

Engineering Design & Materials Assessment

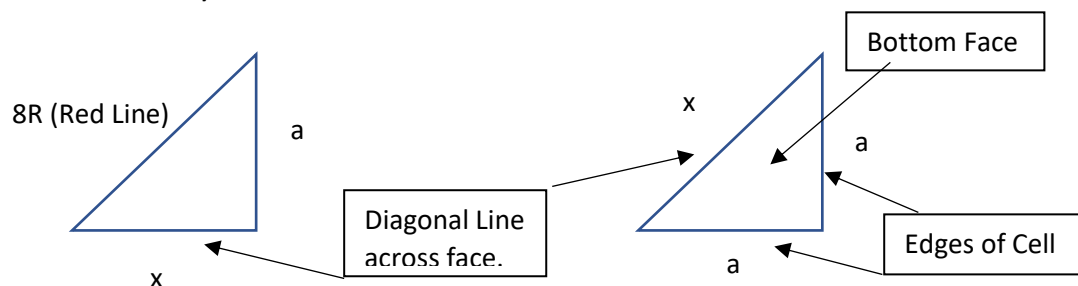
1)

a) 1 hidden + 8 whole + 6 halves + 8 eighths = 13 atoms

$$b) APF = \frac{\text{Volume of Atoms}}{\text{Volume of Unit Cell}} \qquad APF = \frac{13 \times \frac{4}{3} \times \pi \times R^3}{a^3}$$

Red Line = 8 radii

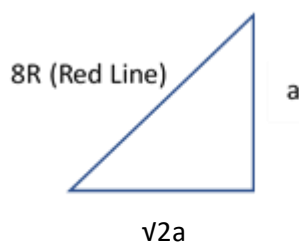
(1 corner + 1 whole + 1 whole (hidden central atom) + 1 whole + 1 corner) = 8



$$a^2 + a^2 = x^2$$

$$2a^2 = x^2$$

$$\sqrt{2}a = x$$



$$a^2 + (\sqrt{2}a)^2 = (8R)^2$$

$$a^2 + 2a^2 = (8R)^2$$

$$3a^2 = 64R^2$$

$$\sqrt{3}a = 8R \qquad R = \frac{\sqrt{3}a}{8}$$

$$APF = \frac{13 \times \frac{4}{3} \times \pi \times \left(\frac{\sqrt{3}a}{8}\right)^3}{a^3}$$

$$APF = \frac{\frac{52\pi}{3} \times \frac{3\sqrt{3}a^3}{512}}{a^3}$$

$$APF = \frac{52\pi \times 3\sqrt{3}}{3 \times 512} \quad \underline{APF = 0.55}$$

$$\begin{aligned} \text{c) } R &= 0.142 \text{ nm} & \sqrt{3}a &= 8R & a &= \frac{8R}{\sqrt{3}} & V &= a^3 \\ V &= \left(\frac{8R}{\sqrt{3}}\right)^3 & V &= \left(\frac{8 \times 0.142 \times 10^{-9}}{\sqrt{3}}\right)^3 & \underline{V} &= \underline{2.82 \times 10^{-28} \text{ m}^3} \end{aligned}$$

$$\text{d) } A = 53.5 \text{ g/mol} \quad n = 13 \text{ atoms} \quad N_A = 6.022 \times 10^{23}$$

$$V = 2.82 \times 10^{-28} \text{ m}^3 \quad (2.82 \times 10^{-22} \text{ cm}^3) \quad \rho = \frac{nA}{VN_A}$$

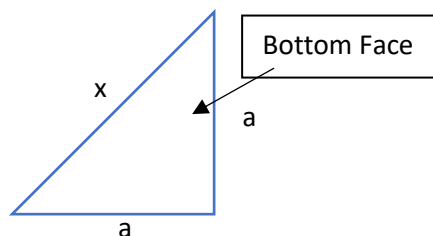
$$\rho = \frac{13 \times 53.5}{2.82 \times 10^{-22} \times 6.022 \times 10^{23}} \quad \underline{\rho = 4.09 \text{ g/cm}^3}$$

e) The material is suitable for its intended purpose as its Theoretical Density is less than 12 g/cm^3 .

2) Molybdenum:

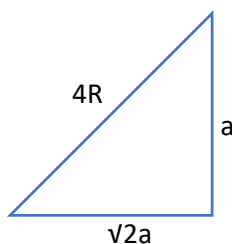
Atomic Weight = 95.96 g/mol

Body Centred Cubic: 2 Atoms per cell, 1 whole (central) + 8 eighths (one per corner), 4 radii across to opposing corners.



$$a^2 + a^2 = x^2 \quad 2a^2 = x^2$$

$$\sqrt{2}a = x$$



$$a^2 + (\sqrt{2}a)^2 = (4R)^2 \quad a^2 + 2a^2 = (4R)^2$$

$$3a^2 = (4R)^2 \quad \sqrt{3}a = 4R \quad R = \frac{\sqrt{3}a}{4}$$

$$APF = \frac{\text{Volume of Atoms}}{\text{Volume of Unit Cell}} \quad APF = \frac{2 \times \frac{4}{3} \times \pi \times R^3}{a^3}$$

$$APF = \frac{\frac{8\pi}{3} \times \left(\frac{\sqrt{3}a}{4}\right)^3}{a^3} \quad APF = \frac{\frac{8\pi}{3} \times \frac{3\sqrt{3}a^3}{64}}{a^3} \quad APF = \frac{8\pi \times 3\sqrt{3}}{3 \times 64}$$

$$\underline{APF = 0.68}$$

$$\sqrt{3}a = 4R \quad a = \frac{4R}{\sqrt{3}} \quad V = a^3 \quad V = \left(\frac{4R}{\sqrt{3}}\right)^3$$

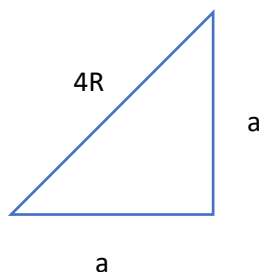
$$R = 0.1363nm \quad V = \left(\frac{4 \times 0.1363 \times 10^{-9}}{\sqrt{3}}\right)^3 \quad \underline{V = 3.12 \times 10^{-29}m^3}$$

$$\rho = \frac{nA}{VN_A} \quad \rho = \frac{2 \times 95.96}{3.12 \times 10^{-23} \times 6.022 \times 10^{23}} \quad \underline{\rho = 10.21g/cm^3}$$

Nickel:

Atomic Weight = 58.69g/mol

Face Centred Cubic: 4 atoms per cell, 6 halves (one per face) + 8 eighths (one per corner), 4 radii diagonally across each face.



$$a^2 + a^2 = (4R)^2 \quad 2a^2 = (4R)^2$$

$$\sqrt{2}a = 4R \quad R = \frac{\sqrt{2}a}{4}$$

$$APF = \frac{4 \times \frac{4}{3} \times \pi \times R^3}{a^3} \quad APF = \frac{4 \times \frac{4}{3} \times \pi \times \left(\frac{\sqrt{2}a}{4}\right)^3}{a^3} \quad APF = \frac{\frac{16\pi}{3} \times \frac{2\sqrt{2}a^3}{64}}{a^3}$$

$$APF = \frac{16\pi \times 2\sqrt{2}}{3 \times 64} \quad \underline{APF = 0.74}$$

$$\sqrt{2}a = 4R \quad a = \frac{4R}{\sqrt{2}} \quad V = a^3 \quad V = \left(\frac{4R}{\sqrt{2}}\right)^3$$

$$R = 0.1246nm \quad V = \left(\frac{4 \times 0.1246 \times 10^{-9}}{\sqrt{2}}\right)^3 \quad \underline{V = 4.38 \times 10^{-29}m^3}$$

$$\rho = \frac{nA}{VN_A} \quad \rho = \frac{4 \times 58.69}{4.38 \times 10^{-23} \times 6.022 \times 10^{23}} \quad \underline{\rho = 8.90g/cm^3}$$

Molybdenum is not as densely packed as nickel, but it has a smaller cell volume and a greater density than nickel.

$$3) \quad \text{Simple Cubic: } a = 2R \quad FCC: a = \frac{4R}{\sqrt{2}} \quad BCC: a = \frac{4R}{\sqrt{3}}$$

$$V = a^3$$

$$SC: V = (2R)^3 \quad FCC: V = \left(\frac{4R}{\sqrt{2}}\right)^3 \quad BCC: V = \left(\frac{4R}{\sqrt{3}}\right)^3$$

Simple Cubic: 1 atom FCC: 4 atoms BCC: 2 atoms

$$\rho = \frac{nA}{VN_A} \quad SC: \rho = \frac{1 \times A}{(2R)^3 \times 6.022 \times 10^{23}}$$

$$FCC: \rho = \frac{4 \times A}{\left(\frac{4R}{\sqrt{2}}\right)^3 \times 6.022 \times 10^{23}} \quad BCC: \rho = \frac{2 \times A}{\left(\frac{4R}{\sqrt{3}}\right)^3 \times 6.022 \times 10^{23}}$$

Alloy A:

$$\rho = 8.22g/cm^3 \quad A = 77.4g/mol \quad R = 0.125nm$$

$$SC: \rho = \frac{1 \times 77.4}{(2 \times 0.125 \times 10^{-7})^3 \times 6.022 \times 10^{23}} = 8.22g/cm^3$$

Alloy A is a Simple Cubic structure.

Alloy B:

$$\rho = 13.42 \text{ g/cm}^3 \quad A = 107.6 \text{ g/mol} \quad R = 0.133 \text{ nm}$$

$$SC: \rho = \frac{1 \times 107.6}{(2 \times 0.133 \times 10^{-7})^3 \times 6.022 \times 10^{23}} = 9.50 \text{ g/cm}^3 \text{ Not Simple Cubic.}$$

$$FCC: \rho = \frac{4 \times 107.6}{\left(\frac{4 \times 0.133 \times 10^{-7}}{\sqrt{2}}\right)^3 \times 6.022 \times 10^{23}} = 13.43 \text{ g/cm}^3$$

Alloy B is a Face Centred Cubic Structure.

Alloy C:

$$\rho = 9.23 \text{ g/cm}^3 \quad A = 127.3 \text{ g/mol} \quad R = 0.142 \text{ nm}$$

$$BCC: \rho = \frac{2 \times 127.3}{\left(\frac{4 \times 0.142 \times 10^{-7}}{\sqrt{3}}\right)^3 \times 6.022 \times 10^{23}} = 11.99 \text{ g/cm}^3 \text{ Not BCC.}$$

$$FCC: \rho = \frac{4 \times 127.3}{\left(\frac{4 \times 0.142 \times 10^{-7}}{\sqrt{2}}\right)^3 \times 6.022 \times 10^{23}} = 13.05 \text{ g/cm}^3 \text{ Not FCC.}$$

$$SC: \rho = \frac{1 \times 127.3}{(2 \times 0.142 \times 10^{-7})^3 \times 6.022 \times 10^{23}} = 9.23 \text{ g/cm}^3$$

Alloy C is a Simple Cubic structure.

$$4) \quad A: [0\bar{1}\bar{1}] \quad B: (-1, -\frac{1}{2}, 0) > [\bar{2}10]$$

$$C: (\frac{1}{2}, \frac{1}{2}, 1) > [112] \quad D: (\frac{1}{2}, \frac{1}{2}, -1) > [11\bar{2}]$$

$$5) \quad a_1 = u' \quad a_2 = v' \quad z = w'$$

$$u = \frac{1}{3}(2u' - v') \quad v = \frac{1}{3}(2v' - u') \quad t = -(u + v) \quad w = w'$$

$$A: \quad u' = \frac{1}{2} \quad v' = -\frac{1}{2} \quad w' = 0$$

$$u = \frac{1}{3}\left(1 + \frac{1}{2}\right) = \frac{1}{2} \quad v = \frac{1}{3}\left(-1 - \frac{1}{2}\right) = -\frac{1}{2} \quad t = -\left(\frac{1}{2} - \frac{1}{2}\right) = 0$$

$$(uvtw) = \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right) \quad \underline{[1\bar{1}00]}$$

$$B: \quad u' = 1 \quad v' = 1 \quad w' = 0$$

$$u = \frac{1}{3}(2 - 1) = \frac{1}{3} \quad v = \frac{1}{3}(2 - 1) = \frac{1}{3} \quad t = -\left(\frac{1}{3} + \frac{1}{3}\right) = -\frac{2}{3}$$

$$(uvtw) = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, 0\right) \quad \underline{[11\bar{2}0]}$$

$$C: \quad u' = 0 \quad v' = 0 \quad w' = 1$$

$$u = \frac{1}{3}(0 - 0) = 0 \quad v = \frac{1}{3}(0 - 0) = 0 \quad t = -(0 + 0) = 0$$

$$(uvtw) = (0, 0, 0, 1) \quad \underline{[0001]}$$

6) Left to right, top to bottom.

$$x = a \qquad y = b \qquad z = c$$

$$a) \infty a \qquad \infty b \qquad c$$

$$\infty \qquad \infty \qquad 1$$

$$\frac{1}{\infty} \qquad \frac{1}{\infty} \qquad \frac{1}{1}$$

$$\underline{(001)}$$

$$b) \infty a \qquad b \qquad \infty c$$

$$\infty \qquad 1 \qquad \infty$$

$$\frac{1}{\infty} \qquad \frac{1}{1} \qquad \frac{1}{\infty}$$

$$\underline{(010)}$$

c) As the plane goes through the origin, a second cube has been placed in front and the co-ordinates have been taken from O-Primed.

$$-a \qquad \infty b \qquad \infty c$$

$$-1 \qquad \infty \qquad \infty$$

$$-\frac{1}{1} \qquad \frac{1}{\infty} \qquad \frac{1}{\infty}$$

$$\underline{(-100)}$$

d) ∞a	b	c
∞	1	1
$\frac{1}{\infty}$	$\frac{1}{1}$	$\frac{1}{1}$

(0 1 1)

e) As the plane goes through the origin, a second cube has been placed to the right and the co-ordinates have been taken from O-Primed.

a	$-b$	∞c
1	-1	∞
$\frac{1}{1}$	$-\frac{1}{1}$	$\frac{1}{\infty}$

(1 -1 0)

f) As the plane goes through the origin, a second cube has been placed in front and the co-ordinates have been taken from O-Primed.

$-a$	∞b	c
-1	∞	1
$-\frac{1}{1}$	$\frac{1}{\infty}$	$\frac{1}{1}$

(-1 0 1)