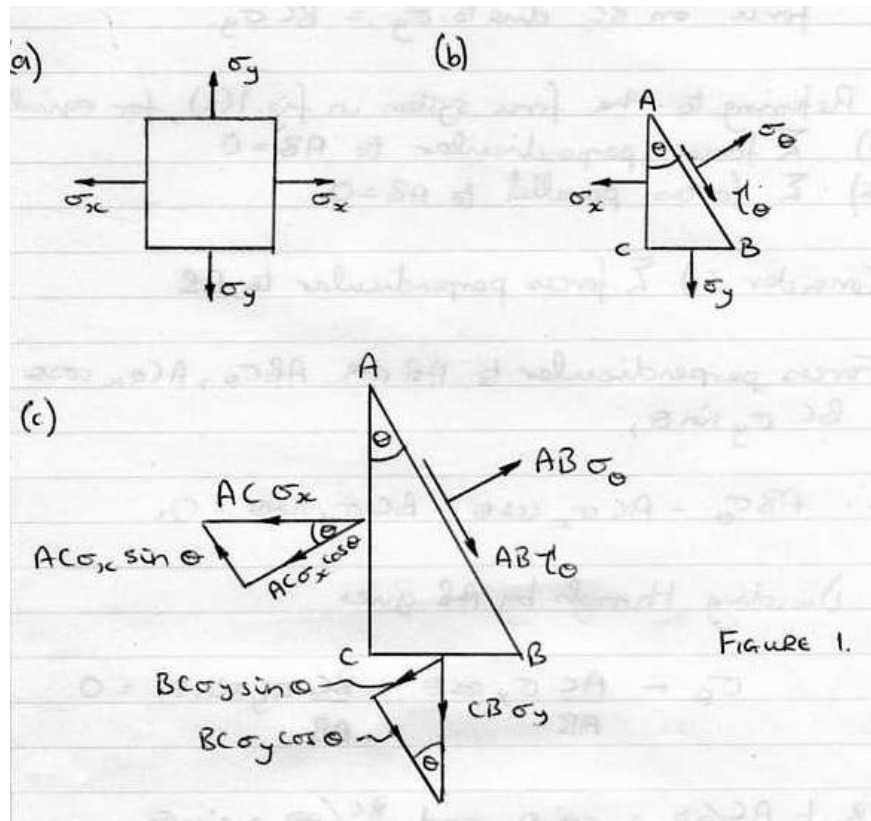


Normal and Shear Stresses on any plane in an element subject to NORMAL stresses only

Consider the following stressed element of unit thickness in equilibrium



Stress normal to the plane;

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta \quad [1]$$

Shear stress on the plane;

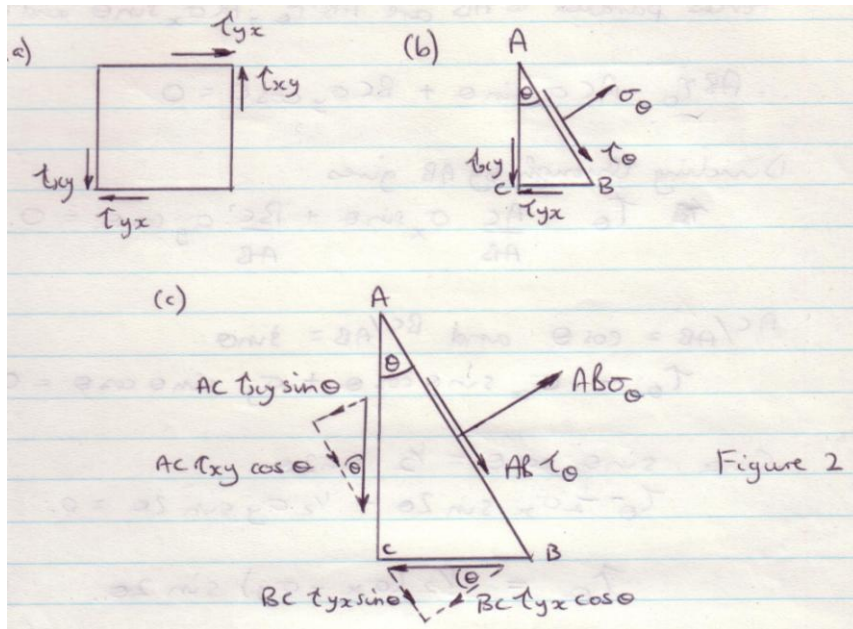
$$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta \quad [2]$$

Maximum shear stress when $\theta = 45^\circ$

$$\tau_{\max} = \frac{1}{2}(\sigma_x - \sigma_y) \quad [3]$$

Normal and Shear Stresses on any plane in an element subject to SHEAR stresses only

Consider the following stressed element of unit thickness in equilibrium



Stress normal to the plane;

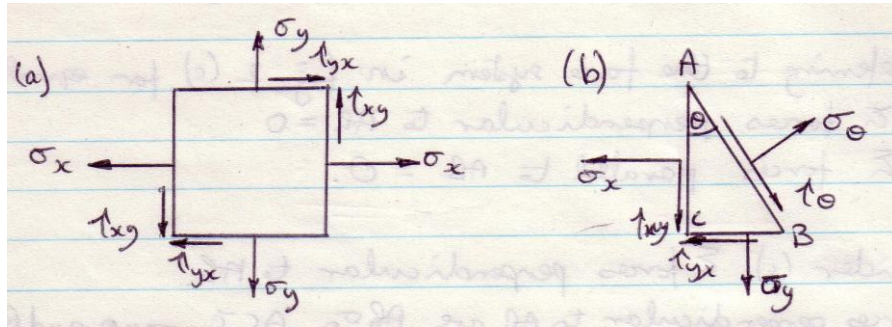
$$\sigma_\theta = \tau_{xy} \sin 2\theta \quad [4]$$

Shear stress on the plane;

$$\tau_\theta = -\tau_{xy} \cos 2\theta \quad [5]$$

Normal and Shear Stresses on any plane in a GENERAL 2D STRESS SYSTEM

Consider the following stressed element of unit thickness in equilibrium



Combining [1] and [4], Stress normal to the plane;

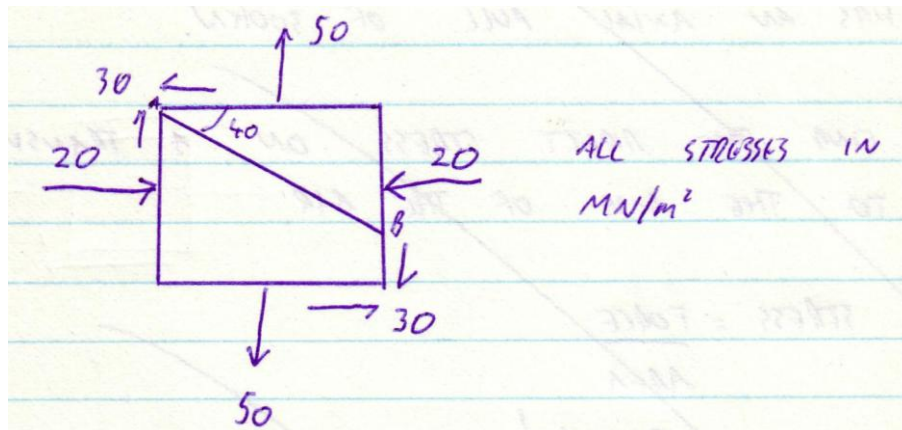
$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy} \sin 2\theta \quad [6]$$

Combining [2] and [5], Shear stress on the plane;

$$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy} \cos 2\theta \quad [7]$$

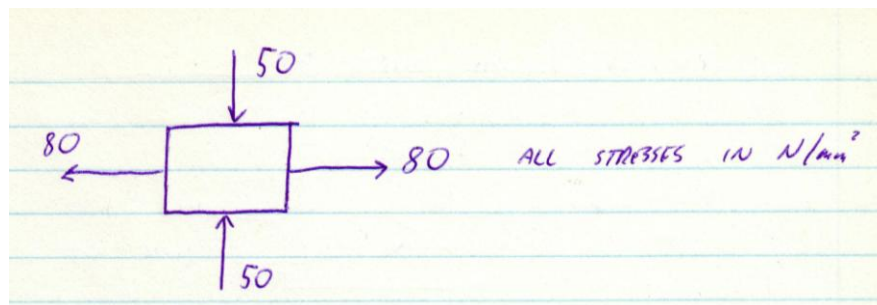
Complex Stress Examples

1.



What are the normal and shear stresses on plane AB?

2.



What are the normal and shear stresses acting on a plane inclined at 60° to the vertical?

3) A bar of cross sectional area 850mm^2 is acted upon by tensile forces of 60kN applied at each end of the bar. Determine the normal and shearing stresses on a plane inclined at 30° to the direction of loading.

[17.65MPa, -30.6MPa]

PRINCIPAL PLANES and PRINCIPAL STRESSES

PRINCIPAL STRESS – The maximum and the minimum normal stresses

PRINCIPAL PLANE – The plane that the principal stress acts on

Stress at any plane;

$$\sigma_{\theta} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy} \sin 2\theta$$

To find max, σ_1 , or min, σ_2 ;

$$\frac{d\sigma_{\theta}}{d\theta} = 0$$

$$\sigma_{1,2} = \frac{1}{2} \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$$

At a principal plane the shear stress, $\tau_{\theta} = 0$, therefore our maximum shear stress is given by [3];

$$\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_2)$$

$$\tau_{\max} = \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Shear stress is a max on planes at 45° to planes of principal stress.

The angle of the face that the principal stress is on can be calculated by;

$$\tan \theta = \frac{\sigma_p - \sigma_x}{\tau_{xy}}$$

Principal Stress and Principal Plane examples.

1.

Under certain loading conditions the stresses in the walls of a cylinder are as follows:

- (a) 80 MN/m^2 tensile;
- (b) 30 MN/m^2 tensile at right angles to (a);
- (c) shear stresses of 60 MN/m^2 on the planes on which the stresses (a) and (b) act; the shear couple acting on planes carrying the 30 MN/m^2 stress is clockwise in effect.

Calculate the principal stresses and the planes on which they act. What would be the effect on these results if owing to a change of loading (a) becomes compressive while stresses (b) and (c) remain unchanged?

$$[120 \text{ MN/m}^2, -10 \text{ MN/m}^2, 33.69^\circ, 123.7^\circ]$$

$$[56.5 \text{ MN/m}^2, -106.5 \text{ MN/m}^2, 66.27^\circ, 156.2^\circ]$$

2.

A low cost race car structure is composed of circular members of diameter d . At a certain position along one member the loading is found to consist of a shear force of 10 kN , together with an axial tensile load of 20 kN .

- a) Show that the principal stresses σ_1 and σ_3 are given by

$$\sigma_1 = \frac{30.7}{d^2} \text{ kNm}^{-2} \quad \text{and} \quad \sigma_3 = -\frac{5.27}{d^2} \text{ kNm}^{-2}$$

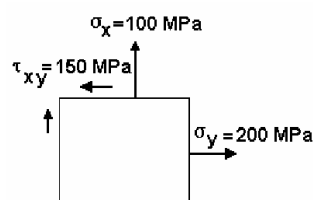
3.

A material is subjected to two mutually perpendicular direct stresses of 80 MN/m^2 tensile and 50 MN/m^2 compressive, together with a shear stress of 30 MN/m^2 . The shear couple acting on planes carrying the 80 MN/m^2 stress is clockwise in effect. Calculate

- (a) the magnitude and nature of the principal stresses;
- (b) the magnitude of the maximum shear stresses in the plane of the given stress system;
- (c) the direction of the planes on which these stresses act.

$$[86.55 \text{ MN/m}^2, -56.59 \text{ MN/m}^2, 71.6 \text{ MN/m}^2, 77.61^\circ, 167.6^\circ]$$

4. Find the principal stresses and the position of the principal planes for the below example



$$[-8.1 \text{ MPa}, 308.1 \text{ MPa}, 54.2^\circ, 144.2^\circ]$$