

Semester 2**Transmissions**

By the end of this semester you should be able to:

1. List the components in the powertrain and explain the basic operation of each.
2. Explain why a clutch is needed.
3. Define the term gear ratio and explain the relationship between gear ratio and torque.
4. Describe the types of gears used in a powertrain.

Powertrain

- The powertrain is the mechanism that transmits power from the engine crankshaft to the drive wheels.
- It can provide several gear ratios between the engine crankshaft and the drive wheels.
- For example a 3 speed manual transmission, the gear ratio can be changed so that the engine crankshaft will rotate 4, 8, or 12 times to cause the wheels to rotate once.
- These transmissions are performed by the driver by hand.

Manual powertrains consist of:

- ◆ Power plant (engine)
- ◆ Clutch
- ◆ Transmissions
- ◆ Drive shafts
- ◆ Differential
- ◆ Wheel Axle

Clutch**Purpose of the Clutch.**

- The clutch transmits rotary motion from one shaft to another while permitting engagement or disengagement of two coaxial shafts during rotation of one or both.
- These two shafts are the engine crankshaft and the transmission input or clutch shaft.

- With the clutch engaged power is delivered from the engine to the clutch shaft via the crankshaft, when the clutch is disengaged the crankshaft is uncoupled from the clutch shaft and no power is delivered to the transmission.
- The clutch uncouples the shafts to enable transmission.
- Without this interruption of power meshing and de-meshing of gears would be difficult and would damage the transmission.
- With the gears de meshed through operation of the clutch, gears can be re-engaged at the appropriate speed by use of synchronizing devices in the transmission.
- The clutch also enables power to be applied through the transmission gradually to enable a vehicle to accelerate from a stand still.
- Works using the friction between two contact surfaces.
- Operated by driver.

Clutch Construction.

- Between the rear of the engine and the front of the transmission there is a cast housing called the clutch housing, this protects the components from dirt and water and supports the clutch release lever and sometimes houses part of the starter motor.
- The single plate clutch usually consists of 3 basic elements, the **Crankshaft flywheel**, the **friction plate/disk** and the **pressure plate assembly**.

Friction Plate.

- This is a driven plate/disk with a splined hub that slides along the splines of the clutch shaft, these force the clutch shaft to rotate at the same speed as the flywheel when the disk is clamped on to the flywheel.
- It is faced on both sides with a heat resistant frictional material and is made of spring steel.
- When the clutch is engaged the plate is clamped tightly between the machined metal surfaces of the flywheel and the pressure plate, which are usually made from cast iron with a high graphite content offering lubrication enabling slip when the clutch is engaged.
- When a force is applied to the pressure plate the friction between the metal surfaces of the flywheel and the pressure plate causes the three to rotate as a solid unit.
- The friction plate usually has a flexible damping device to absorb the torsional vibration of the crankshaft.

- There is also a clutch release bearing located between the pressure plate and the clutch housing activated by the clutch lever.

Operation of the Clutch.

- When the clutch is engaged the clutch lever moves against the release bearing moving it inward.
- This operates release levers that overcome the spring force between the friction and pressure plates slightly separating the two.
- With this force removed from the friction plate it is free to move away from the flywheel face, the friction disk and the clutch shaft begin to slow and stop, whilst the flywheel and pressure plate continue to rotate.
- This stops the power being driven to the transmission and enables gear changes before the clutch is released and the friction plate is clamped by the pressure plate again.
- For engine to deliver usable power the crankshaft must be rotating faster than the engines minimum idle speed (600rpm or higher). An engine may start at speeds below 100rpm but will stall when load is applied.

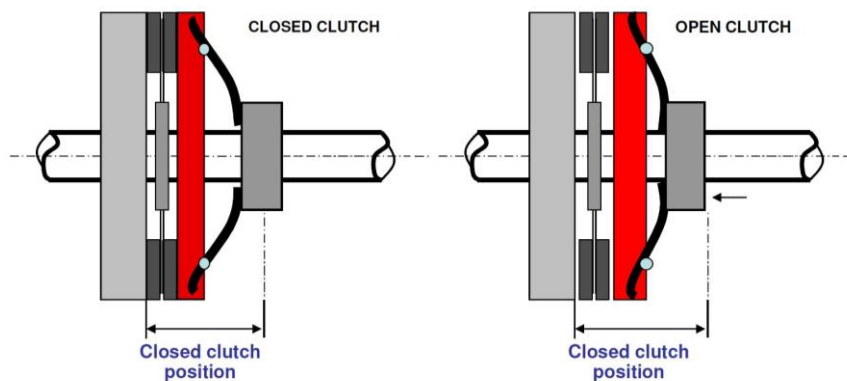


Figure 1: Simple clutch mechanism, (aldousvoice.com, 2019)

Constant Pressure Theory

- There are two methods to calculate the torque, T , transmitted by a flat plate clutch
- The first method is the Constant Pressure Theory which assumes that the pressure is constant over the contact surfaces
- This is best applied to new/unworn clutches

$$T = \frac{2}{3} \mu W \frac{(r_o^3 - r_i^3)}{(r_o^2 - r_i^2)}$$

Where T	=	torque (Nm)
μ	=	coefficient of friction (no units)
W	=	axial load (N)
r_o	=	outer radius (m)
r_i	=	inner radius (m)

Constant Wear Theory

- The Constant Wear Theory assumes the clutch wears by equal amounts over its surface
- This is best applied to worn clutches

$$T = \frac{\mu W (r_o + r_i)}{2}$$

Where T	=	torque (Nm)
μ	=	coefficient of friction (no units)
W	=	axial load (N)
r_o	=	outer radius (m)
r_i	=	inner radius (m)

Use the constant wear theory in all cases unless the clutch is unworn.

Example 1

A flat plate clutch has an inner radius of 20mm and an outer radius of 80mm. It is required to transmit a power of 200W at 600 rpm. The friction material has a coefficient of friction of 0.3. What is the required axial force? Use both the constant pressure and the constant wear theories.

$$T = \frac{2}{3} \mu W \frac{(r_o^3 - r_i^3)}{(r_o^2 - r_i^2)}$$

$$T = \frac{\mu W(r_o + r_i)}{2}$$

[illegible]

Example 2

Calculate the maximum torque which can be transmitted by a new clutch if it has an outer diameter of 150mm and an inner diameter of 30mm, a coefficient of friction of 0.4. The axial force may be taken as 970N. (Assume constant pressure)

$$T = \frac{2}{3} \mu W \frac{(r_o^3 - r_i^3)}{(r_o^2 - r_i^2)}$$

[illegible]

Multi Plate Clutches

- As the name suggests, a number of flat annular plates are held in contact by a normal axial force.
- Each pair of contact surfaces carries the full load with each side of each plate generally effective.
- Thus in such cases the total torque transmitted is n times the torque transmitted by one pair of contact surfaces, where n is the number of plates.

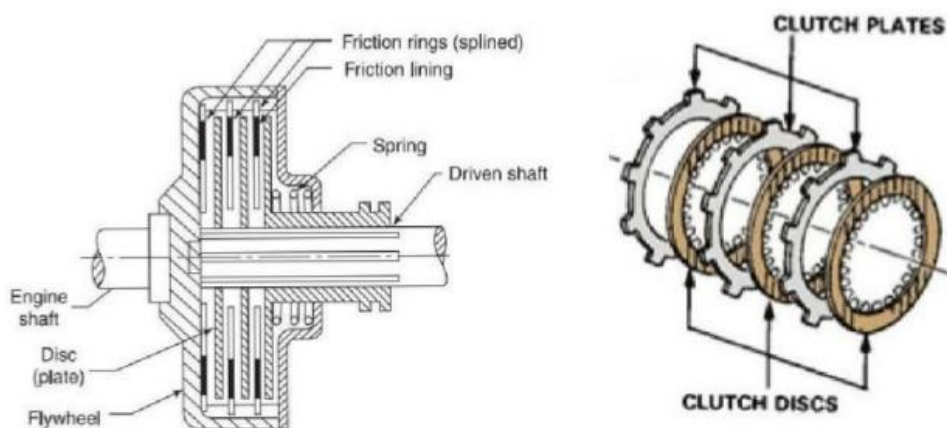


Figure 2: Multi plate clutches, (mechanicalbooster.com, 2019)

Example 5.

A single plate clutch plate of 0.15m effective diameter is lined with a material of coefficient of friction of 0.35. If the total spring force is 2.5 kN calculate:

a) The maximum torque transmitted.

b) The power transmitted at 3000 rev/min.

[131.25Nm, 41.23kW]

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Example 6.

The frictional disc of a single plate clutch has a mean radius of 0.12m. The spring force is 3000N and the coefficient of friction between the friction material and the driving surface is 0.3. Calculate the maximum power that could be transmitted by the clutch at 2000 rev/min. [22.62kW]

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Flywheels

- Many machines and engines produce erratic torques which are used to drive rotating shafts.
- These torques often occur in a cyclic fashion, where torque varies against angle or rotation.
- Examples:
 - Reciprocating piston engines
 - Compressors
 - Turbines
 - Reciprocating pumps
 - Press tools
- Flywheels are used to smooth the motion of the output shaft and keep the shaft speed to within acceptable limits.
- To do this it must reduce the effect of fluctuating torques applied to the shaft.



Figure 3: Flywheel, (areamotorsport.co.uk, 2019)

- Flywheels are essentially a device for storing rotary kinetic energy.
- They store the energy from fluctuations of high torque and release it during times of low torque.
- Acceleration leads to an increase in the kinetic energy.
- An objects inertia is a measure of its resistance to change in its velocity.

$$\text{Linear Kinetic Energy} = \frac{1}{2} mv^2$$

$$\text{Angular Kinetic Energy} = \frac{1}{2} I \omega^2$$

$$I = mk^2$$

When a rotating body changes speed, the angular acceleration is related to the moment of inertia and the applied torque by the formula;

$$T = I\alpha$$

If you can increase the inertia of a component you can reduce the angular acceleration which will result from a given change in torque

Transmission

- A transmission is an assembly of gears and shafts that transmits the engines power from the engine to the final drive.
- Generally transmission has at least 3 forward gear ratios, reverse and a neutral location.
- Also road cars currently have 5- 6 forward gears, 1 being 'low' and 6 being 'high'.
- Through correct gear ratio selection the engine torque can be increased, this permits the car motion without stalling or excessive clutch slip.
- Most modern gearboxes operate on the **constant mesh** principle, meaning all the gear pairs (1st, 2nd, 3rd etc.) are constantly meshed together
- Gears are mounted on two shafts
 - Permanently fixed to input shaft
 - Free to rotate around output shaft unless selected
- Dog clutches (or synchromesh units on most road cars) are used to connect these meshed pairs to the gearbox output shaft as the driver selects each gear

Gears and Torque

Gears transmit power between shafts. These gears are usually attached to a shaft and are '**meshed**' with another gear fastened to a different shaft.

Gears are '**meshed**' when their teeth are interlinked. In any set of gears the smallest is called the '**Pinion**' gear.

When one gear of a two gear set is turned the other gear turns the opposite way. The relative speed of the two meshing gears is determined by the number of teeth in the gears. This is called the '**Gear Ratio**'.

For example when two gears have the same number of teeth, they both turn at the same speed. However when the driven gear has more teeth than the driving gear, the smaller driving gear will turn faster than the larger driven gear.

This means that a driven gear with 12 teeth will turn twice as fast as a gear with 24 teeth. The gear ratio in this case being 2:1.

If a 12 tooth gear is meshed with a 36 toothed gear the 12 tooth gear turns three times for every one revolution of the larger gear. The gear ratio between these being 3:1

The gear ratio may also be determined by comparing the diameters of the two gears.

In a circle the distance across the centre to the opposite outside edge is called the 'diameter'...On a gear the diameter is the distance across the centre from the point on which the tooth where the force is applied to the equivalent on the opposite side of the gear. This average diameter is known as the pitch circle diameter or PCD.

Fill in the blanks on Figure 4:

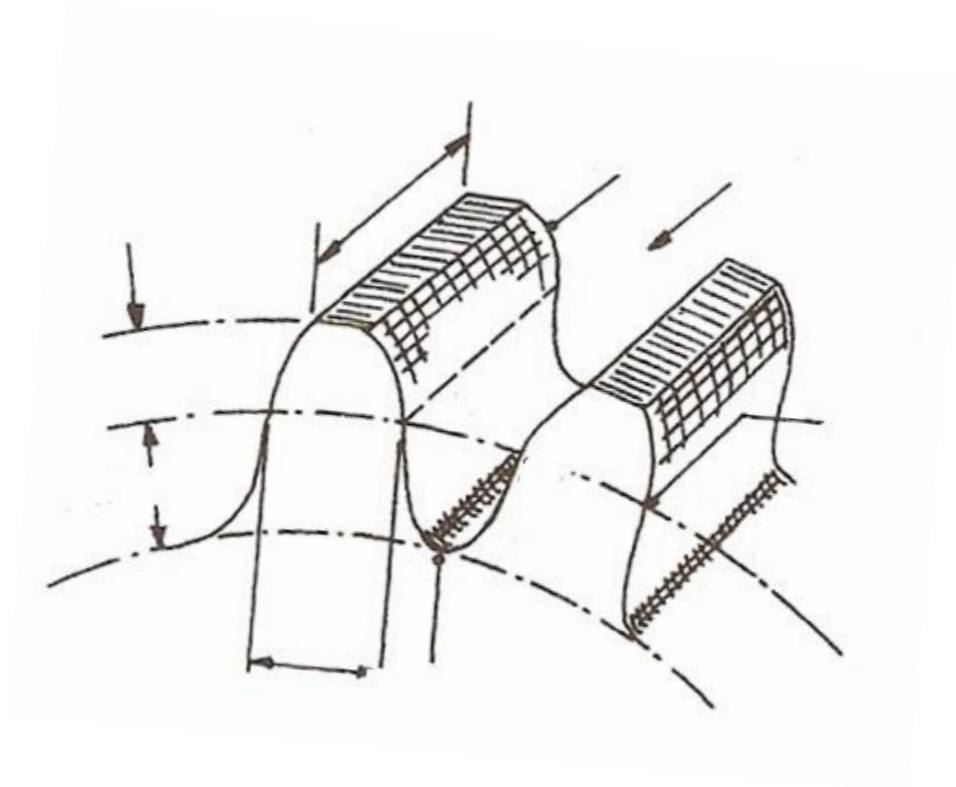


Figure 4: Gearing Terminology (me-mechanical engineering.com, 2019)

1. The..... is the part of the tooth at a greater radius than the pitch circle.
2. The is the part of the tooth at a lesser radius than the pitch circle.
3. The is the thickness of the tooth and is important in determining a gear's load capacity.
4. The is the shape of the tooth, most gears have what is known as an INVOLUTE profile.
5. The Pitch is the linear distance between corresponding points on two adjacent teeth. For two gears to mesh they must have the same pitch

Torque in gears

- The gear ratio changes as the number of teeth on each mesh changes.
- At the same time torque changes.
- Torque is the turning effort applied to the shaft, not to be confused with the work.
- Any shaft or gear that turns has a torque applied to it.
- The engine pistons and connecting rods push on the journals of the crankshaft, this applies a torque to the shaft causing it to turn.
- This applies torque through the flywheel and clutch to the gears in the transmission. This torque is carried through the powertrain to the wheels.
- Torque in shafts and gears is measured as a straight-line force at a distance from the centre of the shaft or gear.

Torque and Gear Ratio

When the smaller gear drives the larger gear the ratio is 2:1, however when the torque ratio is 1:2 the larger gear turns at half the speed of the smaller gear. As a result the larger gear will have twice the torque of the smaller gear.

In gear systems *speed reduction* means *torque increase*.

For example when a three speed transmission is in first gear, there is a speed reduction (or gear reduction) of 12:1 from the engine to the drive wheels. The crankshaft turns 12 times for the drive wheels to turn once. Ignoring frictional losses this means that the torque increases 12 times. If the engine produces a torque of 60Nm then a torque of 720Nm is applied to the drive wheels.

Note

If the radius of the wheels is 0.25m and then the force of the torque acting on the ground is 720Nm / 0.25m = 2880N. Consequently the push on the wheel axle is 2880N. But this torque is split between the two drive wheels therefore each tyre pushes on the ground with a force of 1440N.

Types of Gears

Spur Gear- The most basic type of gear, teeth are parallel to and align with the centre of the gear or the shaft axis. They are simple to manufacture, have relatively high noises and are not particularly smooth in operation.



Figure 5: Spur gears, (motioncontroltips.com, 2019)

Helical Gear- Similar to spur gears, but the teeth have been twisted at an angle from the centre line of the gear. These gears create sideways forces as the teeth are skewed, but when doubled up as the ones in figure 6, these forces are cancelled out. Smoother and quieter in operation compared with spur gears.

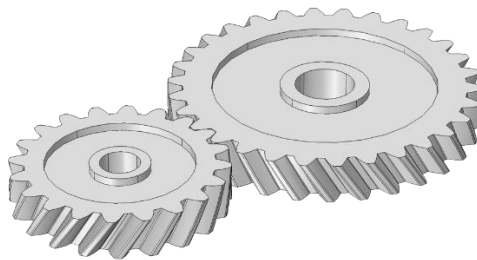


Figure 6: Helical gears, (geartechnology.com, 2019)

Bevel Gear- Shaped like a cone with the top cut off the teeth point inward to the peak of the cone. Designed for shafts whose axis intersect and can turn motion through any angle. Are also capable of having a reduction ratio and can come in both spur and helical tooth formats.



Figure 7: Bevel gear, (accu.co.uk, 2019)

Rack (crown) and Pinion- A rack and pinion comprises a circular gear (the pinion) engaging a linear gear (the rack), which operate to translate the rotational motion of a shaft into linear motion. Driving the pinion into rotation causes the rack to be driven linearly (steering rack). A crown gear has teeth that project at right angles to the face of the wheel or can be a type of bevel gear where the pitch cone angle is 90 degrees. Crown gears normally mesh with other bevel gears, or sometimes spur gears, a typical use being a crown gear and pinion system which allows a rotary motion to be shifted 90 degrees. Most commonly associated with differentials to get the drive through 90 degrees from the output shaft to the driven wheels.



Figure 8: Crown and pinion,
(OEM Gear manufacture co, 2019)



Figure 9: Rack and pinion,
(indiamart.com, 2019)

Internal gears- Have teeth which point inward, and are generally part of a planetary gear set (Automatic and Overdrive Transmission)



Figure 10: Planetary gear set, (designnews.com, 2019)

Worm gears- This is a gear arrangement in which a worm, a gear in the form of a screw, meshes with a worm gear, which is similar in appearance to a spur gear. The two elements are also called the worm screw and worm wheel. Worm gears can generate very high reduction ratios in a single stage and are reasonably compact. However, there are large amounts of sliding contact between the teeth of the worm and the gear wheel which can lead to high wear levels.



Figure 11: Worm gear, (motopncontroltips.com, 2019)

Differential

Function:

- Power flows from the engine to the wheels via a differential.
- When power arrives at the differential it is split and sent to the two driving wheels.
- When the vehicle is moving in a straight line both driving wheels will travel at the same speed.
- When the car come to a corner the outer wheel has to travel faster and further than the inner wheel.
- The action of the differential enables one wheel to turn at a different speed to the other while both are transmitting power from the drive shafts to the wheels.

Construction (RWD):

- The components of a differential are housed in a cast iron casing.
- There are two drive (half) shafts and side gears.
- The case has bearings that enable the free rotation of the shafts.
- There are two pinion gears which support a pinion shaft that fits into the casing.
- These pinion gears mesh with the side gears linking the inner ends of the drive shafts.
- There is a ring gear bolted to a flange at the rear of the differential casing, linked to the drive via a meshing drive pinion free to rotate about the ring gear.

Operation.

- When the car travels in a straight direction, the casing, ring gear, side gears and pinion gears are all freely running.
- At this time the pinion gears do not rotate on the pinion shaft as they exert an equal force on the side gears, as a result they rotate at the same speed as the ring gear, causing the wheels to rotate at the same speed.
- When the car negotiates a corner the pinion gears rotate on the pinion shaft permitting the outer wheel to turn faster than the inner wheel.
- At this time the casing rotates and the pinion gears rotate on their shaft, this is to enable the pinion gears to 'walk around' the slower rotating side gear.

Calculating Gear Ratio's

Essentially drive can be taken from one shaft (the driving shaft) through a set of gears and used to drive another shaft (the driven shaft). The speed difference between the two shafts can be increased or decreased by decreasing or increasing the number of teeth on the gear of the driven shaft respectively. This difference in the number of teeth on each gear is called the **Gear Ratio**.

If we think of gears as two circles rolling together at their pitch circles, then the peripheral speeds at the pitch circles must be the same:

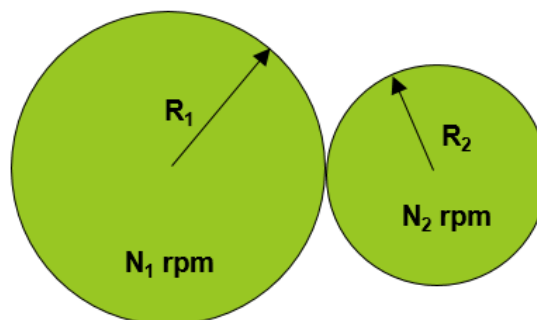


Figure 12: Simple gear set

Consider our bigger gear in figure 12. Its circumference (m) is:

$$2\pi R_1$$

In one second the number of rotations is:

$$\frac{N_1}{60}$$

Therefore, at the pitch circle, it is moving at a velocity (m/s) of:

$$\frac{2\pi R_1 N_1}{60}$$

For the smaller gear, the same logic applies, so:

$$\frac{2\pi R_2 N_2}{60}$$

But these pitch circle velocities must be equal: $\frac{2\pi R_1 N_1}{60} = \frac{2\pi R_2 N_2}{60}$

Therefore: $R_1 N_1 = R_2 N_2$

So we have a relationship between speeds and gear size.

Equation 1:
$$\frac{R_2}{R_1} = \frac{N_1}{N_2}$$

So, if our first gear is larger than the second gear, then the speeds of the second gear will be greater than the first, and vice versa

E.g. if our gear 1 is twice the diameter of gear 2, the rotational speed of gear 2 will be twice that of gear 1.

If the gears are to mesh together, then the teeth on each gear must be the same size (pitch).

In metric gears, the size of a gear is called the *module*

Imperial gears are sized by their *diametral pitch* (or DP)

Gears will mesh if they have the same pitch (i.e module or DP)

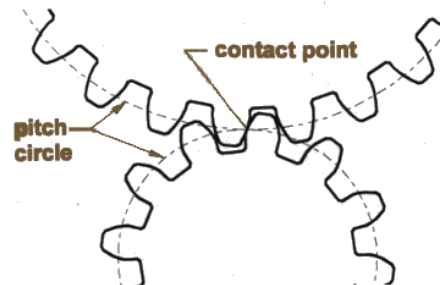


Figure 13: Gear mesh

There is a direct relationship between gear diameter and the number of teeth on the gear (for a fixed DP or module)

Hence, we can replace gear radius with number of teeth (n).

So the gear ratio, r :

Equation 2:

$$r = \frac{n_2}{n_1} = \frac{N_1}{N_2}$$

A **simple gear train** is one where the gear wheels are all on separate shafts and in plane as in Figure 14:

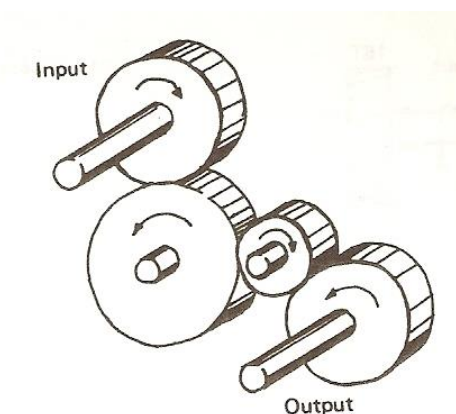
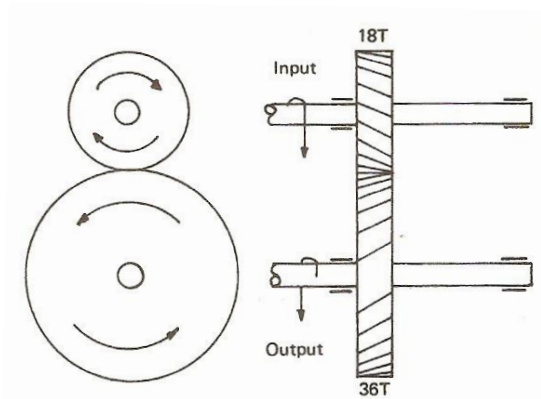


Figure 14: Simple gear train

Example 1

Calculate the gear ratio:



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Compound Gear Train

A compound gear train has two or more sets of gears connected together. The output of the first *stage* is connected via a common shaft to the *input* of the second stage. This is more compact than a multi-stage simple gear train and is a common arrangement in many gear boxes:

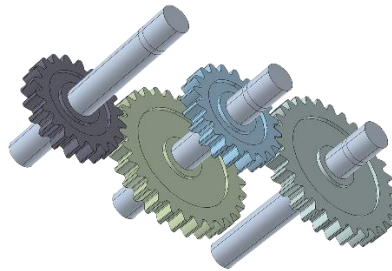
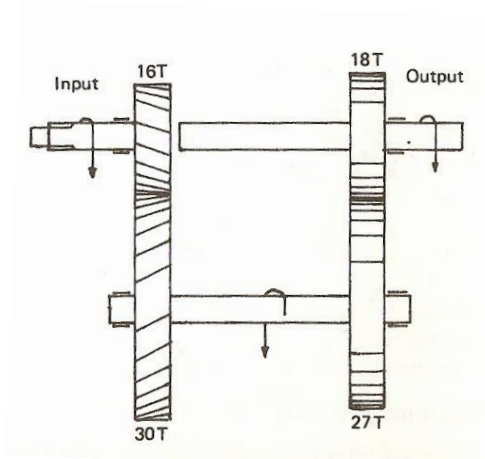


Figure 15: Compound gear train, (comsol.com, 2019)

Example 2

Calculate the overall gear ratio:



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Or:

For compound gear train:

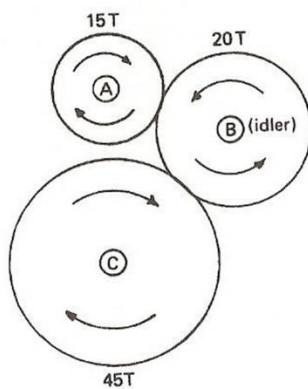
Equation 3
$$\text{Gear Ratio} = \frac{\text{No of teeth on driven gear}}{\text{No of teeth on driver gear}} \times \frac{\text{No of teeth on driven gear}}{\text{No of teeth on driver gear}}$$

A reverse gear train uses an idler gear between two gear wheels.

The idler changes the direction of rotation but does not affect the overall ratio.

Example 3

Calculate the overall reduction of this reverse gear train.



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Torque Relationships

The efficiency of the system is compromised by the losses to friction:

Equation 4:
$$\text{System Efficiency} = \frac{\text{Torque Ratio}}{\text{Gear Ratio}}$$

Equation 5:
$$\text{Torque Ratio} = \frac{\text{Output Torque}}{\text{Input Torque}}$$

Final Drive Ratio

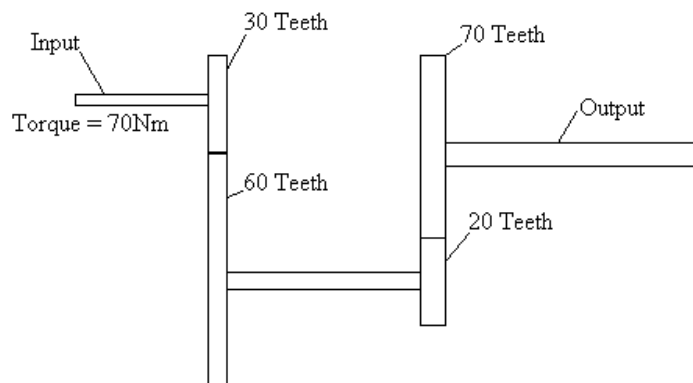
The final drive ratio is a product of the gear box ratio and the rear-axle ratio.

Equation 6:
$$\text{Final Drive Ratio} = \text{Gear box ratio} \times \text{Axle ratio}$$

Equation 7:
$$\text{Speed of the road wheels} = \frac{\text{Engine Speed}}{\text{Overall Gear Ratio}}$$

Example Questions.

1.



Given the above diagram calculate:

- a) The gear ratio,
- b) The output torque if the system efficiency is 75%.

2. A vehicle's gearbox has an input of 90Nm at 3010rpm. The input shaft has 20 teeth and meshes with a lay shaft having 70 teeth. The corresponding lay shaft output has 15 teeth driving a gear on the main shaft with 75 teeth. The gearbox system has an efficiency of 80%.

Calculate:

- a) The speed of the output shaft,
 - b) The torque at the output shaft.
3. At an engine speed of 3675rpm a torque of 150Nm is supplied to a gearbox. If the gearbox and final drive ratios are 2.5 and 4.5 respectively and the vehicle is travelling in a straight line, calculate:
- a) The speed of the road wheels,
 - b) The torque applied to each road wheel if the system is 100% efficient.
4. At an engine speed of 2025rpm a torque of 96Nm is supplied to a gearbox. If the gearbox and final drive ratios are 2.5 and 4.5 respectively, calculate:
- a) The overall gear ratio,
 - b) The torque supplied by the differential to each road wheel (neglecting frictional losses).
 - c) The speed of the road wheel.

HND/BSc Powertrain Technology: Transmission Matching

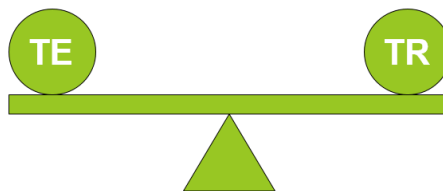
- Aim is to analyse the transmission requirements of a vehicle/engine combination
- Determine the optimum gear ratios
- Starting by analysing the forces on a vehicle which determine whether it accelerates or decelerates

Vehicle Forces

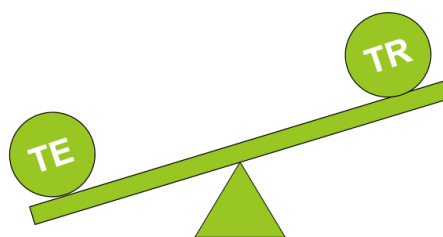
- **Tractive Effort (TE)** is the force applied by the engine via the transmission at the wheels in order to overcome the Tractive Resistance of the vehicle
- **Tractive Resistance (TR)** would slow the vehicle down and bring it to rest

$$TE = \frac{\text{Torque @ Wheels}}{\text{Rolling Radius of Wheel}}$$

The vehicle's behaviour may be thought of as a balance between TE and TR:

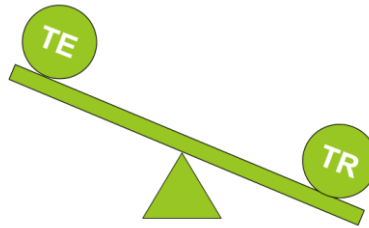


If Tractive Effort is **greater** than Tractive Resistance:



The vehicle will **accelerate**

If Tractive Effort is **less** than Tractive Resistance:



The vehicle will **decelerate**

Tractive Resistance (TR) can be divided into three main parts:

- **Rolling Resistance** due to tyre deformation and inertia in the system
- **Gradient Resistance** due to overcoming a proportion of vehicle weight as the vehicle climbs a slope
- **Aerodynamic Drag** overcoming the air resistance due to the vehicle's shape

Aerodynamic drag

- Air resistance of a vehicle determines how much power is required to overcome aerodynamic drag in road vehicles
- Obvious importance of aerodynamics in racing vehicles
- Power available for acceleration
- Fuel efficiency etc.

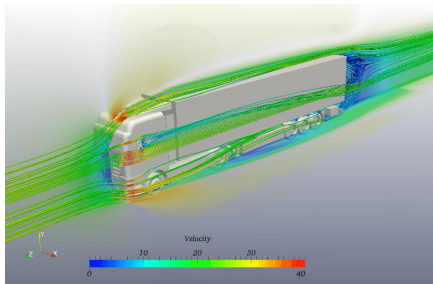


Figure 1: Freight analysis

(Simscale.com, 2019)

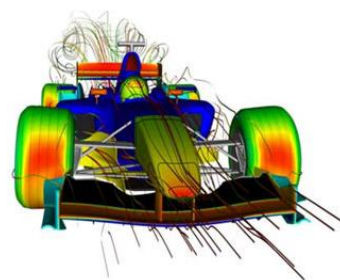


Figure 2: Formula 1 Analysis,

(computationalfluidynamics.com,2019)

- Vehicles have a range of different body types
- Each will have a measured Drag Coefficient, c_d
- We usually take this as an average constant value

The Drag Force equation can be rewritten:

$$F_{drag} = \frac{1}{2} \rho C_d A (V + V_0)^2$$

- Where V is vehicle speed and V_0 is headwind speed (m/s)

Thus, the power required to overcome air resistance is:

$$P_{drag} = F_{drag} \times V$$

$$P_{drag} = \frac{1}{2} \rho C_d A (V + V_0)^2 \cdot V$$

Typical drag power characteristics:

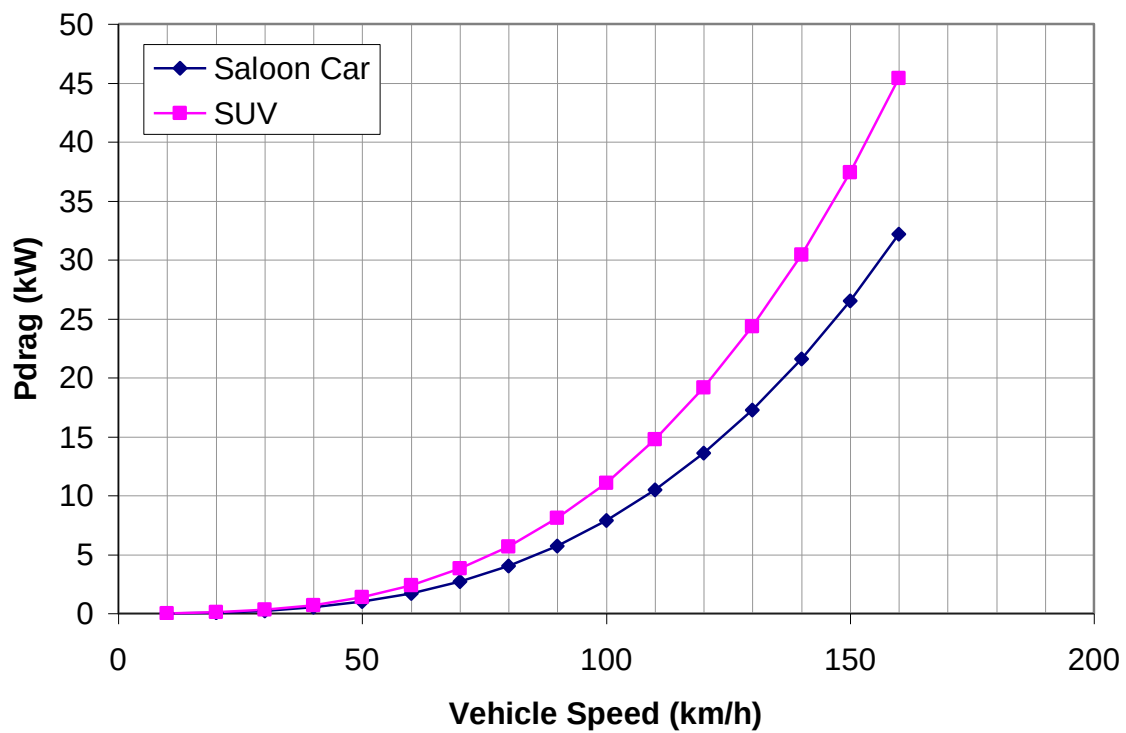


Figure 3: Drag power vs vehicle speed characteristic curve

Rolling resistance

- Rolling resistance is due to deformation processes which occur at the contact patch between the tyre and the road
- Is a weak function of speed and tyre construction

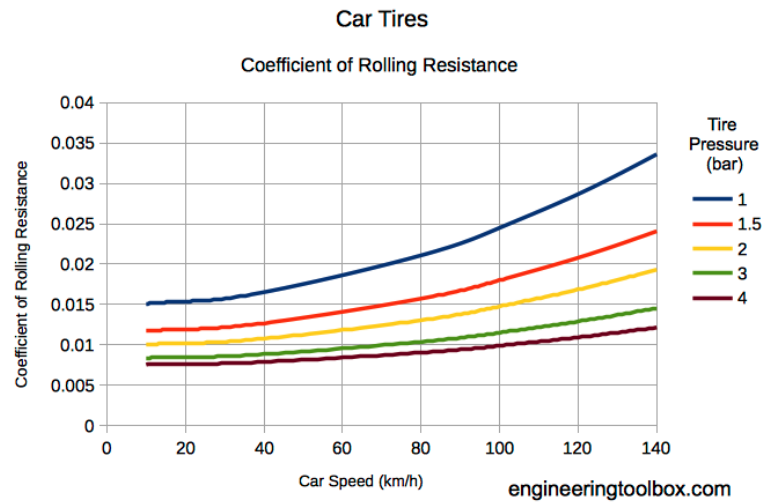


Figure 4: Rolling resistance vs vehicle speed

- Also depends on tyre pressure

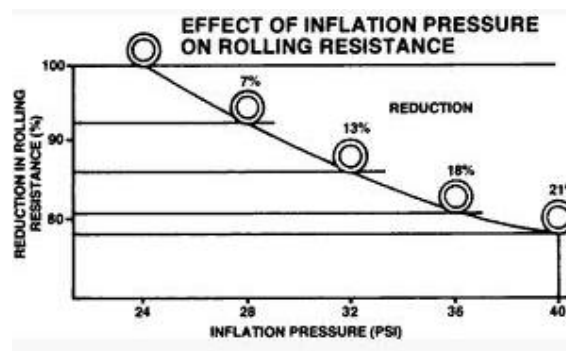


Figure 5: Effect of tyre pressure on rolling resistance, (greenhybrid.com, 2019)

- For our purposes we take it as being a constant and nominally independent of vehicle speed:

$$F_{rr} = C_{rr}mg\cos\theta$$

Where m = mass of vehicle (kg)

g = 9.81 ms^{-2}

θ = angle of slope (degrees)

C_{rr} = Rolling resistance coefficient values range from 0.006 to 0.015 for normal road tyres

Gradient Resistance

This is the proportion of a vehicle's weight which acts along the slope or against the vehicle's motion as it climbs a gradient:

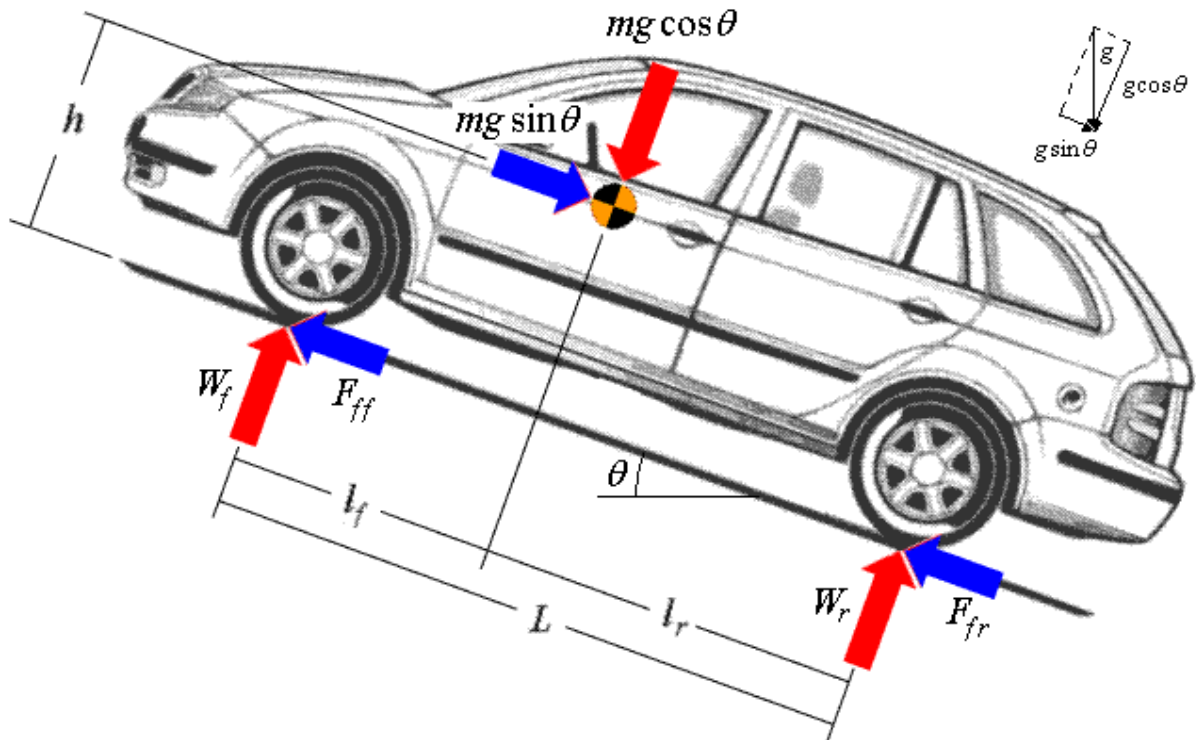


Figure 6: Gradient resistance

It is defined as:

$$F_g = mg \sin \theta$$

- It is **positive** if the vehicle is **climbing** a slope
- It is **negative** if the vehicle is **descending** a slope

So the **Tractive Resistance** by adding the aero, rolling and gradient resistances:

$$TR = F_{drag} + F_{rr} + F_g$$

$$TR = \frac{1}{2} \rho C_d A (V + V_0)^2 + C_{rr} mg \cos \theta + mg \sin \theta$$

$$TR = \frac{1}{2} \rho C_d A (V + V_0)^2 + mg (C_{rr} \cos \theta + \sin \theta)$$

Example

Consider an example below for a 1.4 litre VW Golf

Rolling radius of 175/70R13 tyre	R	0.26 m
Laden mass	m	1107 kg
Aerodynamic properties	A.C _d	0.77
Coefficient of rolling resistance	C _{rr}	0.015
Transmission efficiency	η	0.85
Air density	ρ	1.225 kgm ⁻³
Maximum torque	T _{max}	124 Nm @ 3500 rpm
Maximum power	P _{max}	56.2 kW @ 5000 rpm

Stage 1:

- Plot the graph of drag power versus road speed:

$$P_{drag} = \frac{1}{2} \rho C_d A (V + V_0)^2 \cdot V$$

- Use zero headwind
- Road speed needs to be in m/s,
- Typically, use up to 200 km/h to allow for markets with higher maximum allowable speeds

What is 200km/h in m/s?.....

Velocity (m/s)	Pdrag (kW)	Velocity (km/s)



Figure 7: Drag power vs Road velocity

Stage 2:

- Add to this graph a horizontal line indicating maximum engine power available at the wheels
 - Remember to allow for the transmission efficiency, new power is:.....
 - The maximum vehicle speed is where the available power is equal to the drag power (i.e. the two lines intersect):.....

Stage 3:

- But at 156 kph, is the engine at the correct speed to produce $P_{\max}$?
- Assume top gear is directly driven i.e. 1:1 ratio, such that at 156 kph (43.3 ms^{-1}), the engine is running at 5000 rpm when in top gear
- The relationship between road speed V (m/s) and engine speed N (rpm) is:

$$V = \frac{2\pi RN}{60fn}$$

Where R = tyre rolling radius (m)

f = differential ratio (no units)

n = gear ratio (no units)

- Rearrange to make diff ratio (f) the subject:

$$f = \frac{2\pi RN}{60Vn}$$

- We now substitute our known values to find the diff ratio:

.....

.....

.....

.....

Recalling the definition of tractive effort:

$$TE = \frac{\text{Torque @ Wheels}}{\text{Rolling Radius of Wheel}}$$

Tractive Effort (TE) is the force applied by the engine via the transmission at the wheels in order to overcome the Tractive Resistance of the vehicle.

The torque at the wheels is the Torque at the engine multiplied by the reduction ratio of the gearing system, so:

$$TE = \frac{Tf\eta n}{R}$$

Where:

T = Engine torque (Nm)

R = rolling radius (m)

f = diff ratio

n = gear ratio

Bottom Gear Ratio

- So considering a hill start, let's assume the Golf must be able to pull away in 1st (ratio = n_1) on slope of 18.4°
- At these conditions, use the maximum available engine torque to pull away so:

$$TE = \frac{T_{max}f n_1 \eta}{R}$$

- To pull away, the Tractive Effort must overcome the total Tractive Resistance

$$TR = \frac{1}{2} \rho C_d A (V + V_0)^2 + mg(C_{rr} \cos \theta + \sin \theta)$$

- Simplify the TR formula as the car is not moving at the point when it pulls away, so $V = 0$

$$TR = mg(C_{rr} \cos \theta + \sin \theta)$$

- Since $\theta = 18.4^\circ$, and $m = 1107 \text{ kg}$, so we can calculate the tractive resistance in this case:

.....

- Since $TE = 3582.4 \text{ N}$, $T_{\max} = 124 \text{ Nm}$, $f = 3.144$, $\eta = 0.85$, and $R = 0.26 \text{ m}$, rearrange the TE formula for and solve for n_1

$$TE = \frac{T_{\max} f n_1 \eta}{R}$$

.....

Intermediate Gears

- Firstly, we need to calculate the speed ratio, s , this is the ratio between the engine speed for maximum torque, and that for maximum power, e.g. for our Golf:

.....

- Now calculate the intermediate gears using:

$$n_{\text{gear}} = n_1 \times s^{\text{gear}-1}$$

Where

n_{gear} = The gear ratio to be calculated

n_1 = is 1st gear ratio

s = is the speed ratio

For the Golf example:

2nd gear:

.....

3rd gear:

.....

4th gear (as a check, n_4 should be 1)

.....

Thus the check calculation gives n_4 approximately equal to 1, perfect.

A higher speed ratio will mean that more gears are needed between first and top gear to maintain the engine speed between maximum torque and maximum power speeds

In practice, road testing and modelling will be used to optimise these ratios

In order to find the tractive effort (TE) available across the engine speed range in each gear, consider the torque curve:

- To do this we need the Torque Curve (you will be given this data through testing)

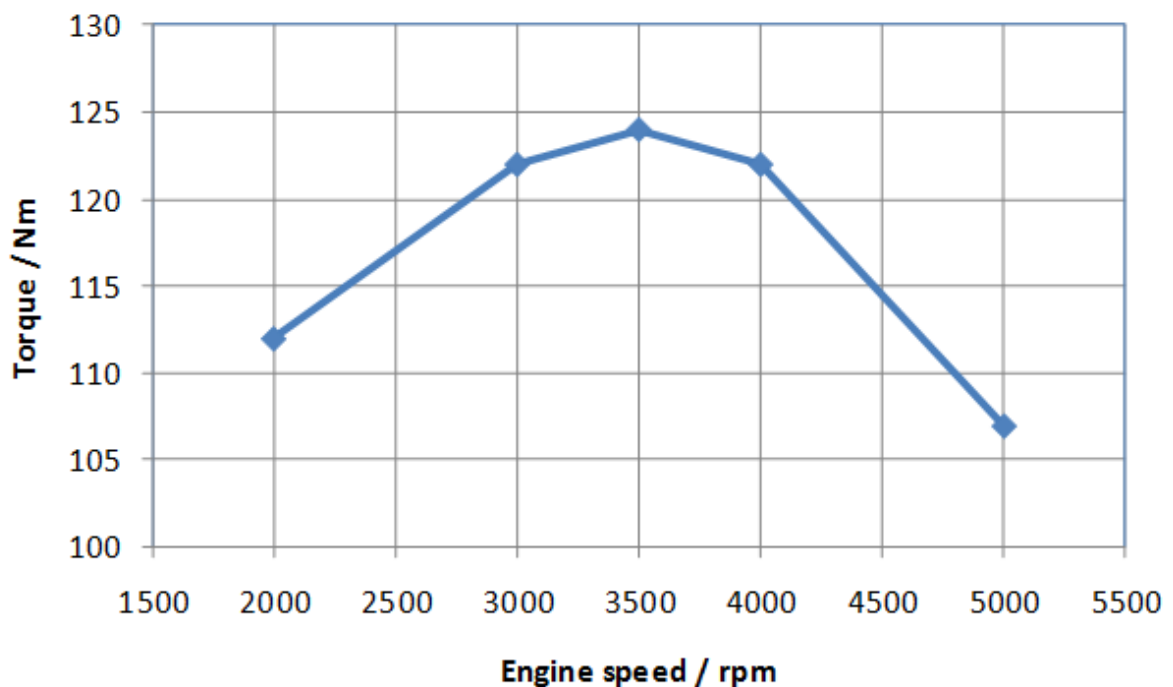


Figure 8: Torque vs Engine Speed

Maximum Gradients

Calculate the relationship between engine speed (N rpm) and road speed (V m/s) using:

$$V = \frac{2\pi RN}{60fn}$$

Where R = rolling radius (m), f = diff ratio, n = gear ratio

Calculate TE for each engine speed:

$$TE = \frac{Tfn\eta}{R}$$

Where η = transmission efficiency, T = engine torque (Nm)

Now plot TE against road speed in each gear:

(R=0.26m, f=3.144, $n_1=2.88$, $n_2=1.97$, $n_3=1.35$, $n_4=1$, $\eta = 0.85$)

1st Gear: 2.88

Engine Speed (N, rpm)	Torque (T, Nm)	Road Speed (V, m/s)	Road Speed (V, km/h)	Tractive Effort (TE, N)
2000				
3000				
3500				
4000				
5000				

2nd Gear: 1.97

Engine Speed (N, rpm)	Torque (T, Nm)	Road Speed (V, m/s)	Road Speed (V, km/h)	Tractive Effort (TE, N)
2000				
3000				
3500				
4000				
5000				

3rd Gear: 1.35

Engine Speed (N, rpm)	Torque (T, Nm)	Road Speed (V, m/s)	Road Speed (V, km/h)	Tractive Effort (TE, N)
2000				
3000				
3500				
4000				
5000				

4th Gear: 1

Engine Speed (N, rpm)	Torque (T, Nm)	Road Speed (V, m/s)	Road Speed (V, km/h)	Tractive Effort (TE, N)
2000				
3000				
3500				
4000				
5000				

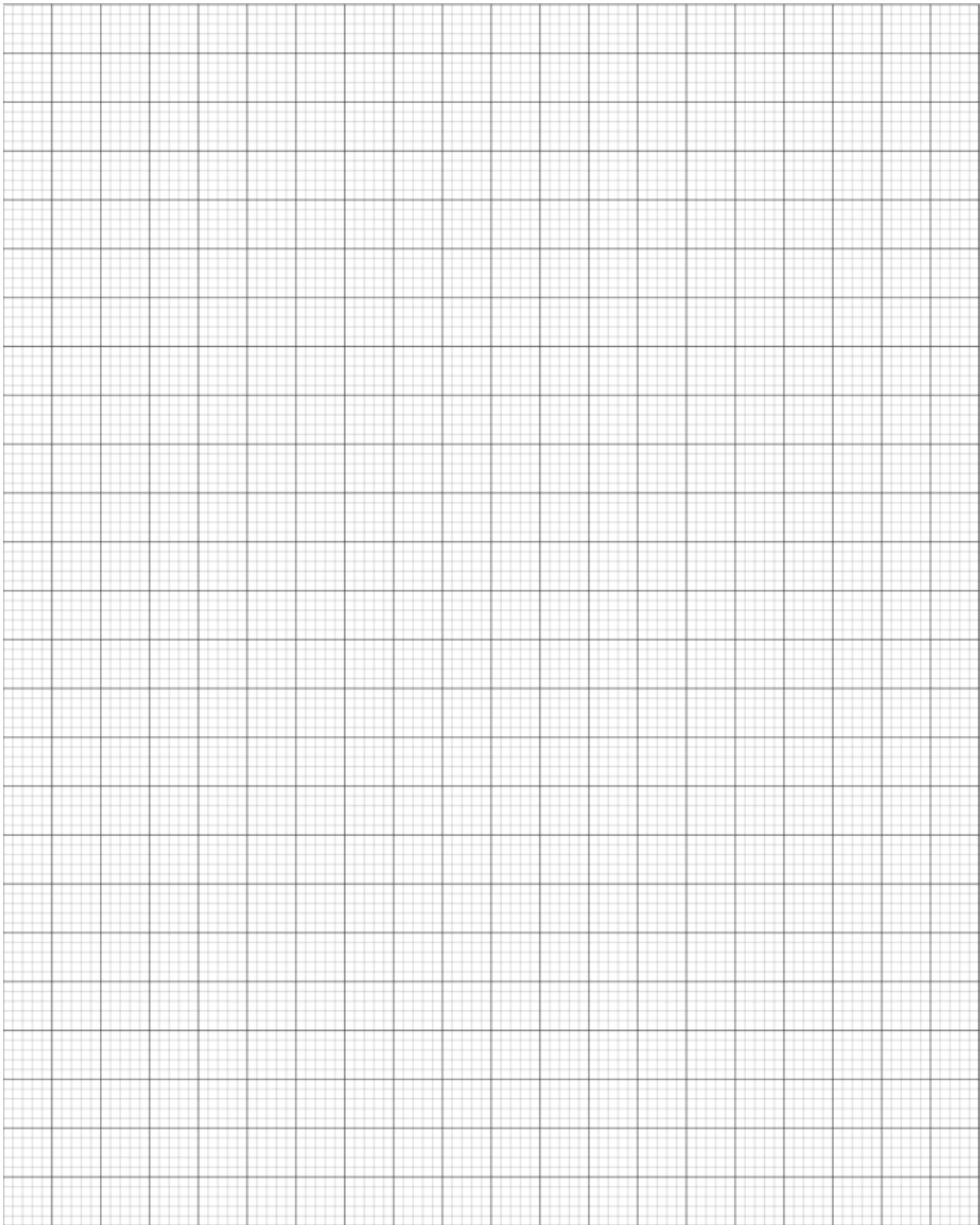


Figure 9: Tractive Effort vs Road velocity

Gear Changing

Any amount of power produced by an engine can be produced at a range of speeds and torques

For example, 25kW can be produced at

- $N = 2000 \text{ rpm}$ $T = 119 \text{ Nm}$
- $N = 5000 \text{ rpm}$ $T = 47 \text{ Nm}$

Similarly, the maximum power available at the wheels can be produced at a range of road speeds and torques (and hence TE). **Ideally you reach the theoretical useful end of a gear as you reach peak power so change up to the next gear and back into the start of the torque curve.** Always making best use of your available torque to accelerate.