

Engineering Science 1 Continued

An Introduction to the Physical Principles and Basic Analysis Associated with Heat Transfer

In physics heat transfer is the process by which energy in the form of heat is exchanged between bodies or parts of the same body at different temperatures. Heat is generally transferred by convection, conduction or radiation. Although these three processes can occur simultaneously, it is not unusual for one mechanism to overshadow the other two.

Conduction

Solids are primarily associated with conductive heat transfer and in some circumstances liquids too. In conductive heat transfer, gases are often considered as insulators due to this type of heat transfer being negligible with gases.

Temperature difference between one part of a substance and another causes a microscopic transfer of heat which is conductive heat transfer. A substance is made up of atoms or molecules which in solids are formed in a lattice structure. More energetic molecules at a higher temperature transfer heat energy to less hot molecules as seen in Figure 1:

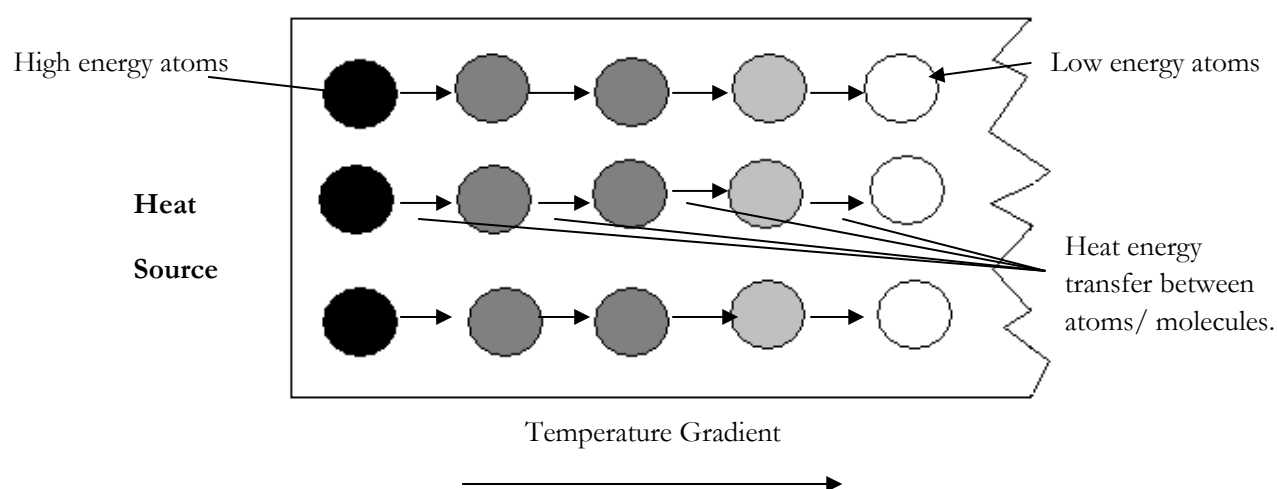


Figure 1: Molecular heat transfer.

Conduction is the only method of heat transfer in opaque solids. If temperature at one end of a metal rod is raised by heating, heat is conducted to the colder end, but the exact mechanism of heat conduction in solids is not entirely understood. It is believed, however, to be partially due to the motion of free electrons in the solid matter, which transport energy if a temperature difference is applied. This theory helps to explain why good electrical conductors also tend to be good heat conductors. Although the phenomenon of heat conduction had been observed for centuries, it was

not until 1882 that the French mathematician Jean Baptiste Joseph Fourier gave it precise mathematical expression in what is now regarded as Fourier's Law of heat conduction. The physical law states that the rate at which heat is conducted through a body per unit cross sectional area is proportional to the negative of the temperature gradient existing in the body.

The 'temperature profile' for the conduction of heat in one direction through a flat 'plate' of uniform thickness 'x' may be illustrated as in Figure 1:

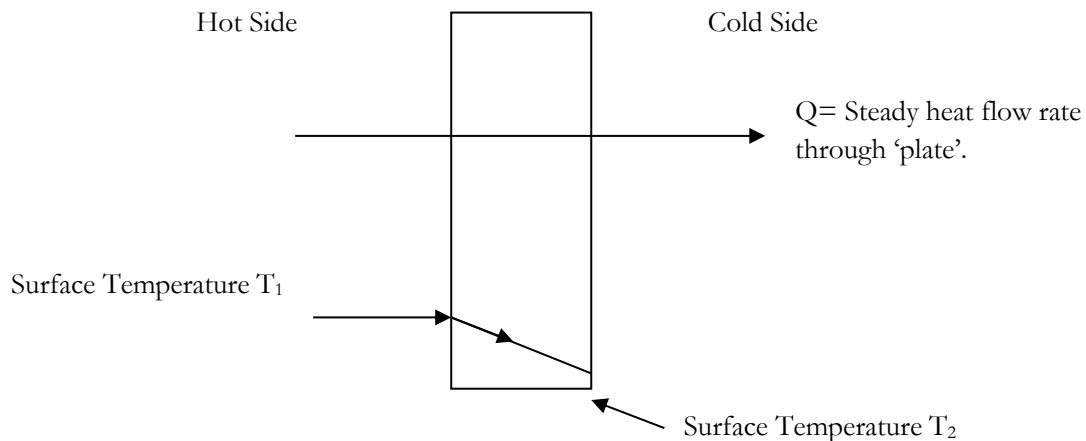


Figure 2: Temperature profile.

The steady rate of conductive heat flow ('transfer') depends on several factors:

1. It is proportional to the temperature difference ($T_1 - T_2$). If $T_1 = T_2$ there will be no heat flow, and the greater the temperature difference the greater the heat flow rate.
2. It is proportional to the surface area 'A'.
3. It is inversely proportional to the thickness 'x' of the material. The thinner the material, the greater the heat flow rate and vice versa.
4. It depends on the material through which the heat is being conducted, as identified by the **thermal conductivity 'k'**, which is a material property.

The above are combined to give us **Fourier's Law**:

$$Q = \frac{-kA(T_2 - T_1)}{x} \quad \text{or} \quad Q = -UA(T_2 - T_1)$$

Where:

Q is the heat flow rate from the surface by convection (W),

T_1 is the hot surface's temperature (K),

T_2 is the cold surface's temperature (K),

A is the area of the surface exposed to the fluid (m²),

k is the thermal conductivity (W /mK or °C),

and $U = \frac{k}{x}$, and is called the heat transfer coefficient.

Substance	k (W/mK)
Alcohol	0.17
Aluminium	250
Building Brick	1.33
Carbon	1.7
Carbon Dioxide	0.0146
Concrete/ Stone	1.7
Copper	401
Engine Oil	0.15
Fiberglass	0.037
Glass	1.05
PVC	0.19
Steel, Carbon 1%	43
Stainless Steel	16
Water	0.65

Table 1: Average values for 'k' for a selection of substances.

Conduction through a composite wall

Consider the composite wall as shown in Figure 3, consisting of the materials 'a', 'b' and 'c':

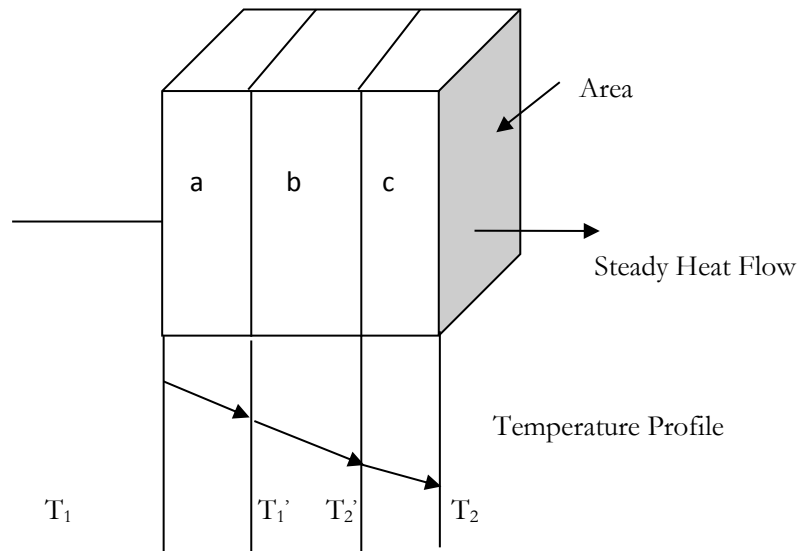


Figure 3: Composite wall.

Important points to remember:

1. The temperature at the material interfaces are common to the surface temperatures of both materials concerned i.e T_1' is common to material 'a' and 'b', T_2' is common to materials 'b' and 'c'.
2. The steady heat flow rate through each layer of the composite will be the same i.e. $Q = Q_a = Q_b = Q_c$.

So for material 'a' applying the basic equation:

$$Q_a = \frac{-kA(T_1' - T_1)}{x_a} \quad \text{or} \quad Q_a = -U_a A(T_1' - T_1) \quad , \text{ where} \quad U_a = \frac{ka}{x_a}$$

$$\text{Transposing: } (T_1 - T_1') = \frac{Q_a}{U_a A}$$

Similarly for materials 'b' and 'c':

$$(T_1' - T_2') = \frac{Qb}{UbA} \quad \text{and} \quad (T_2' - T_2) = \frac{Qc}{UcA}$$

Adding the above we get:

$$(T_1 - T_1') + (T_1' - T_2') + (T_2' - T_2) = \frac{Qa}{UaA} + \frac{Qb}{UbA} + \frac{Qc}{UcA}$$

But since $Q = Qa = Qb = Qc$:

$$(T_1 - T_2) = \frac{Q}{A} \left(\frac{1}{Ua} + \frac{1}{Ub} + \frac{1}{Uc} \right)$$

Hence we can define a value of 'U' for the composite material such that:

$$\frac{1}{U} = \frac{1}{Ua} + \frac{1}{Ub} + \frac{1}{Uc}$$

Substituting back into the formula above:

$$(T_1 - T_2) = \frac{Q}{A} \times \frac{1}{U} \quad \text{or} \quad Q = -UA(T_2 - T_1)$$

The above takes the same format as the basic Fourier's equation but with 'U' being the **overall heat transfer coefficient for the composite material**.

Heat Transfer – Conduction Examples

1. Calculate the heat flow from my office if the window is 2m x 1.2m x 1.25cm if the temperature inside is 25°C and the temperature outside is 2°C.
2. Find the heat transfer coefficient for the material of a door (dimensions 3m x 2m) of a furnace given the heat flow rate is 50000W and that the internal temperature is 500°C and the external temperature is 35°C.
So given that the door is made from steel of thermal conductivity 54W/m°C, Calculate how thick the door is.
3. The wall of an industrial furnace is constructed from 0.15m thick fireclay brick having a thermal conductivity of 1.7 W/mK. Measurements made during the steady state operation reveal temperatures of 1400K and 1150K at the inner and outer surfaces respectively. What is the rate of heat loss through a wall that is 0.5 by 1.2m wide?
4. An air to air intercooler that is fabricated from aluminium of thermal conductivity 220 W/m°C has a core thickness of 62.5mm and dimensions of 0.85m by 0.55m. Given that the charge entering the cooler is a temperature of 40°C and at exit it has been cooled to 25°C, calculate the steady heat flow rate of the cooler.
5. A composite cooling hose running along the back of an engine bay is 0.85m long and has an outer diameter of 70mm. It consists of an inner and outer layer 5mm thick silicone of thermal conductivity 170W/m°C and has a middle layer of nylon weave of 2mm thickness and conductivity of 0.25 W/m°C. Considering that the temperature of the coolant is 85°C and the ambient air within the engine bay is 26°C calculate the steady heat flow from the coolant hose.

Convection

Convection is the process of heat transfer within a fluid (gas or liquid) or from a solid to a fluid. The fluid transfers heat energy by mixing one portion of the fluid with another.

Fluids naturally move through pressure differences caused either by a pump or because of differences in density caused by temperature changes. When convection is facilitated through some kind of device it is known as **forced** convection and when it occurs as a result of changes in density is referred to as **free** or **natural** convection.

For example, the heat transferred from a hot plate to atmosphere is via natural convection, whereas the heat transferred by a fan heater, in which a fan blows air across a heating element is by forced convection.

So:

The transfer of heat through solid bodies is through the process of conduction alone. Whereas the transfer of heat from a solid to a liquid or a gas takes place partly by conduction and partly convection.

Label the following appropriately convection, conduction or both:

- Water boiling in a sauce pan.....
- Water boiling in a bowl in a microwave.....
- Currents in a large body of water.....
- Thermals in the atmosphere.....
- Smoke in a chimney.....
- Oil filled radiators.....

The **rate** at which the **transfer of heat** takes place depends on the velocity of the fluid, the difference of the temperatures between the fluid and the wall and the density and viscosity of the fluid.

Can convection take place within a vacuum?.....

In an engine heat is transferred from hot parts of the engine via conduction to the coolant and then to the radiator by forced convection of the coolant.

Consider the engine cooling system in Figure 4. The pump creates a pressure increase and forces the fluid to circulate around the engine. The fresh water travelling around the engine is a lower temperature and so heat is transferred from the engine to the water. This coolant then flows around

the system to the radiator matrix. Here air flowing through the radiator has heat transferred to it from the coolant enabling it to perform the same process again. Heat is transferred because of the pressure difference created by the pump.

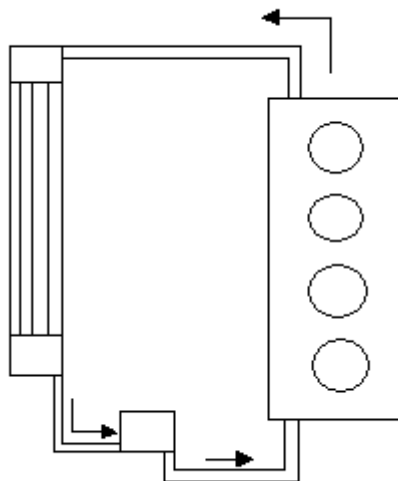


Figure 4: Engine cooling system.

Now consider the water heater and radiator in Figure 5. When water undergoes a temperature increase its density reduces (except at very low temperatures). The water leaving the heater is of a higher temperature than the rest of the water in the system and therefore has a lower density; hence this water rises to the radiator. At the radiator the water loses heat, lowering the temperature and increasing in density thus flowing back down to the heater.

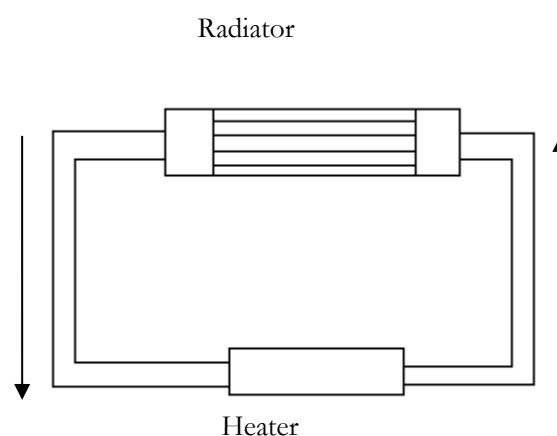


Figure 5: Heater and radiator system.

Label the following appropriately, forced convection or natural convection:

- Currents in a large body of water.....
- Thermals in the atmosphere.....
- Smoke in an industrial fan chimney.....
- Central heating system.....

Heat transfer by convection is basically a surface phenomenon which is expressed by the following formula proposed by Newton commonly known as Newton's Law of Cooling:

$$Q = h A (T_2 - T_1)$$

Where:

Q is the heat flow rate from the surface by convection,

T_2 is the hot surfaces temperature.

T_1 is the fluid stream temperature,

A is the area of the surface exposed to the fluid,

h is the convective heat transfer coefficient (W / m^2K or $^{\circ}C$).

Comparison of this formula with Fourier's Law show that they are essentially the same expression but h in place of U. The vital difference being that where U was dependant on the thermal conductivity k and the thickness x, h depends on many factors determined by other means.

Heat Transfer- Convection Examples

1. A fluid flows over a plane surface 1m by 1m with a bulk temperature of $50^{\circ}C$. The temperature of the surface is $20^{\circ}C$. The convective heat transfer coefficient is $2000W/m^2C$. Calculate the steady heat flow rate from the surface.
2. Water of temperature 350K flows through an air cooled radiator at atmospheric temperature 285K, given the convective heat transfer coefficient is $650W/m^2K$, and the area of the radiator is $0.51m^2$ find the heat transfer.
3. Fluid flows through a pipe with a temperature of $65^{\circ}C$. If the temperature of the outside of the pipe is $32^{\circ}C$, the convective heat transfer coefficient is $70 W/m^2K$ and the heat transfer is 600W find the diameter of the pipe of length 0.5m.
4. Find the temperature difference of a water to water intercooler that has an area $2.5m^2$, convective heat transfer coefficient of $1100W/m^2K$ and heat transfer of 65000W.

Radiation

When dealing with radiation, it is better to consider the transfer as being a transfer of energy in the form of electromagnetic waves. Radiant energy is given off by any body at a temperature above zero K. The radiant energy travels in straight lines and in all directions around the transmitting body. The receipt of radiant energy by another body necessitates it being in the receiving line of radiation. Electromagnetic radiation is in the form of transverse vibration and so may be classified according to frequency (f) or wavelength (λ). There is a very broad spectrum of possible wavelengths, ranging from fractions of a micrometer ($\mu\text{m}=10^{-6}$) up to several kilometres.

The spectrum of the family of electromagnetic radiation waves as seen below in Figure 6, where the extremes range from radio waves (long wave) to very harmful gamma radiation or 'rays'.

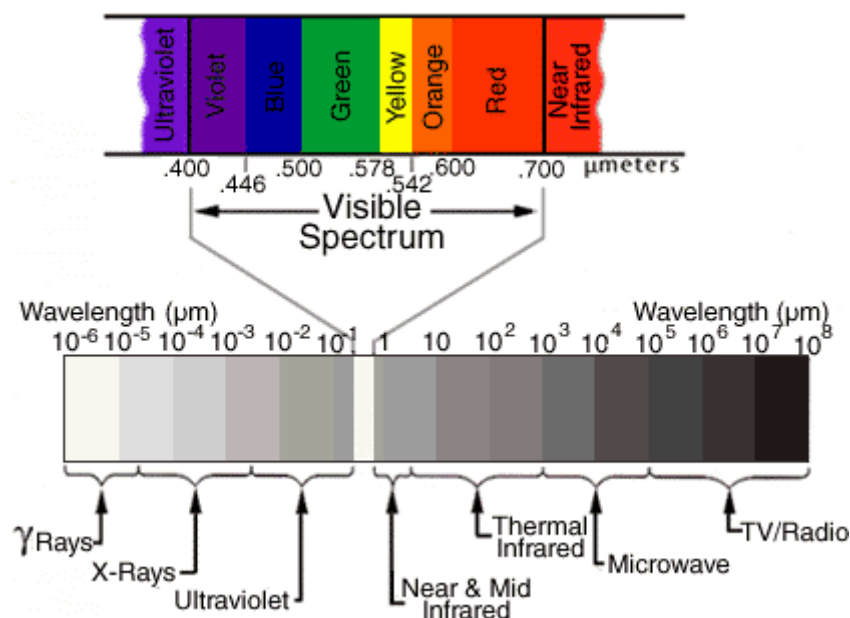


Figure 6: Radiation spectrum.

Since all radiation travels with the same velocity 'c' the speed of light, we have the simple relationship:

$$\lambda = \frac{c}{f}$$

When radiant energy strikes an object, some may be absorbed, some may be reflected and some may pass right through the object and be transmitted again.

So consider the following:

Absorptivity = Fraction absorbed,

Reflectivity = Fraction reflected,

Transmissivity = Fraction transmitted

We can assume:

Absorptivity + Reflectivity + Transmissivity = 1

The fraction absorbed will show up as an increase in the internal energy and it therefore has the same effect as if heat flow had taken place. In this sense, all electromagnetic radiation regardless of the wavelength is a form of heat or thermal radiation. However it turns out that the very short and very long wavelengths are not appreciably absorbed, so that radio waves (long wavelengths) pass readily through solid objects and 'x' and 'γ' rays (short wavelengths) similarly are largely transmitted.

Thus the band of thermal radiation may be considered in the range of 0.1 and 100 μm, which also corresponds to the band of radiation given out by a heated body. A heated body emits radiation in this range and radiation wavelengths outside this range cannot be produced simply by heating the body.

The Stephan-Boltzman Law

As we have already stated, all bodies above absolute zero emit some radiant energy. The amount of energy radiated goes up rapidly with the temperature of the body; in fact it was discovered by Jozef Stephan in 1879 and verified by Ludwig Boltzman in 1884 that the total energy radiated is proportional to the fourth power of the absolute temperature. Mathematically the Law is:

$$Q = \sigma AT^4$$

Where:

Q is the energy radiated from the surface (kW),

A is the surface area of the body (m²),

T is the absolute temperature of the body (K),

σ is the Stephan-Boltzman constant (56.7 x10⁻¹² kW/m²K⁴),

ϵ is the emissivity of the surface (varies between 0 and 1) at the temperature T .

However the equation is more usually seen as:

$$Q = 56.7 \epsilon A \left(\frac{T}{1000} \right)^4$$

Where the 10^{-12} factor is cancelled in the Stephan-Boltzman constant by dividing the temperature by 1000 before raising to the fourth power.

In practice, bodies do not radiate into perfect heat sinks but also receive radiation from the surroundings. Thus if the body is at a temperature T_2 and the surroundings are at a lower temperature T_1 , the net loss of heat is given by the following equation:

$$Q = 56.7 \epsilon A \left[\left(\frac{T_2}{1000} \right)^4 - \left(\frac{T_1}{1000} \right)^4 \right]$$

Difficulty is experienced when the body is outside, since it is hard to determine the sky radiation temperature. **On a cold clear night, the sky radiation temperature will be of the order of 230K.** **During the day, the sky temperature is often taken as about 5°C below ambient.** The re-radiation from the atmosphere is often known as the diffuse radiation and expressed as a percentage of the direct solar radiation.

The emissivity of a surface is a function of the nature of the surface (e.g. colour, texture, and roughness), temperature of the surface, wavelength radiation and angle of emission. For a simplified analysis the emissivity is often taken as an approximate mean value as shown in Table 2:

Material	Emissivity (ϵ)
Polished Aluminium	0.05
Oxidised Aluminium	0.2
Anodised Aluminium	0.95
Polished Steel	0.07
Oxidised Steel	0.8
Brick	0.85
Glass	0.92
Water	0.98
Concrete	0.54

Table 2: Average values for emissivity for a selection of substances.

A surface with an emissivity of 1 at all wavelengths is called a 'black body'. A surface with an emissivity of less than one, but remains constant over all wavelengths is called a 'grey body'. No real surfaces exhibit black or grey behaviour exactly, but some approach it fairly closely.

Heat transfer- Radiation Example

A steel access hatch of dimensions 0.5 by 0.7m covers a generator in the wall of a building. The surface temperature of the hatch is an average of 301K. Find the heat transfer from the hatch:

- At night,
- In the day when the ambient temperature is 292K.

Stress & Strain

Direct Stress & Strain

Having previously considered a load being applied to a material thus putting it into tension or compression the properties of direct stress and strain were found to follow the following relationships:

$$\text{Stress } \sigma = \frac{\text{Force}}{\text{X sectional - area}}$$

$$\text{Strain } \varepsilon = \frac{\text{change - in - length}}{\text{original - length}}$$

The above calculations being based on the assumption that a material remains perfectly elastic and obeys Hooke's Law. The direct stress and strain properties giving way to the calculation of Young's Modulus of Elasticity:

$$E = \frac{\sigma}{\varepsilon}$$

Young's modulus being the resistance of a material to tensile and compressive forces.

Example

A mild steel engine bolt when correctly tightened provides a tensile load of 3kN. The bolt is 300mm long and has a diameter of 15mm. Given that for mild steel $E = 200 \times 10^9 \text{ N/m}^2$ calculate the stress on the material. How much with the bolt extend?

Materials can however deform in other ways:

Shear Stress

If a material is subjected to two forces that are equal, opposite and parallel a shear action is developed as in figure 7.

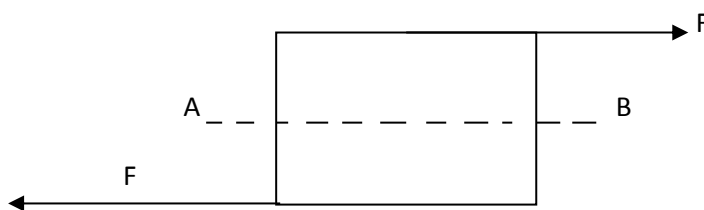


Figure 7.

As the forces 'F' are not direct, there will be a tendency for one part of the material to slide or 'shear' from the other part. The cross sectional area under shear stress is measured parallel to the line of action of the forces:

$$\text{Shear stress: } \tau = \frac{F}{A} = \left[\frac{\text{N}}{\text{m}^2} \right]$$

Example

A steel pivot bolt in a suspension arrangement has a maximum allowable shear stress of 60MN/m^2 and a diameter of 25mm . Find the maximum shear force that can be applied to the pivot.

Shear Strain

Strain takes place in the shear mechanism but deformations takes place as follows:

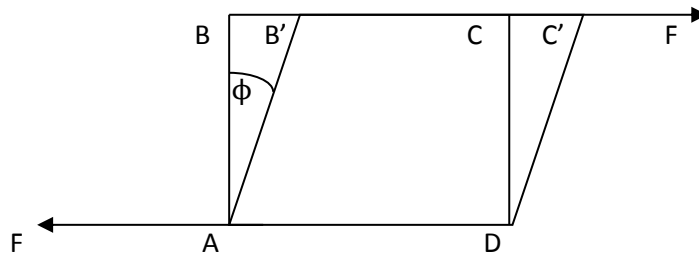


Figure 8.

The square face ABCD is deformed to parallelogram AB'C'D as in figure 8. Shear strain is then defined as the movement of the faces in the direction of the force divided by the distance between the faces. BB' is usually denoted by 'x' and AB by 'L'.

$$\text{Shear strain, } \gamma = \frac{BB'}{AB} = \frac{x}{L} = \tan \phi = \phi \text{ radians}$$

Where ϕ is the angular distortion of the vertical faces and shear strain has no units. Hooke's Law still applies to shear stress and the relationship between shear stress and strain is called the modulus of rigidity and has the quantity symbol G.

$$G = \frac{\tau}{\gamma} = \frac{\tau L}{x}$$

Example

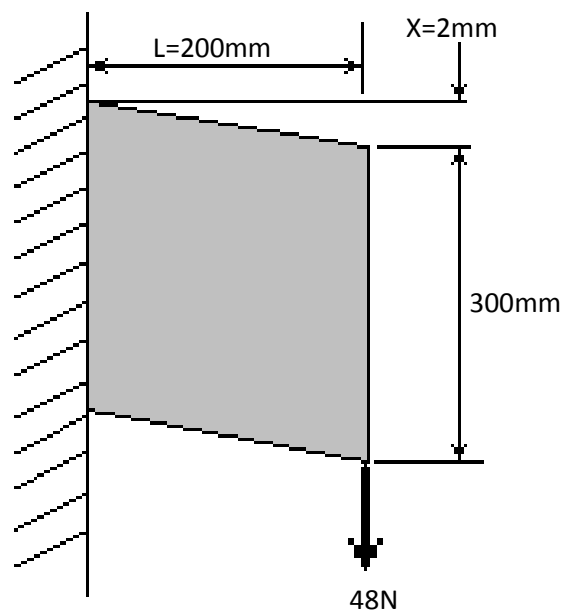
A nickel steel gudgeon pin in an engine piston has an outside diameter of 27mm and an inside diameter of 20mm . The force on the piston is 8kN . The modulus of rigidity for the steel is 90GN/m^2 . Calculate the stress and strain in the material.

Stress & Strain Examples

1. A hole of diameter 5mm needs to be punched in a plate 2.5mm thick. The ultimate shear stress of the plate material is 350MN/m^2 . Calculate the force requires. *Hint: The area resisting shear is equal to the hole circumference multiplied by the thickness.* [13.7kN]
2. Calculate the maximum thickness of the steel plate that can be cut by a guillotine if the ultimate shear stress of the plate is 300MN/m^2 and the maximum guillotine force is 150kN given that the plate is 0.8m wide. [0.625mm]

3. How many 10mm diameter rivets are required for a single lap joint which is to support a load of 100kN? The safe working shear stress of the rivet material is 75MN/m^2 . [17 rivets]
4. Calculate the average value of the shear stress in a solid gudgeon pin of 25mm diameter when the load on the pin is 40kN. *Hint: This is a case of double shear hence the area resisting shear is equal to 'twice' the area of the cross-section of the pin.* [40.7MN/m²]

5.

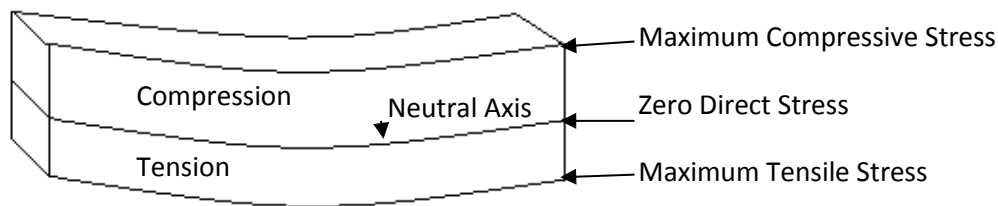


The figure above shows an experiment to determine the modulus of rigidity of a sample of rubber, a rectangular block of the material, measuring 300 x 200 x 20mm, was firmly fastened, so that it projected a distance of 200mm from a vertical wall. It was found that when a downward load of 48N was applied to the free vertical edge, a vertical deflection of 2mm was obtained as shown. Determine the experimental value of the modulus of rigidity of this rubber sample. [800kN/m²]

6. At a certain instant during the stroke of a single cylinder engine of 76.5mm bore the cylinder pressure is 2MN/m^2 . Calculate the resulting stress in each of the four cylinder head studs assuming the core diameter of the studs to be 9mm. [36.1 MN/m²]
7. Calculate the force required to punch an 80mm diameter hole into a steel sheet 1.25mm thick if the sheer strength of the material is 420MN/m^2 . [131.9kN]

Tensile and Compressive Stress and the Bending Equation

Consider the rectangular sectioned beam which is 'sagging' below.



The top part of the beam is in compression whilst the bottom part is in tension. The greatest stresses will occur at the outer edges of the beam, the top having compressive stresses and the bottom having tensile stresses. The stresses reduce uniformly to the centre of the material and where the stresses change from compressive to tensile there is considered to be zero direct stress. This axis where there is zero stress is called the neutral axis. For a rectangular section or any symmetrical section this is considered to be at mid depth. As the greatest stress is at the outer fibres of the beam, a rectangular section is not always an economical use of the material. An 'I' shaped sectioned beam is designed so that the top and bottom flanges resist the tensile and compressive stresses and all the cross section resists the shearing forces. If the beam bends the other way (hogging) then the tensile stresses are at the top and the compressive stresses are at the bottom. The stress in the material is proportional to the distance from the neutral axis. This is known as distance 'y', hence the stress at a distance y from the neutral axis is such that $\sigma_y \propto y$.

Therefore $\sigma_y = y \times \text{constant}$

Or $\sigma_y/y = \text{constant}$

This constant is equal to the Young's Modulus of Elasticity of the material of the beam divided by its radius of curvature.

$E/R = \text{constant}$

So the following can be written:

$$\frac{\sigma}{y} = \frac{E}{R}$$

When the beam is loaded, the bending moment applied, represented by 'M', is resisted by an equal internal moment.

This can be written as $M = (E/R) \times I$

Where I is the second moment of area of the beam section about the neutral axis.

$$\text{Hence: } \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

M = Maximum bending moment [Nm]

I = Second moment of area about the neutral axis [m^4]

σ = Stress at outer fibres of the beam material [N/m^2]

y = distance from the neutral fibres to the outer fibres [m]

E = Young's modulus of elasticity of the beam material [N/m^2]

R = Radius of Curvature to the neutral axis [m]

Assumptions for the Bending Moment Equation

The bending moment equation is based on certain relationships between different quantities such as stress, elasticity, curvature and dimensions. For these relationships to be valid the following five assumptions are necessary:

1. The bending equation assumes that Hooke's Law applies to each layer of the beam and Young's modulus of elasticity has the same value in compression as in tension.
2. The beam material is uniform throughout its volume.
3. Each cross-section of the beam is symmetrical about the plane of bending and the loads are applied to the beam in the plane of bending. If a horizontal beam supports vertical loads, then the plane of bending is vertical.
4. The beam must be initially straight and unstressed.
5. The resultant force perpendicular to any cross-section is zero.

Second Moment of Area

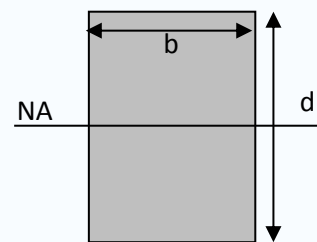
The amount of bending of a beam depends on the shape of the cross section. This shape is expressed by the second moment of area about the natural axis. The second moment of area is a measure of the distribution of an area relative to its axis.

The moment of area about an axis is the area multiplied by the distance from the centre to the axis. This is a difficult concept to visualise because it does not really physically exist, it's just a convenient measure to assist in calculations

Working out the second moment of area of some larger shapes can be complicated. If any area is divided up into small bits, like the circle, and each bit multiplied by the distance to the axis squared, then all these products added up gives the second moment of area. When calculating the second moment of area we consider the dimensions about the neutral axis. The neutral axis of a shape occurs at its geometric [centroid](#). In [physics](#), the word centroid may mean the geometric centre of the object's shape this can be at [centre of mass](#) or the [centre of gravity](#), depending on the context.

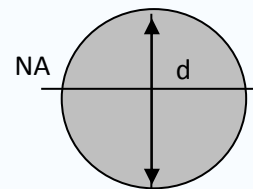
For a rectangular cross-section,

$$I_{NA} = \frac{bd^3}{12}$$



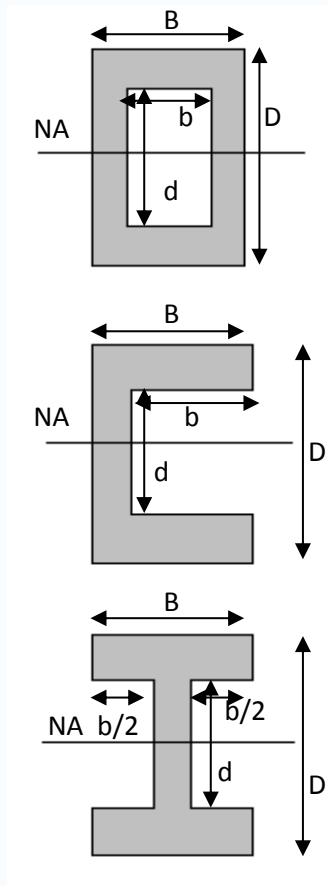
For a circular cross-section,

$$I_{NA} = \frac{\pi D^4}{64} = \frac{\pi r^4}{4}$$



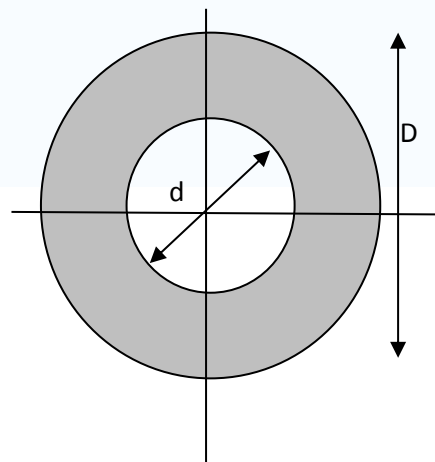
For box section, 'U' section, or 'I' section,

$$I_{NA} = \frac{BD^3}{12} - \frac{bd^3}{12}$$



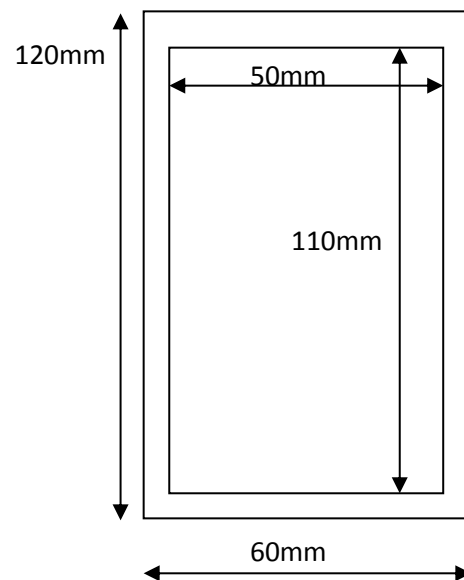
For a hollow circular section,

$$I_{NA} = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{4} (R^4 - r^4)$$



Examples

1. An engine is supported by a hoist hung from an overhead beam. The maximum bending moment of the beam is 72kNm. The second moment of area is $176.68 \times 10^{-6} \text{m}^4$ and the beam is symmetrical. The depth of the beam is 276.8mm. Calculate the maximum stress of the beam and the radius of curvature if Young's modulus of elasticity is 208GN/m^2 .
2. A beam has a rectangular cross-section 100mm x 25mm. It is subjected to a maximum bending moment of 625Nm. Compare the maximum stress in the material when the beam is positioned with the widest edge down and the narrowest edge down.
3. Find the second moment of area about the neutral axis of the shape below:



Torsion

When the moment of a force is applied to a shaft, it will tend to turn or twist. This is referred to as a turning moment of torque. Torsion is the twisting deformation produced when a torque is applied to a shaft. When a shaft twists, if any thin circular cross section is considered it will rotate slightly relative to the next section. The material therefore undergoes shear strain and shear stress, as thin circular cross sections of material tend to slide over each other.

There is a similar equation for the torsion of shafts to that of beam bending. This equation relates to the applied torque to the shear stress and twisting distortion of the shaft in a similar matter that the bending equation relates the bending moment to direct stress and bending distortion of the beam. Torque is the twisting moment applied to the shaft and is a product of applied force and the distance of the applied force to the central axis of the shaft. The symbol for Torque is T and the units are Nm. When the shaft is transmitting power, it is subjected to a torque at the input end from the drive such as an engine and gearbox, and a resisting torque at the output from the machinery being driven such as an axle and wheels.

Angle of Twist

When a torque is applied to a shaft it twists. There is a relative movement between adjacent cross sections of the shaft. If a straight line is drawn down the shaft parallel to the axis, when the torque is applied this deforms to a helix. The relative angular twist between the different cross sections is called the angle of distortion α . See figure 10:

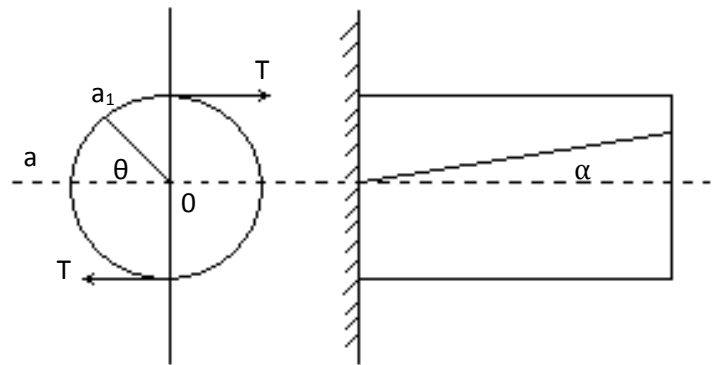


Figure 10.

A torque T is applied to the shaft whilst one end is kept fixed. A point 'a' on the end section is displaced to a position 'a₁'. The angle of twist at the shaft face is $\theta = a\hat{O}a_1$.

The angle θ is measured in radians. If a length of one radius r is laid along the circumference of the circle, then the angle θ , between each end of this length and the circle centre is one radian. As the circumference of a circle is equal to πd , then in 360° there are 2π radians.

Shear Stress

The value of the shear stress is proportional to the radius, zero shear stress occurring at the shaft section centre and maximum shear stress occurring at the outer edge.

$$\tau/r = \text{constant}$$

Torsion Equation

The above constant is used as the basis of the torsion equation, the full equation as follows:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

T = Twisting moment or torque [Nm]

J = Polar second moment of area [m⁴]

τ = Maximum shear stress at the outer fibres [N/m²]

r = radius to the outer fibres [m]

G = Modulus of rigidity [N/m²]

θ = Angle of twist [radians]

l = length of shaft under torsion [m]

Polar Second Moment of Area

In bending theory of beams, the second moment of area is taken from the neutral axis. The resistance to twisting depends on the polar second moment of area, J. The polar second moment of area is the second moment of area of the shaft cross section about the central point rather than an axis. When you are dealing with shafts there are usually only two cases; either a solid circular shaft or a hollow circular shaft with an annular (doughnut shaped) cross section.

For a solid circular shaft,

$$J = \frac{\pi D^4}{32} \quad \text{where D is the diameter.}$$

For a hollow circular shaft,

$$J = \frac{\pi(D^4 - d^4)}{32} \quad \text{where d is the inner diameter.}$$

Examples

1. When a torque of 100 Nm is applied to a solid shaft, the angle of twist at the shaft face is 1° . The shaft is 1m long and for the material, $G = 83 \text{ GN/m}^2$. Calculate the diameter of the shaft.
2. If a drive shaft 2m long is to be subjected to a torque of 3.3kNm and the maximum shearing stress must not exceed 48 MN/m^2 , calculate the solid shaft diameter required to satisfy these conditions. $G = 80 \text{ GN/m}^2$.

Torsional Resilience

Torsional resilience is the elastic strain energy stores in the shaft when work is done to twist the shaft. As with direct resilience this energy is only stored if the strain is below the elastic limit. If a torque is gradually applied from zero to T, then the average torque applied is:

$$\frac{0 + T}{2} = \frac{T}{2}$$

The work done to twist the shaft is therefore equal to:

$$\frac{T}{2} \times \theta$$

As with direct resilience, this can be expressed in terms of stress.

$$\frac{T}{J} = \frac{\tau}{r}$$

$$T = \frac{\tau J}{r}$$

For a solid shaft, $J = \frac{\pi D^4}{32}$ and $r = \frac{D}{2}$

$$\text{Then } T = \tau \times \frac{\pi D^4}{32} \times \frac{2}{D}$$

$$T = \frac{\tau \pi D^3}{16}$$

Substituting from $\frac{\tau}{r} = \frac{G\theta}{l}$

$$\theta = \frac{\tau l}{rG} = \frac{\tau l^2}{GD}$$

Then using the formula for Torsional resistance:

$$\frac{T\theta}{2} = \frac{1}{2} \times \frac{\tau \pi D^3}{16} \times \frac{\pi l}{GD}$$

$$\frac{T\theta}{2} = \frac{\tau^2 \pi D^2 l}{16G}$$

The volume, $V = \frac{\pi D^2}{4} l$

Therefore $\frac{T\theta}{2} = \frac{\tau^2 V}{4G}$

Example

If a drive shaft of diameter 83.3mm and length 2m is to be subjected to a torque of 3.3kNm, calculate the torsional resilience. $G=80\text{GN/m}^2$.

Distortion of Materials

Examples

1. Calculate the average shear stress in a solid gudgeon pin of 30mm diameter if the load on the pin is 22kN. [15.56MN/m²]
2. A round bar has a length of 270mm and diameter 15mm. A tensile load is gradually applied to the bar. A load of 25kN causes an extension of 0.2mm. Find the value of Young's modulus of elasticity for the bar material. [191GN/m²]
3. A push rod has a diameter of 5mm. At a certain point in the operation of the valve the force on the rod is 2.5kN. Calculate the compressive stress in the rod. [127.3MN/m²]
4. A piston has a force of 4kN acting on the crown at TDC. The cross sectional area of the connecting rod is 180mm². Calculate the compressive stress in the material. [22MN/m²]
5. A steel bar has a diameter of 50mm and is 1m long. The bar supports a tensile load of 50kN. Calculate the resilience in the bar if $E=200\text{GN/m}^2$. [3.189Nm]
6. A cast iron cylinder has an outside diameter of 120mm and an inside diameter of 100mm, and supports a load of 100kN. Find the compressive stress in the material. [28.92MN/m²]
7. A beam of length 7m is simply supported at each end and carries a concentrated load 3m from one end. If the bending moment is 50kNm at the section where the load is concentrated, find the magnitude of the load and the reaction forces. [2380.9N, 2916.6N]
8. A beam is 200mm deep by 100mm wide and subject to a maximum bending moment of 300kNm. What is the maximum stress? [0.45GN/m²]
9. A steel rectangular bar of thickness 12mm is bent into an arc until the steel yields at the top surface. The yield stress of the material is 300MN/m² and $E = 200\text{GN/m}^2$. Find the radius of curvature. [4m]

10. A solid steel shaft has a diameter of 300mm and twists through 1° over a length of 5m while rotating at a speed of 100rev/min, The value of $G = 85 \text{ GN/m}^2$. Calculate the torque imposed on the shaft. [235.98kNm]
11. A torque of 100Nm is applied to a shaft 2m in length. The angle of twist is 0.5° . The value of $G=85\text{GN/m}^2$. Calculate the diameter of the shaft. [40mm]
12. A beam of length 5m is simply supported at the ends. It carries concentrated loads of 100N, 200N and 300N at distances of 1m, 2.5m and 4 from one end, respectively. Calculate the reaction forces. [240N, 360N]
13. Find the twisting moment of a solid shaft of diameter 100mm when the angle of twist is 0.75° over a length of 2m. For the shaft material, $G=100\text{GN/m}^2$. [6381.36Nm]
14. A hollow shaft has an outside diameter of 350mm and an inside diameter of 250mm. It transmits a torques of 400kNm. Calculate the shear stress if $G = 90\text{GN/m}^2$. [64.2MN/m²]