

BEng First year
M4X00391 Mathematics
test 1 (90 mins)

choose 2 from 3
Questions

Question 1

a)

$$(i) \frac{4}{2x+1} = \frac{5}{x-1}$$

$$4(x-1) = 5(2x+1)$$

$$4x-4 = 10x+5$$

$$-9 = 6x$$

$$-\frac{9}{6} = x \quad \text{i.e. } -\frac{3}{2}$$

$$(ii) \frac{(x+2)}{6} - \frac{(2x+3)}{3} = \frac{x}{12}$$

$$\times 12 / \quad 2(x+2) - 4(2x+3) = x$$

$$2x+4 - 8x-12 = x$$

$$-6x-8 = x$$

$$-8 = 7x$$

$$\frac{-8}{7} = x$$

$$(iii) 2^{x+1} = 5^x \times 3$$

$$\log 2^{x+1} = \log 5^x \times 3$$

$$(x+1)\log 2 = \log 5^x + \log 3$$

$$(x+1)\log 2 = x\log 5 + \log 3$$

$$0.3x + 0.3 = 0.7x + 0.5$$

$$0.3 - 0.5 = 0.7x - 0.3x$$

$$-0.2 = 0.4x$$

$$\boxed{-0.5 = x}$$

$$(iv) \log_a (x+3) + \log_a (2x+1) = 2\log_a (3x-1)$$

$$\log_a (x+3)(2x+1) = \log_a (3x-1)^2$$

$$2x^2 + x + 6x + 3 = 9x^2 - 3x - 3x + 1$$

$$2x^2 + 7x + 3 = 9x^2 - 6x + 1$$

$$0 = 7x^2 - 13x - 2$$

$$0 = (7x+1)(x-2)$$

$$\boxed{x = -\frac{1}{7}} \quad \boxed{x = 2}$$

(o.k. if used Quadratic formula)

b)

$$2V = \frac{P+Q}{3P-T}$$

$$2V(3P-T) = P+Q$$

$$6PV - 2VT = P+Q$$

$$6PV - P = 2VT + Q$$

$$P(6V-1) = 2VT + Q$$

$$P = \frac{2VT + Q}{6V-1}$$

when $V=2$, $Q=3$, $T=-1$

$$P = \frac{2(2)(-1) + 3}{6(2) - 1}$$

$$= \frac{-1}{11} = 0.09$$

∴ 0.1 to 1 dp

* Must use factorisation Method *

c)

(i) $2x^2 + 11x + 12 = 0$

$$(2x+3)(x+4) = 0$$

$$2x+3=0 \quad x+4=0$$

$$x = -\frac{3}{2}$$

$$x = -4$$

| |

| |

(ii) $x^2 - 28 = 3x$

$$\therefore x^2 - 3x - 28 = 0$$

$$(x+4)(x-7) = 0$$

$$x = -4 \quad x = 7$$

|

| |

| |

(iii) $-4 = -x - 3x^2$

$\therefore$

$$3x^2 + x - 4 = 0$$

$$(3x+4)(x-1) = 0$$

$$3x+4=0 \quad x-1=0$$

$$x = -\frac{4}{3}$$

$$x = 1$$

|

| |

| |

d)

(i) $5T^2 + 16T + 3$

$$(5T+1)(T+3)$$

|

(ii) $u^2 - 4F^2$

$$(u+2F)(u-2F)$$

|

|

|

(iii) $xy - xr + ty - tr \rightarrow x(y-r) + t(y-r)$
 $\therefore = (y-r)(x+t)$

$$e) \quad x=2 \quad x=-3$$

$$x-2=0 \quad x+3=0$$

$$(x-2)(x+3)=0$$

$$(x^2 + x - 6 = 0) \quad \text{i.e.} \quad y = x^2 + x - 6$$

Q1 - 40 marks

Question 2

$$a) \quad 14x^2 + 3x - 5 = 0$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(14)(-5)}}{2(14)}$$

$$x = \frac{-3 \pm \sqrt{9 + 280}}{28}$$

$$x = \frac{-3 \pm \sqrt{289}}{28}$$

$$x = \frac{-3 \pm 17}{28}$$

$$x = \frac{-3 + 17}{28}$$

$$x = \frac{-3 - 17}{28}$$

$$x = \frac{1}{2}$$

$$x = -\frac{5}{7}$$

If discriminant is -ve, equation has complex roots
(no real roots)

$$b) \quad y = 3x^2 + x - 1$$

$$\frac{dy}{dx} = 6x + 1$$

$$(i) \quad x = 2,$$

$$\frac{dy}{dx} = 13$$

$$(ii) \quad x = 0,$$

$$\frac{dy}{dx} = 1$$

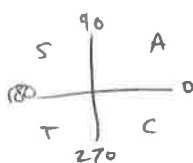
$$(iii) \quad x = -3,$$

$$\frac{dy}{dx} = -17$$

$$c) \quad (i) \quad \sin x = 0.6$$

$$x = 36.87^\circ$$

$$x = 143.13^\circ$$



$$(ii) \quad 7 \cos x = 6$$

$$\cos x = 6/7$$

$$x = 31^\circ$$

$$x = 329^\circ$$

$$(iii) \quad -3 \tan x = 2$$

$$\tan x = -\frac{2}{3}$$

$$x = \tan^{-1}(2/3)$$

$$x = 33.69 \leftarrow \text{not an answer}$$

$$x = 146.31^\circ$$

$$x = 326.31^\circ$$

$$d) \quad x^2 - 7x + 10$$

$$(x-5)(x-2)$$

| |

$$\therefore (\tan x - 5)(\tan x - 2) = 0$$

|

$$\tan x = 5$$

$$\tan x = 2$$

|

|

$$x = \tan^{-1}(5)$$

$$x = \tan^{-1}(2)$$

$$x = 78.69^\circ$$

$$x = 63.43^\circ$$

|

|

$$x = 258.69^\circ$$

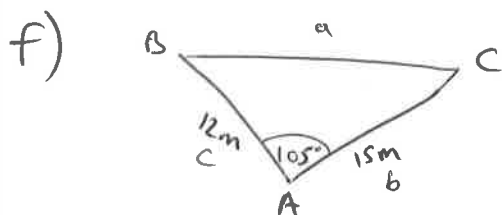
$$x = 243.43^\circ$$

|

|

$$e) = 1$$

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$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

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$$(BC)^2 = 15^2 + 12^2 - 2(15)(12) \cos 105^\circ$$

|

$$(BC)^2 = 225 + 144 + 93.17$$

|

$$BC = \sqrt{461.17}$$

|

$$BC = 21.47^\circ$$

|

Biggest side opposite biggest angle
 Smallest " " Smallest "

|

$$g) \quad y = \tan^{-1}\left(\frac{1}{x+g}\right)$$

$$\tan y = \frac{1}{x+g}$$

$$x+g = \frac{1}{\tan y}$$

$$x = \frac{1}{\tan y} - g$$

Q2 - 40 marks

Question 3

$$a)(i) \quad y = 5x^4 + 3x^2 - x + 1$$

$$\frac{dy}{dx} = 20x^3 + 6x - 1$$

$$(ii) \quad y = (2x+3)^2$$

$$y = 4x^2 + 12x + 9$$

$$\frac{dy}{dx} = 8x + 12$$

$$(iii) \quad y = x\sqrt{x} + \frac{2}{\sqrt[3]{x}}$$

$$y = x^{1.5} + 2x^{-0.33}$$

$$\frac{dy}{dx} = 1.5x^{0.5} - 0.66x^{-1.33}$$

$$\text{i.e. } \frac{3}{2}\sqrt{x} - \frac{2}{3}x^{-4/3}$$

$$(iv) \quad y = \sin 2x + 3 \cos 4x - \tan \frac{x}{2}$$

~~$$\frac{dy}{dx} = \frac{1}{2} \cos 2x + \frac{3}{4} \sin 4x - \frac{1}{2} \sec^2 \frac{x}{2}$$~~

$$\frac{dy}{dx} = 2 \cos 2x + 12 \sin 4x - \frac{1}{2} \sec^2 \frac{x}{2}$$

|||

$$(v) \quad y = \frac{\sin 5x}{e^{2x} + 3x^2} \leftarrow u$$

$$e^{2x} + 3x^2 \leftarrow v$$

$$\frac{du}{dx} = 5 \cos 5x \quad \frac{dv}{dx} = 2e^{2x} + 6x$$

||

$$\therefore \frac{dy}{dx} = \frac{(e^{2x} + 3x^2)(5 \cos 5x) - (\sin 5x)(2e^{2x} + 6x)}{(e^{2x} + 3x^2)^2}$$

||
|

$$(vi) \quad y = (6x^3 - 2x + 1)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} (6x^3 - 2x + 1)^{-1/2} (18x^2 - 2)$$

||

$$\text{i.e. } \frac{9x^2 - 1}{\sqrt{6x^3 - 2x + 1}}$$

$$(vii) \quad y = 3x^2 \ln x$$

$\uparrow$ $\uparrow$
 u v

$$\frac{du}{dx} = 6x$$

$$\frac{dy}{dx}$$

$$\frac{dv}{dx} = \frac{1}{x}$$

$$= \ln x(6x) + 3x^2 \left(\frac{1}{x}\right)$$

$$\therefore 6x \ln x + 3x$$

$$b) \quad y = 4x^2 + 3x - 5$$

$$\frac{dy}{dx} = 8x + 3, \quad \frac{d^2y}{dx^2} = 8$$

$$\text{Sub in } 3 \left(\frac{d^2y}{dx^2} \right) + 2 \left(\frac{dy}{dx} \right) - 30 - 16x = 0$$

$$\Rightarrow 3(8) + 2(8x + 3) - 30 - 16x = 0$$

$$24 + 16x + 6 - 30 - 16x = 0$$

$$30 + 16x - 30 - 16x = 0$$

$$0 = 0$$

$$c) \quad y = \sin 2x$$

$$y' = 2 \cos 2x$$

$$y'' = -4 \sin 2x$$

$$y''' = -8 \cos 2x$$

$$y'''' = 16 \sin 2x$$

For
2nd, 3rd, 4th
derivatives

$$d) \quad z = x^3 y^5 + 2x^4 y + 3x^2 - 2y^3 + 10$$

$\frac{\partial z}{\partial x} = 3x^2 y^5 + 8x^3 y + 6x$ $\begin{matrix} x \\ \swarrow \\ \frac{\partial^2 z}{\partial x^2} \end{matrix}$ $= 6xy^5 + 24x^2 y + 6$ $\begin{matrix} y \\ \swarrow \\ \frac{\partial^2 z}{\partial y \partial x} \end{matrix}$ $= 15x^2 y^4 + 8x^3$	$\frac{\partial z}{\partial y} = 5x^3 y^4 + 2x^4 - 6y^2$ $\begin{matrix} y \\ \swarrow \\ \frac{\partial^2 z}{\partial y^2} \end{matrix}$ $= 20x^3 y^3 - 12y$ $\begin{matrix} x \\ \swarrow \\ \frac{\partial^2 z}{\partial x \partial y} \end{matrix}$ $= 15x^2 y^4 + 8x^3$
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e)

$$y = 5x - 1 \quad (1)$$

$$y = 2x^2 - 2x + 5 \quad (2)$$

equating

$$2x^2 - 2x + 5 = 5x - 1$$

$$2x^2 - 2x + 5 - 5x + 1 = 0$$

$$2x^2 - 7x + 6 = 0$$

Use formula

$$x = \frac{+7 \pm \sqrt{(-7)^2 - 4(2)(6)}}{2(2)}$$

or factorise

$$(2x + 3)(x - 2)$$

$$x = \frac{7 \pm \sqrt{1}}{4}$$

$$x = \frac{7 + 1}{4}$$

$$x = \frac{7 - 1}{4}$$

$$x = 2$$

$$x = 3/2$$

Q3 Total - 40 marks

Paper marked
out of 80