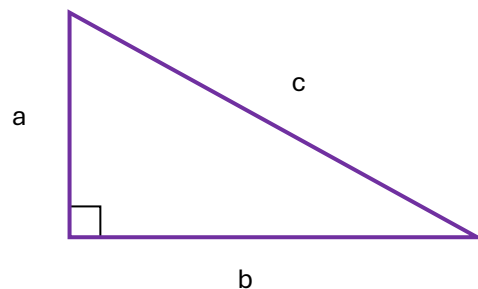

TRIGONOMETRY

Trigonometry is a subject that involves the measurement of sides and angles of triangles and their relationship to each other. A good knowledge of trigonometry and trigonometric functions is very important as it is used in many fields in engineering.

An important property of any triangle is that its angles add up to 180 degrees. Also, in a right angled triangle the side opposite the 90 degree angle is called the 'hypotenuse'. It is also useful to remember that in any triangle, the longest side always lies opposite the largest angle, and the smallest side lies opposite the smallest angle

Pythagorean Theorem:

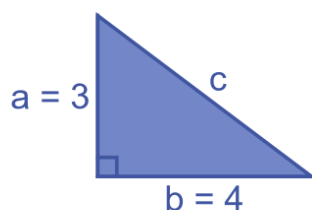
The Pythagorean is a relationship between the sides in a right angle triangle. This is a triangle where one of the three angles is an 90-degree angle. In a right angle triangle the sides are called legs and hypotenuse.



$$a^2 + b^2 = c^2$$

In words, for a right angle triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse.

Examples:



1.

Find the length of C.

$$a^2 + b^2 = c^2$$

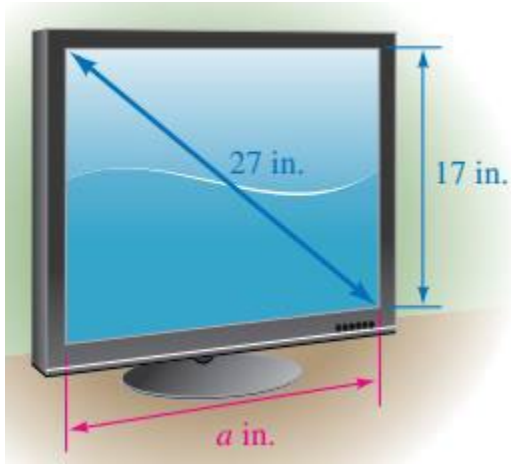
$$3^2 + 4^2 = c^2 \text{ giving } 9 + 16 = c^2$$

Rearrange to $c = \sqrt{9 + 16} = \sqrt{25} = 5$

The length of C is 5.

Remember $\sqrt{25} = \pm 5$ but in a practical example like this the negative answer can be ignored.

- The size of a TV screen is based on the diagonal measure. Find the width of a 27 inch screen that is 17 inches high.



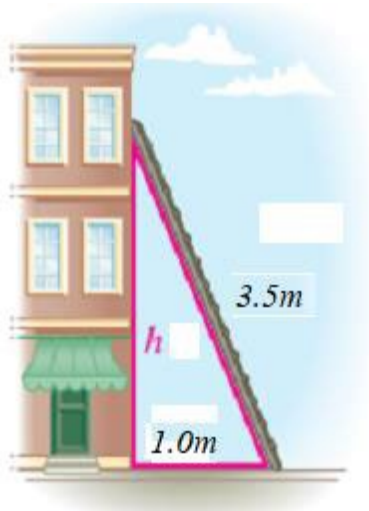
$$a^2 + 17^2 = 27^2$$

$$a^2 = 27^2 - 17^2 \text{ giving } a^2 = 729 - 289 \text{ giving } a^2 = 440$$

$$\text{Therefore } a = \sqrt{440} = 21$$

The width of the TV is 21 inches.

- A ladder 3.5 m long is placed against a perpendicular wall with its foot 1.0 m from the wall. How far up the wall (to the nearest centimetre) does the ladder reach?



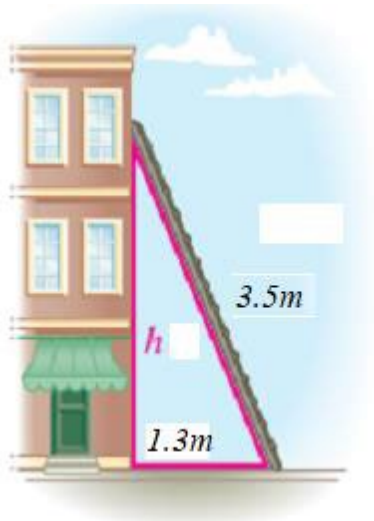
$$1^2 + h^2 = 3.5^2$$

$$h^2 = 3.5^2 - 1^2 \text{ giving } h^2 = 12.25 - 1 \text{ giving } h^2 = 11.25$$

$$\text{Therefore } h = \sqrt{11.25} = 3.35$$

The ladder reaches 3.35m up the wall.

If the foot of the ladder is now moved 30 cm further away from the wall, how far does the top of the ladder fall?



$$1.3^2 + h^2 = 3.5^2$$

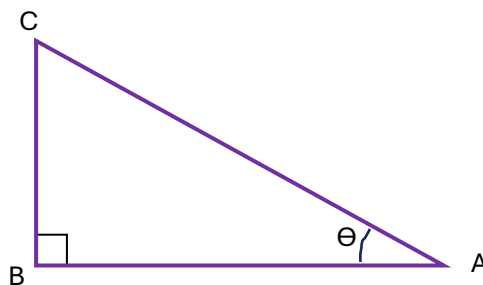
$$h^2 = 3.5^2 - 1.3^2 \text{ giving } h^2 = 12.25 - 1.69 \text{ giving } h^2 = 10.56$$

$$\text{Therefore } h = \sqrt{10.56} = 3.25\text{m}$$

The ladder now reaches 3.25m up the wall. Therefore the top of the ladder has dropped by 10cm.

Trigonometric Ratios

One of the most important things to know about right angled triangles is the relationship between its sides and its angles.



$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

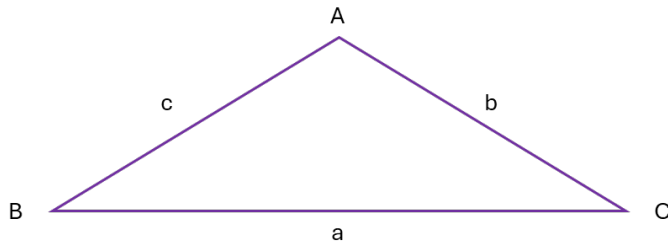
$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$$

So you can see from the above definitions that the side BC is called the 'opposite' side and the side AB is called the 'adjacent' side - but only if you are at angle θ .

If you were at the point C in the triangle, then the side AB would be the opposite side and the side BC would be the adjacent side. However, the side opposite the 90 degree angle is ALWAYS the hypotenuse side.

Standard Notation

We have a different way of denoting the sides of a non-right angled triangle, it is called 'the standard notation of a triangle'.



We simply denote the angles by capital letters and the sides opposite these angles by the corresponding small letters. I.e. side 'a' is opposite angle 'A' and so on.

The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This is an unusual formula due to the presence of two equals signs, however we do not use both at the same time. The following tells you when to use the sine rule -

- 1) Two angles and one side
- 2) Two sides and an angle that is not between the two given sides.

The Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

We use this, rather than the sine rule, when we are given either:

- 1) Two sides and the angle between the two sides (the 'included angle')
- 2) All three sides