	0.01	0	28.4093	0.0007885
0.75	0.1	0.1	0	28.4093	0.0009731
0.75	0.1	1	0	28.4093	0.0010185
0.75	0.1	10	0	28.4093	0.000874
0.75	0.5	0.000001	154428	0.303125323	0.3108126
0.75	0.5	0.00001	88211	0.473395381	0.1734615
0.75	0.5	0.0001	27862	4.82117955	0.0558016
0.75	0.5	0.001	4116.5	15.534132	0.0088235
0.75	0.5	0.01	0	28.4093	0.0007856
0.75	0.5	0.1	0	28.4093	0.000791
0.75	0.5	1	0	28.4093	0.0008949
0.75	0.5	10	0	28.4093	0.0008331
0.75	1	0.000001	174360	0.313304462	0.1735803
0.75	1	0.00001	108144	0.221087151	0.1069052
0.75	1	0.0001	43838	2.57329217	0.0451131
0.75	1	0.001	7879	12.4981	0.0087716
0.75	1	0.01	0	28.4093	0.0007812
0.75	1	0.1	0	28.4093	0.000825
0.75	1	1	0	28.4093	0.0008039
0.75	1	10	0	28.4093	0.0007817
0.75	6	0.000001	225870	0.321779861	0.0378333
0.75	6	0.00001	159660	0.306522049	0.026724
0.75	6	0.0001	93450	0.384946253	0.0165793
0.75	6	0.001	31824	4.09937843	0.0061309
0.75	6	0.01	5022	14.786757	0.0017812
0.75	6	0.1	0	28.4093	0.0007868
0.75	6	1	0	28.4093	0.00079
0.75	6	10	0	28.4093	0.000855
0.75	30	0.000001	272040	0.323135361	0.0096603

0.75	30	0.00001	205860	0.320086281	0.007617
0.75	30	0.0001	139680	0.289550206	0.0053631
0.75	30	0.001	73500	0.846599988	0.0032193
0.75	30	0.01	19770	6.73802669	0.001435
0.75	30	0.1	1860	20.514108	0.0008523
0.75	30	1	0	28.4093	0.000856
0.75	30	10	0	28.4093	0.0007923
0.75	60	0.000001	291840	0.323304933	0.0056275
0.75	60	0.00001	225660	0.32177999	0.0044392
0.75	60	0.0001	159540	0.306539908	0.003476
0.75	60	0.001	93360	0.384984014	0.0022984
0.75	60	0.01	31800	4.0971119	0.0013076
0.75	60	0.1	5040	14.771626	0.0008699
0.75	60	1	0	28.4093	0.000788
0.75	60	10	0	28.4093	0.000787
0.75	100	0.000001	306300	0.323372617	0.0037782
0.75	100	0.00001	240200	0.322458826	0.0031216
0.75	100	0.0001	174100	0.313318831	0.0026383
0.75	100	0.001	108000	0.221255126	0.0018477
0.75	100	0.01	43800	2.57008012	0.0012127
0.75	100	0.1	7800	12.542246	0.0008924
0.75	100	1	0	28.4093	0.0007866
0.75	100	10	0	28.4093	0.0007924
0.75	150	0.000001	317700	0.323406551	0.0028554
0.75	150	0.00001	251700	0.322799151	0.0025428
0.75	150	0.0001	185550	0.316698135	0.0021207
0.75	150	0.001	119550	0.255570185	0.0016565
0.75	150	0.01	53850	1.7780172	0.0011973
0.75	150	0.1	10500	10.6317709	0.0008854
0.75	150	1	0	28.40925	0.0008104
0.75	150	10	0	28.40925	0.0008578
0.75	200	0.000001	325600	0.323423333	0.002411
0.75	200	0.00001	259800	0.322969237	0.002101
0.75	200	0.0001	193800	0.318422072	0.002322
0.75	200	0.001	127800	0.27280876	0.001515
0.75	200	0.01	61800	1.31850691	0.001097
0.75	200	0.1	14200	8.52572233	0.0008898
0.75	200	1	400	26.32659	0.000785
0.75	200	10	0	28.40925	0.0007842
0.75	360	0.000001	341640	0.323446072	0.0016922
0.75	360	0.00001	275760	0.323192567	0.0015335
0.75	360	0.0001	209880	0.320650409	0.0014786
0.75	360	0.001	144360	0.295486192	0.0013291
0.75	360	0.01	78480	0.681596904	0.0010208
0.75	360	0.1	22320	6.02091379	0.0008498
0.75	360	1	2520	18.198472	0.0007939

0.75	360	10	0	28.40925	0.000937
0.75	400	0.000001	344400	0.323448873	0.0016277
0.75	400	0.00001	278800	0.323222715	0.0015078
0.75	400	0.0001	212800	0.320938022	0.0013056
0.75	400	0.001	147200	0.298229328	0.0011461
0.75	400	0.01	81600	0.598156638	0.0009854
0.75	400	0.1	24000	5.60635186	0.000846
0.75	400	1	3200	16.363009	0.0007955
0.75	400	10	0	28.40925	0.0007884
0.75	500	0.000001	350500	0.323454172	0.0014756
0.75	500	0.00001	284500	0.323271797	0.0013548
0.75	500	0.0001	219000	0.321460404	0.0013022
0.75	500	0.001	153500	0.303423323	0.0011556
0.75	500	0.01	87500	0.47237925	0.0009583
0.75	500	0.1	27500	4.83201203	0.0008385
0.75	500	1	4000	15.668624	0.0007925
0.75	500	10	0	28.40925	0.0007834
0.75	600	0.000001	354600	0.323457225	0.001369
0.75	600	0.00001	289200	0.323305606	0.0012907
0.75	600	0.0001	223800	0.321795944	0.0012061
0.75	600	0.001	158400	0.306757555	0.0010406
0.75	600	0.01	93000	0.377001893	0.000936
0.75	600	0.1	31800	4.0337436	0.0008382
0.75	600	1	4800	14.980574	0.0007942
0.75	600	10	0	28.40925	0.0007863
0.75	800	0.000001	361600	0.323461502	0.0012485
0.75	800	0.00001	296800	0.323349809	0.0012086
0.75	800	0.0001	231200	0.322217082	0.0010715
0.75	800	0.001	165600	0.310762421	0.001032
0.75	800	0.01	100800	0.269253779	0.0009327
0.75	800	0.1	38400	3.06909322	0.0008325
0.75	800	1	6400	13.575627	0.0007967
0.75	800	10	0	28.40925	0.0008032
0.75	1000	0.000001	367000	0.323464169	0.0011482
0.75	1000	0.00001	302000	0.323374513	0.0010814
0.75	1000	0.0001	237000	0.322479799	0.0010179
0.75	1000	0.001	172000	0.313549473	0.0009548
0.75	1000	0.01	107000	0.22392341	0.0008917
0.75	1000	0.1	43000	2.58253114	0.0008733
0.75	1000	1	7000	13.03779	0.0008084
0.75	1000	10	0	28.40925	0.0007867
0.75	1500	0.000001	375000	0.323467483	0.0011106
0.75	1500	0.00001	310500	0.323407391	0.0009905
0.75	1500	0.0001	246000	0.322805907	0.0009476
0.75	1500	0.001	181500	0.316785013	0.0009123
0.75	1500	0.01	117000	0.256354673	0.0009407

0.75	1500	0.1	52500	1.78344122	0.00088
0.75	1500	1	10500	10.2724772	0.0007923
0.75	1500	10	0	28.409251	0.0007867
0.75	2000	0.000001	380000	0.323469229	0.0010015
0.75	2000	0.00001	316000	0.323424708	0.0009598
0.75	2000	0.0001	252000	0.322977666	0.0009391
0.75	2000	0.001	188000	0.318488692	0.0010686
0.75	2000	0.01	124000	0.273347115	0.0008638
0.75	2000	0.1	60000	1.31019296	0.0009291
0.75	2000	1	14000	8.39769743	0.0008858
0.75	2000	10	2000	18.086818	0.0008877
0.75	3600	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	3600	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	3600	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	3600	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	3600	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	3600	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	3600	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	3600	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	6000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	6000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	6000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	6000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	6000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met

0.75	6000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	6000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
0.75	6000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
1	0.1	0.000001	109425	1.866753947	0.8835809
1	0.1	0.00001	44072	2.32041478	0.3597271
1	0.1	0.0001	7770.1	11.213936	0.064942
1	0.1	0.001	0	23.579	0.0004005
1	0.1	0.01	0	23.579	0.000408
1	0.1	0.1	0	23.579	0.0004003
1	0.1	1	0	23.579	0.0004039
1	0.1	10	0	23.579	0.0004143
1	0.5	0.000001	156174.5	1.968369128	0.2527704
1	0.5	0.00001	89288	1.7367279	0.1429394
1	0.5	0.0001	28730	4.26407759	0.0471006
1	0.5	0.001	2855.5	17.412492	0.0054911
1	0.5	0.01	0	23.579	0.0004134
1	0.5	0.1	0	23.579	0.000496
1	0.5	1	0	23.579	0.0004125
1	0.5	10	0	23.579	0.0004852
1	1	0.000001	176307	1.980964938	0.1408518
1	1	0.00001	109424	1.866757207	0.0886802
1	1	0.0001	44071	2.32044751	0.0355934
1	1	0.001	7770	11.213664	0.0067704
1	1	0.01	0	23.579	0.0004003
1	1	0.1	0	23.579	0.000401
1	1	1	0	23.579	0.0004061
1	1	10	0	23.579	0.0004088
1	6	0.000001	228336	1.991450854	0.0307461
1	6	0.00001	161460	1.9725731	0.0226597
1	6	0.0001	94578	1.78052374	0.0126899
1	6	0.001	32610	3.61187799	0.0050938
1	6	0.01	4056	15.542731	0.0009381
1	6	0.1	0	23.579	0.0004148
1	6	1	0	23.579	0.0003998
1	6	10	0	23.579	0.0004129
1	30	0.000001	274980	1.993127902	0.008039
1	30	0.00001	208140	1.989357773	0.0060323
1	30	0.0001	141270	1.951554429	0.0042613
1	30	0.001	74430	1.55695743	0.0024584
1	30	0.01	18420	6.92346717	0.0008889
1	30	0.1	0	23.579	0.0003985

1	30	1	0	23.579	0.0004019
1	30	10	0	23.579	0.000436
1	60	0.000001	294960	1.993337477	0.0042863
1	60	0.00001	228120	1.991450591	0.0035122
1	60	0.0001	161340	1.972595144	0.0026059
1	60	0.001	94500	1.78063185	0.0019531
1	60	0.01	32580	3.61254736	0.0008435
1	60	0.1	4080	15.488632	0.000455
1	60	1	0	23.579	0.0004048
1	60	10	0	23.579	0.0004209
1	100	0.000001	309600	1.99342139	0.0028468
1	100	0.00001	242800	1.992290218	0.0023945
1	100	0.0001	176100	1.981005659	0.0017714
1	100	0.001	109300	1.867054439	0.0014477
1	100	0.01	44000	2.32039589	0.0008952
1	100	0.1	7800	11.177414	0.0004766
1	100	1	0	23.579	0.0004229
1	100	10	0	23.579	0.0004937
1	150	0.000001	321150	1.993463437	0.0020786
1	150	0.00001	254400	1.992710927	0.0018425
1	150	0.0001	187650	1.985174238	0.0015256
1	150	0.001	120900	1.909349408	0.0010342
1	150	0.01	54150	1.51079716	0.0006865
1	150	0.1	11100	9.70519278	0.0004565
1	150	1	0	23.57901	0.0003976
1	150	10	0	23.57901	0.0004008
1	200	0.000001	329200	1.993484308	0.0017915
1	200	0.00001	262600	1.992921925	0.0016141
1	200	0.0001	195800	1.987264478	0.0011805
1	200	0.001	129200	1.930660896	0.001001
1	200	0.01	62600	1.32288462	0.0006813
1	200	0.1	13800	8.57535053	0.0004892
1	200	1	0	23.57901	0.0004727
1	200	10	0	23.57901	0.0004012
1	360	0.000001	345240	1.993512119	0.0011642
1	360	0.00001	279000	1.993201628	0.0010457
1	360	0.0001	212400	1.990078054	0.0009062
1	360	0.001	145800	1.958658092	0.0007214
1	360	0.01	79560	1.63732122	0.0005755
1	360	0.1	21960	5.81009773	0.0004489
1	360	1	1080	20.692124	0.0004021
1	360	10	0	23.57901	0.0004006
1	400	0.000001	348000	1.993515557	0.0010905
1	400	0.00001	281600	1.993233505	0.0009589
1	400	0.0001	215200	1.990415044	0.0008271
1	400	0.001	148800	1.962220387	0.000697

1	400	0.01	82400	1.67264064	0.0006049
1	400	0.1	24000	5.26705726	0.0004468
1	400	1	1600	19.607965	0.0004039
1	400	10	0	23.57901	0.0004013
1	500	0.000001	354000	1.993521987	0.0009614
1	500	0.00001	287500	1.993295883	0.0008596
1	500	0.0001	221500	1.991063039	0.0007596
1	500	0.001	155000	1.968522579	0.0006665
1	500	0.01	89000	1.74153833	0.0005531
1	500	0.1	28500	4.24128192	0.0005749
1	500	1	3000	17.012217	0.0006034
1	500	10	0	23.57901	0.0006468
1	600	0.000001	358800	1.993526271	0.0008796
1	600	0.00001	292200	1.993337441	0.0009148
1	600	0.0001	226200	1.991465916	0.000708
1	600	0.001	160200	1.972863377	0.0006135
1	600	0.01	94200	1.78530861	0.0005251
1	600	0.1	32400	3.60273275	0.0004448
1	600	1	4200	15.110361	0.000405
1	600	10	0	23.57901	0.000402
1	800	0.000001	365600	1.993531336	0.0007766
1	800	0.00001	300000	1.993392983	0.0007323
1	800	0.0001	233600	1.991983593	0.0007097
1	800	0.001	168000	1.978102992	0.0005681
1	800	0.01	101600	1.835319274	0.0005094
1	800	0.1	38400	2.84725395	0.0005037
1	800	1	6400	12.256416	0.000405
1	800	10	0	23.57901	0.0004004
1	1000	0.000001	371000	1.993534592	0.0006989
1	1000	0.00001	305000	1.993422421	0.0006395
1	1000	0.0001	239000	1.992289878	0.0005885
1	1000	0.001	174000	1.981289413	0.0005665
1	1000	0.01	108000	1.869040063	0.0005473
1	1000	0.1	43000	2.3532474	0.0004469
1	1000	1	8000	11.095585	0.0004297
1	1000	10	0	23.57901	0.00041
1	1500	0.000001	379500	1.993538797	0.0006185
1	1500	0.00001	313500	1.993463156	0.0005924
1	1500	0.0001	249000	1.99272803	0.0005617
1	1500	0.001	184500	1.985540883	0.0005248
1	1500	0.01	118500	1.910775091	0.0004895
1	1500	0.1	54000	1.43651164	0.0004462
1	1500	1	10500	9.89250443	0.000423
1	1500	10	0	23.579006	0.0004276
1	2000	0.000001	384000	1.993540798	0.000713
1	2000	0.00001	320000	1.993486866	0.0005664

1	2000	0.0001	256000	1.992958097	0.0005599
1	2000	0.001	190000	1.987346665	0.0004989
1	2000	0.01	126000	1.932658564	0.0004732
1	2000	0.1	62000	1.36701637	0.0004629
1	2000	1	14000	8.27809558	0.0005607
1	2000	10	0	23.579006	0.0004361
1	3600	0.000001	388800	1.993543539	0.000621
1	3600	0.00001	327600	1.993514829	0.0004866
1	3600	0.0001	262800	1.993199557	0.0004725
1	3600	0.001	201600	1.990253265	0.0005321
1	3600	0.01	140400	1.962307819	0.0004439
1	3600	0.1	75600	1.65044446	0.0004286
1	3600	1	21600	5.38125725	0.0004176
1	3600	10	3600	14.502885	0.0004296
1	6000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met
1	6000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
1	6000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
1	6000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
1	6000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
1	6000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
1	6000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
1	6000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
1.2	0.1	0.000001	106328.3	0.390776202	0.7718304
1.2	0.1	0.00001	42755.9	2.52314914	0.3114988
1.2	0.1	0.0001	6345.3	12.695564	0.0493309
1.2	0.1	0.001	0	21.7846	0.0002826
1.2	0.1	0.01	0	21.7846	0.000309
1.2	0.1	0.1	0	21.7846	0.0002799
1.2	0.1	1	0	21.7846	0.0002812
1.2	0.1	10	0	21.7846	0.0002933
1.2	0.5	0.000001	153297.5	0.535938521	0.2282077
1.2	0.5	0.00001	86399	0.437672502	0.1358401

1.2	0.5	0.0001	26156	4.72170024	0.037798
1.2	0.5	0.001	32	21.7205	0.0003295
1.2	0.5	0.01	0	21.7846	0.0003763
1.2	0.5	0.1	0	21.7846	0.0003007
1.2	0.5	1	0	21.7846	0.0004071
1.2	0.5	10	0	21.7846	0.0002792
1.2	1	0.000001	173613	0.554074479	0.1249182
1.2	1	0.00001	106327	0.390778592	0.0768148
1.2	1	0.0001	42755	2.52316636	0.0315741
1.2	1	0.001	6345	12.695575	0.0047195
1.2	1	0.01	0	21.7846	0.0002706
1.2	1	0.1	0	21.7846	0.0002711
1.2	1	1	0	21.7846	0.0002732
1.2	1	10	0	21.7846	0.0002834
1.2	6	0.000001	226158	0.569187282	0.0274334
1.2	6	0.00001	158628	0.541987433	0.0190892
1.2	6	0.0001	91602	0.367465059	0.0122033
1.2	6	0.001	29952	4.09461786	0.0041311
1.2	6	0.01	1602	18.851878	0.0004599
1.2	6	0.1	0	21.7846	0.0002681
1.2	6	1	0	21.7846	0.0002703
1.2	6	10	0	21.7846	0.000272
1.2	30	0.000001	273270	0.571605016	0.0066975
1.2	30	0.00001	205740	0.566165382	0.0052395
1.2	30	0.0001	138300	0.511811109	0.003514
1.2	30	0.001	72000	0.808669929	0.0021198
1.2	30	0.01	18870	6.21546193	0.0007367
1.2	30	0.1	0	21.7846	0.0002729
1.2	30	1	0	21.7846	0.0002852
1.2	30	10	0	21.7846	0.0002661
1.2	60	0.000001	293460	0.571907293	0.0036214
1.2	60	0.00001	225960	0.569188337	0.0030677
1.2	60	0.0001	158520	0.542027128	0.0021471
1.2	60	0.001	91560	0.366972852	0.0013293
1.2	60	0.01	29940	4.09194474	0.000641
1.2	60	0.1	1620	18.813578	0.0003167
1.2	60	1	0	21.7846	0.0002661
1.2	60	10	0	21.7846	0.0002646
1.2	100	0.000001	308300	0.57202858	0.0024944
1.2	100	0.00001	240800	0.57039869	0.0020827
1.2	100	0.0001	173400	0.554124285	0.001523
1.2	100	0.001	106200	0.391142347	0.0010662
1.2	100	0.01	42700	2.52083709	0.000563
1.2	100	0.1	6400	12.60867	0.0003806
1.2	100	1	0	21.7846	0.0002637
1.2	100	10	0	21.7846	0.0002649

1.2	150	0.000001	319950	0.572089018	0.0019761
1.2	150	0.00001	252450	0.571001787	0.0015373
1.2	150	0.0001	185100	0.560147466	0.0011402
1.2	150	0.001	117900	0.45169555	0.0008264
1.2	150	0.01	53100	1.69179558	0.0005187
1.2	150	0.1	10500	9.23473227	0.0003205
1.2	150	1	0	21.78461	0.0002702
1.2	150	10	0	21.78461	0.0002716
1.2	200	0.000001	328000	0.57211882	0.0014053
1.2	200	0.00001	260800	0.571308418	0.0011607
1.2	200	0.0001	193400	0.563177982	0.0009752
1.2	200	0.001	126200	0.481961026	0.0007524
1.2	200	0.01	60800	1.25300952	0.0004778
1.2	200	0.1	13800	7.59149792	0.0003132
1.2	200	1	0	21.78461	0.0002643
1.2	200	10	0	21.78461	0.0002654
1.2	360	0.000001	344520	0.572159521	0.000928
1.2	360	0.00001	277200	0.571708387	0.0009185
1.2	360	0.0001	210240	0.567231826	0.0012391
1.2	360	0.001	142920	0.522055141	0.0010229
1.2	360	0.01	76680	0.654186189	0.0006745
1.2	360	0.1	21240	5.63937722	0.0005368
1.2	360	1	0	21.78461	0.000428
1.2	360	10	0	21.78461	0.0004607
1.2	400	0.000001	347200	0.572164265	0.0012482
1.2	400	0.00001	280000	0.571757249	0.0007468
1.2	400	0.0001	213200	0.567734914	0.0006383
1.2	400	0.001	146000	0.527249129	0.0005396
1.2	400	0.01	79600	0.576159934	0.0004041
1.2	400	0.1	22800	5.31923765	0.0003029
1.2	400	1	0	21.78461	0.0002645
1.2	400	10	0	21.78461	0.0002645
1.2	500	0.000001	353000	0.572173161	0.0007511
1.2	500	0.00001	286000	0.571847555	0.000663
1.2	500	0.0001	219000	0.568589783	0.0005709
1.2	500	0.001	152000	0.535958324	0.0004821
1.2	500	0.01	86000	0.430606037	0.0003938
1.2	500	0.1	26000	4.70899097	0.0003361
1.2	500	1	500	20.78085	0.0003029
1.2	500	10	0	21.78461	0.0002637
1.2	600	0.000001	357600	0.5721791	0.000829
1.2	600	0.00001	291000	0.571909916	0.0006254
1.2	600	0.0001	224400	0.569242922	0.0005241
1.2	600	0.001	157200	0.542182626	0.0004474
1.2	600	0.01	91200	0.361409863	0.0003718
1.2	600	0.1	30000	4.03669656	0.0002987

1.2	600	1	1800	18.421515	0.0002656
1.2	600	10	0	21.78461	0.0002646
1.2	800	0.000001	364800	0.572186745	0.00058
1.2	800	0.00001	298400	0.571985421	0.0006024
1.2	800	0.0001	232000	0.569988642	0.0004662
1.2	800	0.001	164800	0.549555583	0.000412
1.2	800	0.01	99200	0.351230578	0.000406
1.2	800	0.1	36800	3.09514228	0.000298
1.2	800	1	4000	15.117257	0.0002672
1.2	800	10	0	21.78461	0.0002649
1.2	1000	0.000001	370000	0.572191284	0.0005952
1.2	1000	0.00001	304000	0.572031479	0.0005709
1.2	1000	0.0001	238000	0.570455722	0.0004304
1.2	1000	0.001	171000	0.554301859	0.0004012
1.2	1000	0.01	105000	0.39422456	0.0003404
1.2	1000	0.1	42000	2.51810396	0.0002942
1.2	1000	1	6000	12.710214	0.0002715
1.2	1000	10	0	21.78461	0.0002721
1.2	1500	0.000001	379500	0.572197776	0.0004499
1.2	1500	0.00001	313500	0.572093154	0.0004435
1.2	1500	0.0001	247500	0.57104038	0.0003809
1.2	1500	0.001	181500	0.560444772	0.000353
1.2	1500	0.01	115500	0.45333932	0.0003347
1.2	1500	0.1	52500	1.64353479	0.0002893
1.2	1500	1	10500	8.77255833	0.0002709
1.2	1500	10	0	21.784609	0.000263
1.2	2000	0.000001	384000	0.572200581	0.000403
1.2	2000	0.00001	318000	0.572119538	0.0003828
1.2	2000	0.0001	254000	0.571349758	0.0003587
1.2	2000	0.001	188000	0.563377297	0.0003363
1.2	2000	0.01	124000	0.48746371	0.000326
1.2	2000	0.1	60000	1.19326028	0.0002899
1.2	2000	1	14000	7.44488188	0.0002707
1.2	2000	10	0	21.784609	0.0002677
1.2	3600	0.000001	388800	0.572204439	0.0005498
1.2	3600	0.00001	327600	0.572164142	0.0003313
1.2	3600	0.0001	262800	0.571732704	0.0003201
1.2	3600	0.001	198000	0.567181131	0.0003073
1.2	3600	0.01	136800	0.525647238	0.0002994
1.2	3600	0.1	72000	0.659721698	0.0003448
1.2	3600	1	18000	6.14346321	0.0002757
1.2	3600	10	0	21.784609	0.0003134
1.2	6000	0.000001	Stability criterion not met	Stability criterion not met	Stability criterion not met

1.2	6000	0.00001	Stability criterion not met	Stability criterion not met	Stability criterion not met
1.2	6000	0.0001	Stability criterion not met	Stability criterion not met	Stability criterion not met
1.2	6000	0.001	Stability criterion not met	Stability criterion not met	Stability criterion not met
1.2	6000	0.01	Stability criterion not met	Stability criterion not met	Stability criterion not met
1.2	6000	0.1	Stability criterion not met	Stability criterion not met	Stability criterion not met
1.2	6000	1	Stability criterion not met	Stability criterion not met	Stability criterion not met
1.2	6000	10	Stability criterion not met	Stability criterion not met	Stability criterion not met
1.5	0.1	0.000001	109520.4	0.623265311	0.7255051
1.5	0.1	0.00001	40816.9	2.60807043	0.2770974
1.5	0.1	0.0001	960.8	16.84616	0.0065233
1.5	0.1	0.001	0	17.8405	0.0001564
1.5	0.1	0.01	0	17.8405	0.0001581
1.5	0.1	0.1	0	17.8405	0.0001608
1.5	0.1	1	0	17.8405	0.0001622
1.5	0.1	10	0	17.8405	0.0001755
1.5	0.5	0.000001	157638.5	0.744730426	0.2106755
1.5	0.5	0.00001	88779	0.466565525	0.1177467
1.5	0.5	0.0001	26895.5	4.41317112	0.0357623
1.5	0.5	0.001	0	17.8405	0.0001656
1.5	0.5	0.01	0	17.8405	0.000164
1.5	0.5	0.1	0	17.8405	0.0001627
1.5	0.5	1	0	17.8405	0.000167
1.5	0.5	10	0	17.8405	0.0001781
1.5	1	0.000001	178359	0.759720892	0.1197617
1.5	1	0.00001	109519	0.623266874	0.0736503
1.5	1	0.0001	40816	2.6081034	0.0271811
1.5	1	0.001	961	16.84593	0.0008468
1.5	1	0.01	0	17.8405	0.0001643
1.5	1	0.1	0	17.8405	0.000165
1.5	1	1	0	17.8405	0.0001662
1.5	1	10	0	17.8405	0.0001694
1.5	6	0.000001	231906	0.772188757	0.0261093
1.5	6	0.00001	163074	0.749731073	0.0186655
1.5	6	0.0001	94230	0.519488286	0.0105431

1.5	6	0.001	30360	3.77933701	0.0034743
1.5	6	0.01	0	17.8405	0.0001654
1.5	6	0.1	0	17.8405	0.0001931
1.5	6	1	0	17.8405	0.0002368
1.5	6	10	0	17.8405	0.0002463
1.5	30	0.000001	279930	0.77418225	0.0065516
1.5	30	0.00001	211110	0.769698334	0.0046767
1.5	30	0.0001	142320	0.724725761	0.0033826
1.5	30	0.001	73440	0.666810041	0.0018423
1.5	30	0.01	17820	6.76766316	0.000543
1.5	30	0.1	0	17.8405	0.0001572
1.5	30	1	0	17.8405	0.0001575
1.5	30	10	0	17.8405	0.0001585
1.5	60	0.000001	300480	0.774431049	0.0034231
1.5	60	0.00001	231720	0.772190725	0.002852
1.5	60	0.0001	162960	0.749759444	0.0019886
1.5	60	0.001	94200	0.520018311	0.0012088
1.5	60	0.01	30360	3.77527613	0.0004818
1.5	60	0.1	0	17.8405	0.0001575
1.5	60	1	0	17.8405	0.0001575
1.5	60	10	0	17.8405	0.0001586
1.5	100	0.000001	315600	0.774531016	0.0022705
1.5	100	0.00001	246800	0.773185985	0.0018088
1.5	100	0.0001	178100	0.759739499	0.0013113
1.5	100	0.001	109400	0.623619315	0.0008616
1.5	100	0.01	40800	2.60376885	0.0004213
1.5	100	0.1	1000	16.80347	0.0001665
1.5	100	1	0	17.8405	0.0001622
1.5	100	10	0	17.8405	0.0001608
1.5	150	0.000001	327450	0.774580763	0.0015882
1.5	150	0.00001	258750	0.773686216	0.0012894
1.5	150	0.0001	190050	0.764725499	0.0010094
1.5	150	0.001	121350	0.674299787	0.0008813
1.5	150	0.01	52350	1.67113746	0.0004295
1.5	150	0.1	6900	11.957688	0.0002444
1.5	150	1	0	17.84048	0.0002651
1.5	150	10	0	17.84048	0.000164
1.5	200	0.000001	335800	0.77460571	0.0012419
1.5	200	0.00001	267200	0.773937066	0.0010971
1.5	200	0.0001	198600	0.767249722	0.0007961
1.5	200	0.001	130000	0.70002342	0.0005732
1.5	200	0.01	61000	1.1665775	0.0003573
1.5	200	0.1	11200	9.45690725	0.0001966
1.5	200	1	0	17.84048	0.0001586
1.5	200	10	0	17.84048	0.0001624
1.5	360	0.000001	352440	0.774638845	0.0008712

1.5	360	0.00001	284040	0.774268571	0.0006779
1.5	360	0.0001	215640	0.770567885	0.0006955
1.5	360	0.001	147240	0.73349224	0.0004772
1.5	360	0.01	78480	0.572061959	0.0003613
1.5	360	0.1	20880	5.77792382	0.0003209
1.5	360	1	0	17.84048	0.0001856
1.5	360	10	0	17.84048	0.0002491
1.5	400	0.000001	355200	0.774642799	0.0007557
1.5	400	0.00001	286800	0.774307506	0.0008028
1.5	400	0.0001	218400	0.770951283	0.0005503
1.5	400	0.001	150000	0.737284562	0.0004893
1.5	400	0.01	81600	0.52727073	0.0002922
1.5	400	0.1	22800	5.2663662	0.0001998
1.5	400	1	0	17.84048	0.0001571
1.5	400	10	0	17.84048	0.000157
1.5	500	0.000001	361500	0.774650529	0.0006249
1.5	500	0.00001	293000	0.774382716	0.0005644
1.5	500	0.0001	225000	0.771732606	0.0004544
1.5	500	0.001	156500	0.744920436	0.0003686
1.5	500	0.01	88500	0.472108861	0.0002813
1.5	500	0.1	26500	4.41998465	0.0001983
1.5	500	1	0	17.84048	0.0001652
1.5	500	10	0	17.84048	0.0001649
1.5	600	0.000001	366000	0.774655209	0.0005787
1.5	600	0.00001	298200	0.774434779	0.0004956
1.5	600	0.0001	229800	0.772206083	0.0004229
1.5	600	0.001	162000	0.75019761	0.0003526
1.5	600	0.01	93600	0.522708507	0.0002765
1.5	600	0.1	30000	3.79463209	0.000201
1.5	600	1	0	17.84048	0.0001736
1.5	600	10	0	17.84048	0.0001859
1.5	800	0.000001	373600	0.774661638	0.0004765
1.5	800	0.00001	305600	0.774495658	0.0004533
1.5	800	0.0001	237600	0.772830766	0.0004351
1.5	800	0.001	169600	0.756117219	0.0004339
1.5	800	0.01	101600	0.585915488	0.0002676
1.5	800	0.1	36000	3.0576817	0.0002295
1.5	800	1	0	17.84048	0.0002785
1.5	800	10	0	17.84048	0.0001828
1.5	1000	0.000001	379000	0.774665375	0.0004253
1.5	1000	0.00001	311000	0.774531967	0.0003625
1.5	1000	0.0001	244000	0.773233249	0.0003229
1.5	1000	0.001	176000	0.760045195	0.0002775
1.5	1000	0.01	108000	0.625238066	0.0002343
1.5	1000	0.1	40000	2.62702857	0.0001902
1.5	1000	1	1000	16.7698	0.0001634

1.5	1000	10	0	17.84048	0.0001637
1.5	1500	0.000001	387000	0.774670072	0.0003342
1.5	1500	0.00001	321000	0.774584216	0.0003068
1.5	1500	0.0001	253500	0.773708561	0.0002763
1.5	1500	0.001	186000	0.76482654	0.0002472
1.5	1500	0.01	120000	0.679288295	0.0002154
1.5	1500	0.1	52500	1.58372079	0.0001845
1.5	1500	1	7500	11.336334	0.0001637
1.5	1500	10	0	17.840482	0.0001681
1.5	2000	0.000001	392000	0.774672579	0.0002999
1.5	2000	0.00001	326000	0.774606915	0.0002806
1.5	2000	0.0001	260000	0.7739617	0.0002852
1.5	2000	0.001	194000	0.767620781	0.00023
1.5	2000	0.01	126000	0.700091973	0.0002141
1.5	2000	0.1	60000	1.12370938	0.0002506
1.5	2000	1	12000	8.71506076	0.0001685
1.5	2000	10	0	17.840482	0.0001633
1.5	3600	0.000001	399600	0.774676215	0.0002452
1.5	3600	0.00001	334800	0.774641714	0.0002359
1.5	3600	0.0001	270000	0.774294275	0.0002202
1.5	3600	0.001	205200	0.770795282	0.0002086
1.5	3600	0.01	140400	0.735524278	0.0001942
1.5	3600	0.1	75600	0.544721289	0.0001776
1.5	3600	1	21600	5.01278088	0.0001667
1.5	3600	10	0	17.840482	0.000163
1.5	6000	0.000001	396000	0.774677808	0.0002354
1.5	6000	0.00001	336000	0.774659251	0.0002403
1.5	6000	0.0001	270000	0.774435908	0.0003113
1.5	6000	0.001	210000	0.77238654	0.0001948
1.5	6000	0.01	150000	0.75312967	0.0002005
1.5	6000	0.1	90000	0.571281372	0.0002199
1.5	6000	1	30000	3.45769416	0.0002778
1.5	6000	10	0	17.840482	0.0001813
2	0.1	0.000001	107310.5	3.428785114	0.6645256
2	0.1	0.00001	43629.5	1.78705318	0.2736399
2	0.1	0.0001	0	12.933	0.0000869
2	0.1	0.001	0	12.933	0.0000875
2	0.1	0.01	0	12.933	0.0000857
2	0.1	0.1	0	12.933	0.000085
2	0.1	1	0	12.933	0.0000876
2	0.1	10	0	12.933	0.0000993
2	0.5	0.000001	156384	3.671714463	0.1943916
2	0.5	0.00001	86908.5	3.13503606	0.1122591
2	0.5	0.0001	27522.5	3.63727224	0.0346114
2	0.5	0.001	0	12.933	0.0000864
2	0.5	0.01	0	12.933	0.0000887

2	0.5	0.1	0	12.933	0.0000897
2	0.5	1	0	12.933	0.0000913
2	0.5	10	0	12.933	0.0000995
2	1	0.000001	177827	3.702644293	0.1138466
2	1	0.00001	107309	3.428786267	0.074407
2	1	0.0001	43629	1.78703631	0.0270793
2	1	0.001	0	12.933	0.0000837
2	1	0.01	0	12.933	0.0000842
2	1	0.1	0	12.933	0.000084
2	1	1	0	12.933	0.000088
2	1	10	0	12.933	0.0000899
2	6	0.000001	233478	3.728521557	0.0239128
2	6	0.00001	162006	3.682018249	0.0168491
2	6	0.0001	92196	3.23184871	0.0097164
2	6	0.001	31596	3.06291603	0.0032742
2	6	0.01	0	12.933	0.0000841
2	6	0.1	0	12.933	0.0000872
2	6	1	0	12.933	0.000088
2	6	10	0	12.933	0.0000872
2	30	0.000001	283470	3.732670982	0.0059022
2	30	0.00001	211860	3.723345134	0.005294
2	30	0.0001	140610	3.630741826	0.0029524
2	30	0.001	72390	2.75776139	0.0015665
2	30	0.01	16710	5.64362372	0.000446
2	30	0.1	0	12.933	0.0000839
2	30	1	0	12.933	0.0000841
2	30	10	0	12.933	0.0000844
2	60	0.000001	304920	3.733189957	0.0031941
2	60	0.00001	233280	3.728522345	0.0024655
2	60	0.0001	161880	3.682044235	0.0018268
2	60	0.001	92160	3.23266999	0.0010218
2	60	0.01	31560	3.06357774	0.0004154
2	60	0.1	0	12.933	0.0000834
2	60	1	0	12.933	0.0000841
2	60	10	0	12.933	0.0000836
2	100	0.000001	320600	3.733397148	0.0020233
2	100	0.00001	249000	3.730596274	0.0016087
2	100	0.0001	177600	3.702701459	0.0011754
2	100	0.001	107200	3.429474015	0.000743
2	100	0.01	43600	1.78296775	0.0003529
2	100	0.1	0	12.933	0.0000836
2	100	1	0	12.933	0.000084
2	100	10	0	12.933	0.0000842
2	150	0.000001	333000	3.733501191	0.0014404
2	150	0.00001	261450	3.731637508	0.001151
2	150	0.0001	190050	3.713052277	0.0008618

2	150	0.001	119250	3.52928171	0.0005791
2	150	0.01	53700	1.86921294	0.000304
2	150	0.1	3450	10.403075	0.0001027
2	150	1	0	12.93296	0.0000849
2	150	10	0	12.93296	0.0000902
2	200	0.000001	341600	3.73355259	0.0011292
2	200	0.00001	270200	3.732156844	0.0009147
2	200	0.0001	198800	3.71820976	0.0006955
2	200	0.001	128000	3.580530854	0.0004805
2	200	0.01	61200	2.30685269	0.0002735
2	200	0.1	8800	7.691327	0.0001112
2	200	1	0	12.93296	0.0000849
2	200	10	0	12.93296	0.0000868
2	360	0.000001	358920	3.733621674	0.0010041
2	360	0.00001	287640	3.732844747	0.000573
2	360	0.0001	216360	3.725068801	0.0004582
2	360	0.001	145440	3.647847164	0.000332
2	360	0.01	77040	2.91327525	0.0002183
2	360	0.1	20520	4.81990307	0.0001213
2	360	1	0	12.93296	0.0000836
2	360	10	0	12.93296	0.0000836
2	400	0.000001	362000	3.733630438	0.0007197
2	400	0.00001	290800	3.73293327	0.0005307
2	400	0.0001	219600	3.725963647	0.0004392
2	400	0.001	148800	3.656969103	0.0003139
2	400	0.01	80000	2.99311688	0.0002054
2	400	0.1	22800	4.38827464	0.0001195
2	400	1	0	12.93296	0.0000834
2	400	10	0	12.93296	0.0000838
2	500	0.000001	368500	3.733646338	0.0005379
2	500	0.00001	297500	3.733093833	0.0004578
2	500	0.0001	226000	3.727486157	0.0003645
2	500	0.001	155500	3.672484854	0.0002769
2	500	0.01	86500	3.1419337	0.0001899
2	500	0.1	27500	3.6024676	0.0001177
2	500	1	0	12.93296	0.0000837
2	500	10	0	12.93296	0.0000835
2	600	0.000001	373800	3.733657072	0.0004699
2	600	0.00001	302400	3.733192247	0.0003962
2	600	0.0001	231600	3.728581514	0.0003242
2	600	0.001	160800	3.682613009	0.0002494
2	600	0.01	91800	3.24091051	0.0001805
2	600	0.1	31200	3.07009206	0.0001155
2	600	1	0	12.93296	0.0000834
2	600	10	0	12.93296	0.0000833
2	800	0.000001	381600	3.733670019	0.0003815

2	800	0.00001	310400	3.733323032	0.0003254
2	800	0.0001	240000	3.729902831	0.0002719
2	800	0.001	168800	3.695026196	0.0002243
2	800	0.01	100000	3.36307088	0.0001627
2	800	0.1	37600	2.30033059	0.0001135
2	800	1	0	12.93296	0.0000835
2	800	10	0	12.93296	0.0000838
2	1000	0.000001	387000	3.733677429	0.0003238
2	1000	0.00001	316000	3.733397852	0.0002791
2	1000	0.0001	246000	3.730659423	0.0002375
2	1000	0.001	175000	3.702698567	0.0001927
2	1000	0.01	106000	3.433734524	0.0001508
2	1000	0.1	43000	1.77396443	0.0001127
2	1000	1	0	12.93296	0.0000891
2	1000	10	0	12.93296	0.0000839
2	1500	0.000001	396000	3.733687531	0.0002493
2	1500	0.00001	327000	3.733509873	0.0002189
2	1500	0.0001	256500	3.731690341	0.0001919
2	1500	0.001	186000	3.713139702	0.0001625
2	1500	0.01	117000	3.532222073	0.0001322
2	1500	0.1	52500	1.88424637	0.0001056
2	1500	1	4500	9.6143075	0.0000852
2	1500	10	0	12.932959	0.0000832
2	2000	0.000001	402000	3.73369297	0.0002095
2	2000	0.00001	332000	3.733554781	0.0001887
2	2000	0.0001	264000	3.732243572	0.0001679
2	2000	0.001	194000	3.718739337	0.0001465
2	2000	0.01	124000	3.579659299	0.0001227
2	2000	0.1	60000	2.35197868	0.0001029
2	2000	1	10000	7.2450965	0.0000868
2	2000	10	0	12.932959	0.000084
2	3600	0.000001	410400	3.733700238	0.0001981
2	3600	0.00001	342000	3.733628159	0.0001423
2	3600	0.0001	273600	3.732883762	0.0001313
2	3600	0.001	205200	3.725194392	0.0001192
2	3600	0.01	140400	3.655750839	0.000108
2	3600	0.1	75600	2.99863039	0.0000998
2	3600	1	21600	4.35360295	0.0000906
2	3600	10	0	12.932959	0.000084
2	6000	0.000001	408000	3.733703504	0.0001262
2	6000	0.00001	342000	3.733660946	0.0001191
2	6000	0.0001	276000	3.733212485	0.0001132
2	6000	0.001	216000	3.729492826	0.0001069
2	6000	0.01	150000	3.689259126	0.0001012
2	6000	0.1	84000	3.25880155	0.0000939
2	6000	1	30000	2.77881605	0.0000966

2	6000	10	0	12.932959	0.0001521
3	0.1	0.000001	115131.3	1.157065579	0.7077138
3	0.1	0.00001	34484.6	1.99512875	0.2165915
3	0.1	0.0001	0	7.8677	0.0000627
3	0.1	0.001	0	7.8677	0.0000602
3	0.1	0.01	0	7.8677	0.0000655
3	0.1	0.1	0	7.8677	0.000067
3	0.1	1	0	7.8677	0.0000728
3	0.1	10	0	7.8677	0.0000989
3	0.5	0.000001	171500	1.437260891	0.2167366
3	0.5	0.00001	90854	0.806827553	0.1119596
3	0.5	0.0001	10207.5	5.4975591	0.0123277
3	0.5	0.001	0	7.8677	0.0000376
3	0.5	0.01	0	7.8677	0.0000406
3	0.5	0.1	0	7.8677	0.0000399
3	0.5	1	0	7.8677	0.0000401
3	0.5	10	0	7.8677	0.0000502
3	1	0.000001	195776	1.472285708	0.1218332
3	1	0.00001	115130	1.157067371	0.0708116
3	1	0.0001	34485	1.99504444	0.0215045
3	1	0.001	0	7.8677	0.0000384
3	1	0.01	0	7.8677	0.0000382
3	1	0.1	0	7.8677	0.0000422
3	1	1	0	7.8677	0.00004
3	1	10	0	7.8677	0.0000513
3	6	0.000001	258516	1.50147295	0.0277032
3	6	0.00001	177876	1.448942839	0.0199467
3	6	0.0001	97236	0.92364063	0.0110387
3	6	0.001	16596	4.32939237	0.0019496
3	6	0.01	0	7.8677	0.000041
3	6	0.1	0	7.8677	0.000041
3	6	1	0	7.8677	0.0000441
3	6	10	0	7.8677	0.0000563
3	30	0.000001	314790	1.50614285	0.00723
3	30	0.00001	234180	1.495642723	0.0053504
3	30	0.0001	153570	1.390648317	0.0035008
3	30	0.001	72960	0.340772982	0.0017784
3	30	0.01	0	7.86769	0.0000414
3	30	0.1	0	7.86769	0.0000412
3	30	1	0	7.86769	0.0000413
3	30	10	0	7.86769	0.0000509
3	60	0.000001	338940	1.50672671	0.0036239
3	60	0.00001	258360	1.501480179	0.0031897
3	60	0.0001	177780	1.449011478	0.0019935
3	60	0.001	97200	0.924290547	0.0010812
3	60	0.01	16620	4.32325795	0.0002258

3	60	0.1	0	7.86769	0.0000432
3	60	1	0	7.86769	0.0000415
3	60	10	0	7.86769	0.0000436
3	100	0.000001	356600	1.506959753	0.0022807
3	100	0.00001	276100	1.503814146	0.0019116
3	100	0.0001	195500	1.472286536	0.0013143
3	100	0.001	115000	1.157395619	0.000854
3	100	0.01	34500	1.98866514	0.0003157
3	100	0.1	0	7.86769	0.0000407
3	100	1	0	7.86769	0.000042
3	100	10	0	7.86769	0.0000635
3	150	0.000001	370500	1.50707627	0.0018471
3	150	0.00001	290100	1.50498109	0.0014016
3	150	0.0001	209550	1.48397368	0.0011466
3	150	0.001	129150	1.274443948	0.0007171
3	150	0.01	48600	0.826415212	0.0002761
3	150	0.1	0	7.86769	0.0000417
3	150	1	0	7.86769	0.0000409
3	150	10	0	7.86769	0.0000436
3	200	0.000001	380400	1.507135199	0.0013506
3	200	0.00001	300000	1.505566257	0.0010754
3	200	0.0001	219600	1.489884184	0.0007494
3	200	0.001	139200	1.333136888	0.0004934
3	200	0.01	58600	0.243600318	0.0002394
3	200	0.1	0	7.86769	0.0000404
3	200	1	0	7.86769	0.0000614
3	200	10	0	7.86769	0.0000404
3	360	0.000001	399960	1.507212586	0.000825
3	360	0.00001	319680	1.506337968	0.0006094
3	360	0.0001	239760	1.497679653	0.0004724
3	360	0.001	159480	1.41087628	0.0003278
3	360	0.01	79200	0.541637381	0.0002069
3	360	0.1	0	7.86769	0.00004
3	360	1	0	7.86769	0.0000404
3	360	10	0	7.86769	0.0000405
3	400	0.000001	403600	1.507222792	0.0007229
3	400	0.00001	323200	1.506436013	0.0006028
3	400	0.0001	243200	1.49861998	0.0005154
3	400	0.001	162800	1.419876207	0.0003102
3	400	0.01	82800	0.637618939	0.0001808
3	400	0.1	2400	7.2433472	0.0000444
3	400	1	0	7.86769	0.0000437
3	400	10	0	7.86769	0.0000434
3	500	0.000001	410500	1.5072396	0.0005682
3	500	0.00001	330500	1.506610864	0.0005932
3	500	0.0001	250500	1.500336108	0.0005407

3	500	0.001	170500	1.437714394	0.0002946
3	500	0.01	90500	0.81275314	0.0003426
3	500	0.1	10500	5.42432573	0.0000565
3	500	1	0	7.86769	0.0000429
3	500	10	0	7.86769	0.0000418
3	600	0.000001	416400	1.507251542	0.0004935
3	600	0.00001	336600	1.506731448	0.0003999
3	600	0.0001	256800	1.50155353	0.0003417
3	600	0.001	176400	1.449004775	0.0003004
3	600	0.01	96600	0.926842492	0.0002414
3	600	0.1	16800	4.27166307	0.0001367
3	600	1	0	7.86769	0.0000693
3	600	10	0	7.86769	0.0000464
3	800	0.000001	425600	1.507266619	0.0004372
3	800	0.00001	345600	1.506876184	0.0003262
3	800	0.0001	265600	1.502940388	0.0003714
3	800	0.001	186400	1.464271451	0.0001955
3	800	0.01	106400	1.073461549	0.0001867
3	800	0.1	26400	2.86611438	0.0000607
3	800	1	0	7.86769	0.0000421
3	800	10	0	7.86769	0.0000424
3	1000	0.000001	432000	1.507275155	0.0003168
3	1000	0.00001	352000	1.50695988	0.0002684
3	1000	0.0001	273000	1.503861544	0.0002164
3	1000	0.001	193000	1.472315708	0.0001636
3	1000	0.01	114000	1.162302274	0.0001133
3	1000	0.1	34000	1.99411221	0.0000638
3	1000	1	0	7.86769	0.0000395
3	1000	10	0	7.86769	0.00004
3	1500	0.000001	442500	1.507286489	0.0002334
3	1500	0.00001	364500	1.507084386	0.0002021
3	1500	0.0001	285000	1.50501802	0.000168
3	1500	0.001	205500	1.483993863	0.0001325
3	1500	0.01	127500	1.280244112	0.0000994
3	1500	0.1	48000	0.802959259	0.0000624
3	1500	1	0	7.8676904	0.0000406
3	1500	10	0	7.8676904	0.0000399
3	2000	0.000001	450000	1.507292772	0.0001924
3	2000	0.00001	372000	1.507142765	0.0001686
3	2000	0.0001	294000	1.505656811	0.0003034
3	2000	0.001	214000	1.489945601	0.0001174
3	2000	0.01	136000	1.335303895	0.0000869
3	2000	0.1	58000	0.196557191	0.0000646
3	2000	1	0	7.8676904	0.0000661
3	2000	10	0	7.8676904	0.0000406
3	3600	0.000001	460800	1.507300852	0.0001201

3	3600	0.00001	381600	1.507214339	0.0001122
3	3600	0.0001	306000	1.506380297	0.000103
3	3600	0.001	230400	1.498245182	0.0000883
3	3600	0.01	154800	1.418896568	0.0000718
3	3600	0.1	75600	0.546149765	0.0000556
3	3600	1	0	7.8676904	0.0000433
3	3600	10	0	7.8676904	0.000041
3	6000	0.000001	462000	1.507304738	0.0001014
3	6000	0.00001	384000	1.507253498	0.0000894
3	6000	0.0001	312000	1.506774567	0.0000815
3	6000	0.001	240000	1.502208316	0.0000732
3	6000	0.01	168000	1.45867248	0.0000722
3	6000	0.1	90000	0.947729366	0.0000783
3	6000	1	18000	3.82787555	0.0001044
3	6000	10	0	7.8676904	0.005339