

1809812

by Liam George

FILE	7593_LIAM_GEORGE_1809812_177947_984652367.PDF (2.03M)		
TIME SUBMITTED	05-MAY-2020 03:39PM (UTC+0100)	WORD COUNT	664
SUBMISSION ID	126308072	CHARACTER COUNT	2325

EXAM

COURSE - HNC ELECTRICAL and ELECTRONIC Engineering

MODULE - SGEE5001 further ENGINEERING Mathematics

NAME - LIAM GEORGE

UNIVERSITY number - 1809812

DATE submitted - 05/05/2020

SECTION A

- a) A sample of 100 components were tested to failure to find its life time, the results, to the nearest 100 hours

X	F	XF	$f(x-\bar{x})^2$
6	7	42	46.957
7	21	147	53.090
8	25	200	8.703
9	19	171	3.194
10	14	140	27.833
11	9	99	52.272
12	5	60	58.140

$$\Sigma f = 100 \quad \Sigma xf = 859 \quad \Sigma (x-\bar{x})^2 = 250.19$$

$$\text{mean } \bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{859}{100} = 8.59$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{250.19}{100}} = 1.58$$

$$\text{mean } \bar{x} = 8.59$$

7/7

$$\text{Standard deviation } \sigma = 1.58$$

b) Binomial distribution

Probability of $\frac{3}{5}$ for producing a resistor with an ohmic value of exactly 1K

(i) 1 resistor out of a batch of 3

$$\text{If } p = \frac{3}{5} = 0.6$$

$$q = 1 - 0.6 = 0.4$$

2/2

$$n = 3$$

$$\begin{aligned} P(1 \text{ resistor}) &= \frac{3}{1} p^1 q^{3-1} = 3(0.6)^1 (0.4)^{3-1} \\ &= 3(0.6) (0.16) \\ &= 0.288 \end{aligned}$$

5/5

(ii) At least 3 resistors out of a batch of 4

$n = 4$, either 3 or 4 resistors

$$\begin{aligned} P(3) &= \binom{4}{3} p^3 q^{4-3} = 4(0.6)^3 (0.4) \\ &= 0.346 \end{aligned}$$

4/4

$$\begin{aligned} P(4) &= \binom{4}{4} p^4 q^{4-4} = 1(0.6)^4 (0.4) \\ &= 0.207 \end{aligned}$$

x

$$\begin{aligned} \therefore P(\text{at least 3}) &= 0.346 + 0.207 \\ &= 0.553 \end{aligned}$$

1/3

section A total
= 19/25

Section B

1) R-C circuit

$$R = 20 \Omega \quad E = 20e^{-3t} \quad C = 0.5 \quad \text{and} \quad I(0) = 0$$

c should be 0.05

$$R-C = \quad E(t) = RI + \frac{1}{C} \int I dt$$

$$E(t) = RI + \frac{1}{C} \int I dt$$

$$20e^{-3t} = 20I + \frac{1}{0.5} \int I dt$$

$$20e^{-3t} = 20I + 2 \int I dt$$

Differentiate :

$$20e^{-3t} = 20I + 2 \int I dt$$

$$-60e^{-3t} = 20 \frac{dI}{dt} + 2I$$

Divide by 20 to make $\frac{dI}{dt} =$

$$\frac{-60e^{-3t}}{20} = \frac{20}{20} \frac{dI}{dt} + \frac{2}{20} I$$

$$-3e^{-3t} = 1 \frac{dI}{dt} + 0.1I$$

Integrate

$$\frac{dI}{dt} + 0.1I = -3e^{-3t}$$

$$P = 0.1 \quad \text{and} \quad Q = -3e^{-3t}$$

$$\therefore \int P dt = \int Q dt = 0.1t$$

$$\text{Integrating factor} = e^{0.1t}$$

$$Ie^{0.1t} = \int e^{0.1t} \times -3e^{-3t} dt$$

$$Ie^{0.1t} = \int -3e^{-2.9t} dt$$

$$Ie^{0.1t} = \frac{-3}{-2.9} e^{-2.9t} + C$$

G.S

when $I = 0$ $t = 0$

$$I e^{0.1t} = \frac{-3}{-2.9} e^{-2.9t} + C$$

$$I e^{0.1(0)} = \frac{-3}{-2.9} e^{-2.9(0)} + C$$

$$0 = \frac{-3}{-2.9} e^0 + C$$

$$\frac{3}{-2.9} = C$$

$$C = -1.03$$

P.S. =

$$I e^{0.1t} = \frac{-3}{-2.9} e^{-2.9t} - 1.03$$

$$\frac{I e^{0.1t}}{e^{0.1t}} = \frac{-3}{-2.9} e^{-2.9t} - \frac{1.03}{e^{0.1t}}$$

$$I = \frac{1.03 e^{-2.9t}}{e^{0.1t}} - \frac{1.03}{e^{0.1t}}$$

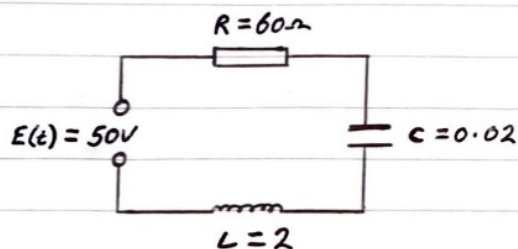
$$I = 1.03 e^{-3t} - 1.03 e^{-t}$$

OR $I = 1.03 e^{-t} - 1.03 e^{-3t}$ x

answer correct
for the values
used

Section B

- 3) Determine the current $I(t)$ in the circuit below given that $I(0) = 0$ and $I'(0) = 10$



$$R = 60$$

$$L = 2$$

$$C = 0.02$$

$$E(t) = 50$$

$$E = E_R + E_L + E_C$$

$$E = RI + L \frac{dI}{dt} + \frac{1}{C} \int I dt$$

Differentiate w.r.t

$$\frac{dE}{dt} = R \frac{dI}{dt} + L \frac{d^2 I}{dt^2} + \frac{1}{C} I$$

When E is constant $\frac{dE}{dt} = 0$

$$0 = R \frac{dI}{dt} + L \frac{d^2 I}{dt^2} + \frac{1}{C} I$$

Rearrange :- $L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{1}{C} I = 0$

$$R = 60$$

$$L = 2$$

$$C = 0.02$$

$$2 \frac{d^2 I}{dt^2} + 60 \frac{dI}{dt} + 50 I = 0$$

Divide through by 2 to make $\frac{d^2 I}{dt^2} = 1$

$$\frac{2}{2} \frac{d^2 I}{dt^2} + \frac{60}{2} \frac{dI}{dt} + \frac{50}{2} I = 0$$

$$\frac{d^2 I}{dt^2} + 30 \frac{dI}{dt} + 25 I = 0$$

$$m^2 + 30m + 25 = 0$$

$$m = \frac{-30 \pm \sqrt{30^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{-30 \pm \sqrt{800}}{2}$$

$$= -0.858 \quad \text{OR} \quad -29.14$$

Real and Different

G.S $I = Ae^{-t} + Be^{-t}$ -2

since $I(0) = 0$, $t = 0$ and $I = 0$

$$0 = Ae^0 + Be^0$$

$$0 = A + B - 0$$

$$\frac{dI}{dt} = -0.858 A e^{-0.858t} - 29.14 B e^{-29.14t}$$

since $I'(0) = 10$

$$10 = -0.858 A e^0 - 29.14 B e^0$$

$$10 = -0.858 A - 29.14 B \quad \text{--- (2)}$$

$$+ \quad 0 = 0.858 A + 0.858 B$$

$$10 = -29.99 B \quad \times \quad -1$$

$$B = \frac{10}{-29.99}$$

$$B = -0.333 \quad \times$$

Therefore $\therefore A = 0.333$ -1

so $I = 0.333e^{-0.858t} - 0.333e^{-29.14t}$ -1

Section B

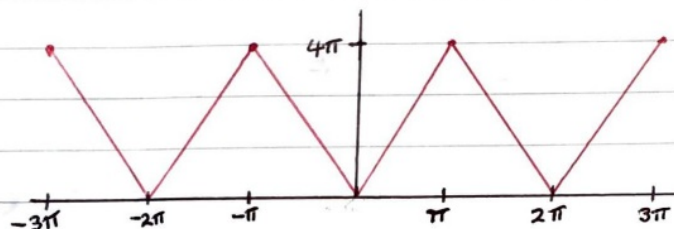
4) Sketch the graph and determine the Fourier series expansion of the function

$$\begin{aligned} f(x) &= -4x & \text{when } -\pi < x < 0 \\ f(x) &= 4x & \text{when } 0 < x < \pi \\ \text{and } f(x) &= f(x + 2\pi) \end{aligned}$$

$$-4(-\pi) = 4\pi$$

$$-4(0) = 0$$

$$4(\pi) = 4\pi$$



symmetrical so = Even

$$b_n = 0$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} 4x dx$$

$$a_0 = \frac{2}{\pi} \left[\frac{4x^2}{2} \right]_0^{\pi}$$

$$a_0 = \frac{2}{\pi} \left[\frac{4(\pi)^2}{2} - 0 \right]$$

$$a_0 = 4\pi$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx$$

$$= \frac{2}{\pi} \int_0^{\pi} 4x \cos nx dx$$

$$\text{let } u = 4x$$

$$\frac{dv}{dx} = \cos nx$$

$$\frac{du}{dx} = 4$$

$$v = \frac{1}{n} \sin nx$$

$$\int (4x) (\cos nx) dx = (4x) \left(\frac{1}{n} \sin nx \right) - \int \frac{1}{n} \sin nx (4) dx$$

$$= \frac{4x}{n} \sin nx - \int \frac{4}{n^2} \cos nx$$

$$\therefore a_n = \frac{2}{\pi} \left[\frac{4x}{n} \sin nx + \frac{4}{n^2} \cos nx \right]$$

$$= \frac{2}{\pi} \left[\frac{4\pi}{n} \sin(n\pi) + \frac{4}{n^2} \cos(n\pi) \right] \pi$$

$$= \frac{2}{\pi} \left[\frac{4(0)}{n} \sin(0) + \frac{4}{n^2} \cos(0) \right] 0$$

$$= \frac{2}{\pi} \left[\frac{4}{n^2} \cos(n\pi) \right] - \frac{2}{\pi} \left[\frac{4}{n^2} \right]$$

$$a_n = \frac{8}{\pi n^2} [\cos(n\pi) - 1]$$

$$n=1, \quad a_1 = \frac{4}{\pi(1)^2} [-1(-1)] = \frac{-8}{\pi(1)^2}$$

$$n=2, \quad a_2 = \frac{4}{\pi(2)^2} [1(-1)] = 0$$

$$n=3, \quad a_3 = \frac{4}{\pi(3)^2} [-1(-1)] = \frac{-8}{\pi(3)^2}$$

$$n=4, \quad a_4 = \frac{4}{\pi(4)^2} [1(-1)] = 0$$

$$n=5, \quad a_5 = \frac{4}{\pi(5)^2} [-1(-1)] = \frac{-8}{\pi(5)^2}$$

$$n=6, \quad a_6 = \frac{4}{\pi(6)^2} [1(-1)] = 0$$

$$n=7, \quad a_7 = \frac{4}{\pi(7)^2} [-1(-1)] = \frac{-8}{\pi(7)^2}$$

$$f(x) \approx \frac{1}{2} a_0 + \sum_{n=1}^{\infty} [a_n \cos nx + \cancel{b_n \sin nx}] \quad b_n = 0$$

$$= \frac{1}{2} (4\pi) - \frac{8}{\pi(1)^2} \cos 1x - \frac{8}{\pi(3)^2} \cos 3x - \frac{8}{\pi(5)^2} \cos 5x - \frac{8}{\pi(7)^2} \cos 7x$$

-2 (only 4 terms required)

$$= \frac{5\pi}{2} - \frac{8}{\pi} \cos x - \frac{8}{9\pi} \cos 3x - \frac{8}{25\pi} \cos 5x - \frac{8}{49\pi} \cos 7x \dots$$

question 4 total =

1809812

GRADEMARK REPORT

FINAL GRADE

GENERAL COMMENTS

Instructor

84/100

PAGE 1

PAGE 2

Text Comment. 7/7

PAGE 3

Text Comment. 2/2

Text Comment. 5/5

Text Comment. 4/4

Text Comment. x

Text Comment. 1/3

Text Comment. section A total = 19/25

PAGE 4

Text Comment. c should be 0.05

PAGE 5

Text Comment. answer correct for the values used

Text Comment. x

Text Comment. 23/25

PAGE 6

PAGE 7

Text Comment. -2

Text Comment. x

Text Comment. -1

Text Comment. x

Text Comment. -1

Text Comment. -1

Text Comment. question 3 total = 20/25

PAGE 8

PAGE 9

Text Comment. -2 (only 4 terms required)

Text Comment. -1

Text Comment. question 4 total= 22/25