

Dynamic StressStrain investigation

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This report investigates the relationship between Direct and Shear Stress and Strain within a material by conducting FEA Analysis on a deformable plate being impacted by a non-deformable projectile at various velocities. The report also comments on the limits of the exercise and discusses avenues for future study.

Dynamic Stress/Strain investigation

OME Physics Year 1
Assignment

Martin Dand - 1810440

Resonance

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1 Introduction

1.1 Task Description

In this assignment, we were tasked with analysing the dynamics of stress and strain on a deformable flat surface being impacted by a non-deformable projectile. In order to do this, we were given access to a piece of Finite Element Modelling software and instructed to research and obtain appropriate material property values for the plate in order to suitably analyse the shear and direct stresses of the projectile striking at different velocities. The software we have been instructed to use is Abaqus CAE.

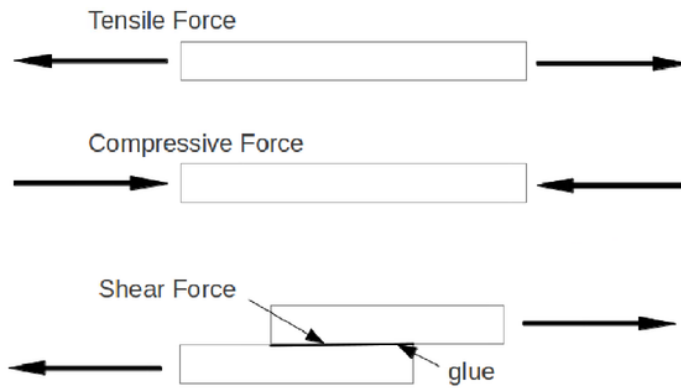


Figure 1 - Forces acting on a solid [1]

1.2 What is Stress?

If a force is applied to a material in such a way to cause it to deform, it puts the material under *stress*. This deforming effect will cause the material to extend (however small the extension). Stress can be defined as the force per unit area of the subject material, and can be mathematically defined by the formula:

$$\sigma = \frac{F}{A}$$

Where:

$$\sigma = \text{stress}$$

$$F = \text{force applied}$$

$$A = \text{cross sectional area of the object}$$

This is known as '*normal*' or '*direct*' stress.

Stress in parallel to a plane can be mathematically defined by the formula:

$$T = \frac{F_p}{A}$$

Where:

$$T = \text{shear stress}$$

$$F_p = \text{shear force in the plane of the area}$$

$$A = \text{area}$$

This is the force acting in the plane between two objects in contact sliding past one another. [1]

Both types of stress are measured in Nm^{-2} or Pascals (Pa).

1.3 What is Strain?

Where Stress causes a material to deform (however slightly), the actual extension of the material is known as *strain*. Also 'the deformation of a solid due to stress'. [1] This value a ratio of two lengths, and therefore dimensionless. It can be mathematically defined by the formula:

$$\varepsilon = \frac{\delta L}{L}$$

Where:

$$\varepsilon = \text{Strain}$$

$$\delta L = \text{change in length} = L - L_0$$

$$L = \text{extended length}$$

$$L_0 = \text{original length}$$

1.4 Relationship between Stress and Strain

These definitions of stress and strain can be applied to both compressive and tensile forces. We can refer to compressive stress and compressive strain being applied to material under compression, and tensile stress and tensile strain being applied to a material under tension. Hooke's Law states that 'for relatively small deformations of an object, the displacement or size of the deformation is directly proportional to the deforming force or load'. [2] This law applies for a material's *elastic* properties, which is a range of force that a material can be under and still return to its original size once the force is removed.

So for a materials *elastic* behaviour:

$$\text{stress} \propto \text{strain}$$

Or:

$$\text{stress} = E\varepsilon$$

Where E is a constant.

This gives rise to a numerical constant known as 'Young's Modulus'. This can be mathematically defined as: [3]

$$E \text{ (modulus of elasticity)} = \frac{\text{stress } (\sigma)}{\text{strain } (\varepsilon)} = \frac{FL}{A\delta L}$$

Young's Modulus is only valid to a certain force however. This force differs by material, and once the material deforms past this point, it will no longer return to its original size once the force is removed. If we plot a bilinear graph of stress against strain, every ductile material so graphed will show a shape similar to that displayed below, with both an *elastic* region (the region where a material can deform and still return to its original size) and a *plastic* region (region of non-recoverable deformation).

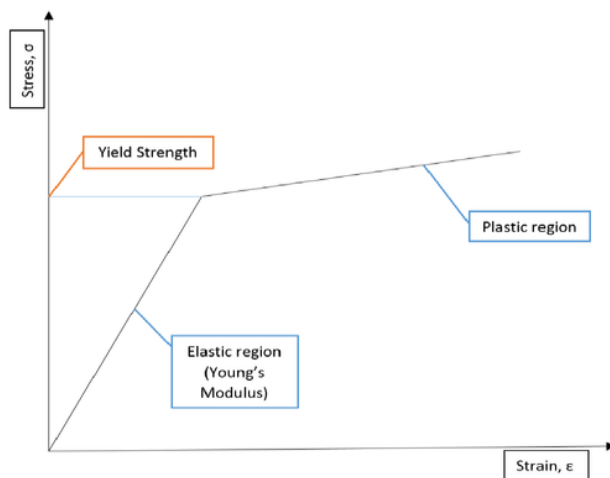


Figure 2 - Bi-Linear model of a Stress-Strain graph

If we look at a slightly more advanced model, we can introduce the idea that every material has a breaking point.

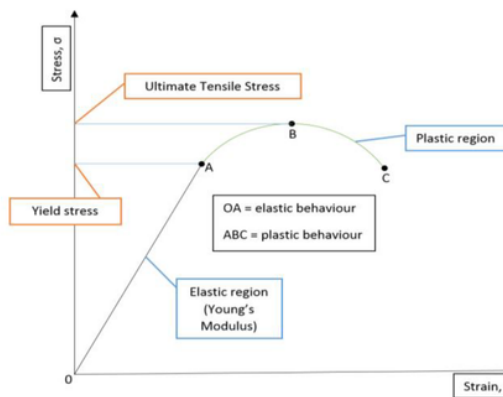


Figure 3 - Stress/Strain curve incorporating plastic behaviour (non-linear)

Once a material reaches the plastic region of the graph, Hooke's law is no longer obeyed and as a result, stress is no longer proportional to strain. If a material is plastically deformed, it will no longer return to its original form when the force is removed. At section AB in Figure 3, the material undergoes a process known as 'strain hardening'. The materials crystalline structure deforms during this portion, and increases hardness [4]. At Point B, the material reaches a maximum stress that it can withstand as can be seen in Figure 3, referred to as the Ultimate Tensile Stress. After this point, the material permanently deforms in a process known as 'necking' where one part of the material narrows. The overall stress required to increase strain then drops (although whilst it actually increases at the necking point, overall it decreases) until the material breaks at the fracture point or *breaking stress point* (point C on Figure 3) [5].

This process applies to *ductile* materials (such as most unhardened steels, copper, and thermoplastics). *Brittle* materials (such as concrete, glass or ceramic) behave slightly differently, in

that there is no real plastic deformation. Once the brittle material has undergone its elastic behaviour, it will move immediately to the fracture point as seen below.

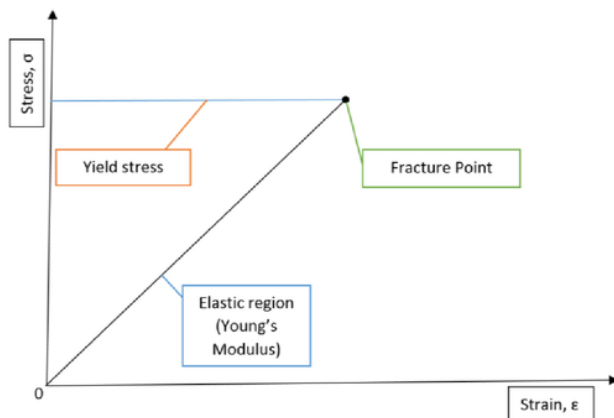


Figure 4 - Stress/strain relationship for Brittle materials

1.5 Poisson's Ratio

When a material is stretched in one plane, it gets thinner in the direction lateral to the stretching force. Likewise if a material is compressed in one plane, it gets thicker. Materials undergoing this phenomenon are seen to have a uniform reduction in size for tensile stresses or increase in size for compressive stresses. This ratio is known as 'Poisson's Ratio', and is defined as *'the negative ratio of the lateral strain to the axial strain for a uniaxial stress state'*. [6] It can be expressed mathematically as:

$$\nu \text{ (Poisson's Ratio)} = \frac{\epsilon_{lateral}}{\epsilon_{longitudinal}}$$

As it is a function of strain, it likewise has no units. This ratio is true if the volume of the material is constant, and this ratio is true only in the elastic portion of the stress/strain relationship. As the molecules move due to the force, the volume of the material changes, which causes 'weak spots' in the material. This gives rise to the phenomenon of necking in the plastic region as previously discussed.

Most common materials hold a Poisson's Ratio between 0-0.5. For example, the Poisson's Ratio for Aluminium 6061-T91 (the material covered in this report as discussed below) is 0.33. This means that for every cm per cm of deformation in the axial plane, there will be 0.33cm of perpendicular deformation if the volume of the material is constant.

2 Method

The group was supplied with a predefined Abaqus model comprising of a deformable plate of an unspecified material fixed in place at all edges and a non-deformable projectile with a pre-defined trajectory. The model was fully meshed. We were instructed to research suitable material properties for the plate. See Table 1 in the Appendix for the appropriate figures, all of which are the parameters discussed in the Introduction to this report.

During the practical part of this experiment a discussion was had regarding the fact that the model was programmed to convert parts of the mesh to particles upon the application of 150 MPa (1.5×10^8

Pa) of stress (a function known as Smooth Particle Hydrodynamics). If this function is not present the mesh distorts which can cause the program to crash, and this allows for the particles to still interact with the model but not endanger the stability of the program. I amended this function to convert at 300MPa (3×10^8 Pa (the UTS of the material is 4.05×10^8)) in order to more closely demonstrate the behaviour of the material model, whilst still maintaining the stability of the program.

Once the material values were entered, I changed the projectile velocity to 100ms^{-1} and ran the program. The tip of the projectile was assigned as a tracked reference point in order to plot the velocity of the projectile throughout the simulation. I ran the program twice more with the velocity changed to 200ms^{-1} and 400ms^{-1} , with the results shown in Figures 5, 6 and 7 respectively. During Job 3 (400ms^{-1} , projectile perforated plate) it was noted that the velocity slightly increased after perforation. The parameters were then adjusted to increase the t_{max} of the job in order to analyse the residual velocity.

3 Results

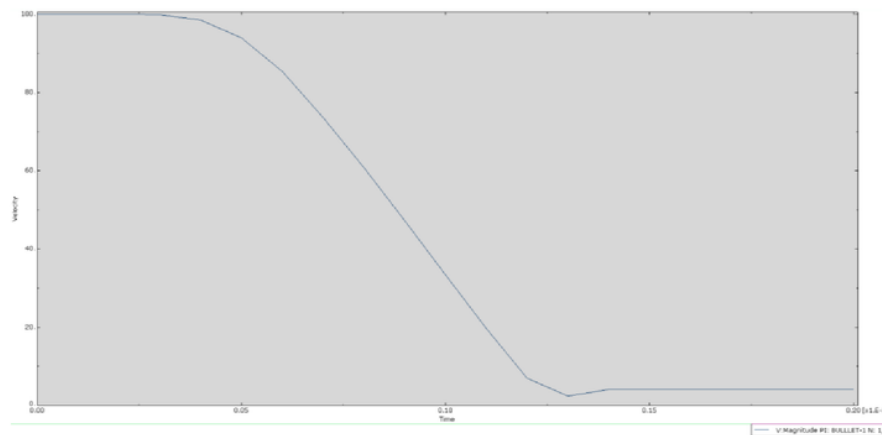


Figure 5 - Velocity vs Time - 100m/s

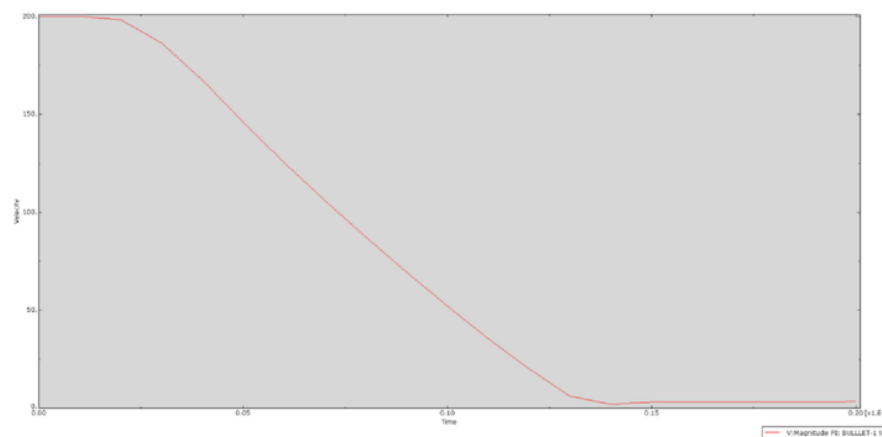


Figure 6 - Velocity vs Time - 200m/s

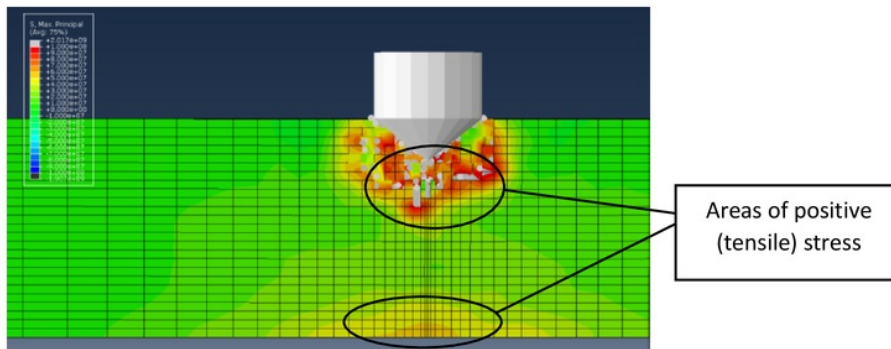


Figure 10 - Job 1 Time Step 6.0022e-05 seconds – Max Principle view

As can be seen in Figure 10, most of the applied stress applied to the plate is tensile. This is shown by the coloured areas of the mesh shown in yellow/orange/red. As can be seen, a stress wave has travelled through the plate and is reflected off the lower surface. Stress waves moving through a solid are directly related to the speed of sound travelling through a material [7] and as a result would require a much smaller time step in this simulation to track accurately. As this was not part of the brief, it was decided not to pursue this.

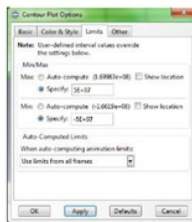


Figure 11 - Stress parameters - Job 1 - S23 view

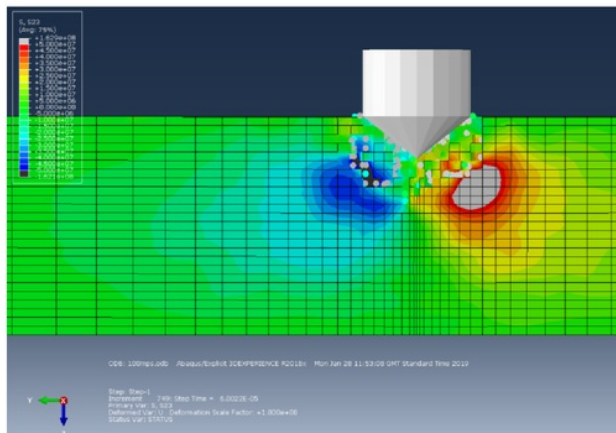


Figure 12 - Job 1 Time Step 6.0022E-05 seconds – S23 View

If the stresses are viewed in a different plane to analyse the Shear stress, it can be seen that the shear stresses are much less prevalent than the direct tensile stresses (max parameters set to +/- 5e07 Pa rather than 1e08 Pa as the view was set to in Figure 10).

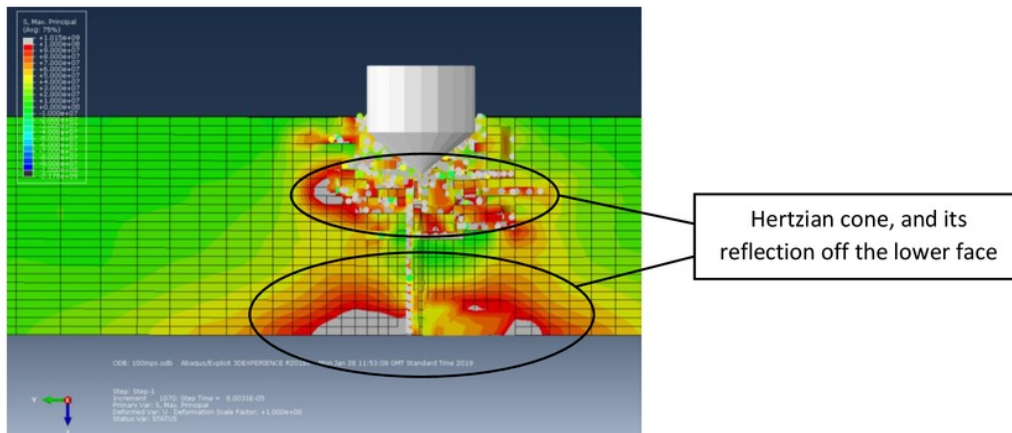


Figure 13 - Job 1 Time Step 8.0031e-05 seconds – Max Principle view

As the job progresses, a Hertzian cone [8] can be seen to be forming. There are higher lines of force radiating out and down from the reference point of the projectile indicating failure lines which will form into cracks in the material. The maximum applied direct stress in this frame (in grey) has been calculated at 1.015×10^9 Pa, and the cracks are appearing within the plate as this force is higher than the Ultimate Tensile Stress of the material.

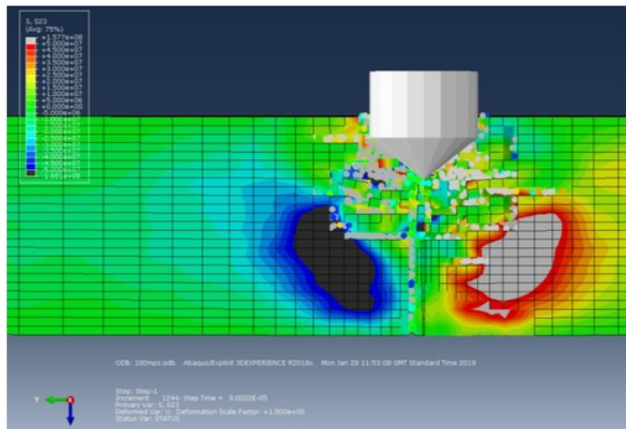


Figure 14 - Job 1 Time Step 9.0022E-05 seconds – S23 View

As can be seen in Figure 14, the shear stresses are still communicating along the Hertzian Cone lines and are shown in the absence of Max Principle Stress (comparing to Figure 13). In this job, the projectile penetrated the plate but did not perforate it. Upon inspection of Figure 5, the velocity of the projectile is observed to have decelerated upon contact with the plate as expected, come to a rest then moved backwards. This could be due to the elastic behaviour of the plate as it deforms under stress (flexing in the direction of the force), then returns to its original form, taking the stationary (compared to the plate) projectile up with it.

4.2 Job 2 – 200ms⁻¹

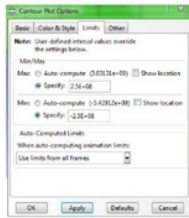


Figure 15 – Stress Parameters – Job 2 – Max Principle view

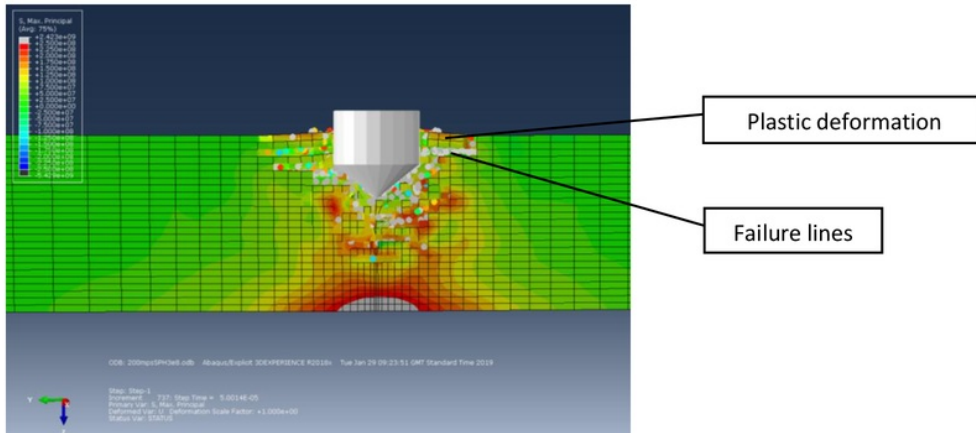


Figure 16 – Job 2 Time Step 5.0014e-05 seconds – Max Principle view

In Figure 16, the tensile wave can be seen to have reflected off the lower face. Where there have been lines of failure near the surface of the upper face, this wave has caused plastic deformation of the material and caused it to deform outwards. In this image, there are also areas of compressive stress (indicated by the blue particles).

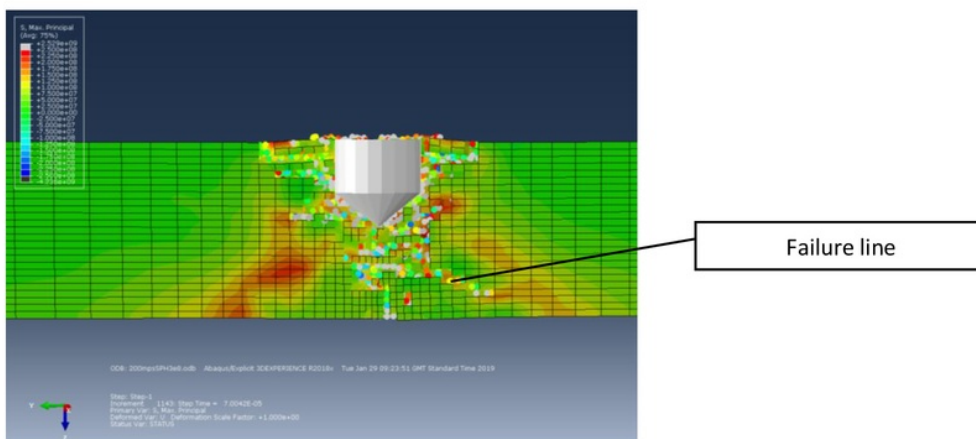


Figure 17 – Job 2 Time Step 7.0042e-05 seconds – Max Principle view

Figure 17 shows failure lines along the Hertzian Cone, and the stress dissipating in the area under it. This section shows the potential for material spalling off the bottom face.

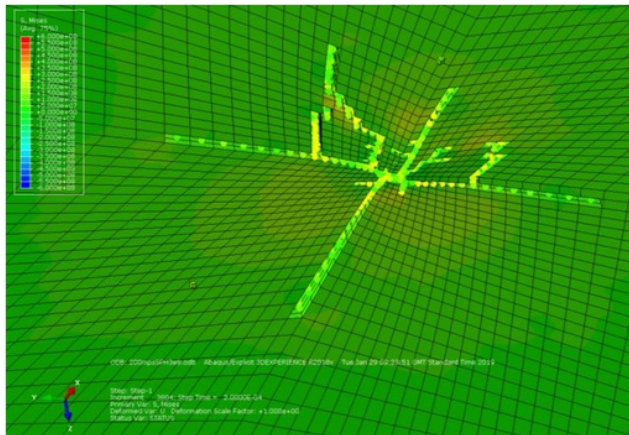


Figure 18 - Job 2 Time Step 2.0000e-04 seconds – Radial Fracture

As can be seen, if the base of the plate is inspected after the job has completed a radial fracture can be seen to have formed. The projectile penetrated the plate but did not perforate it. A similar trajectory is observed to Job 1, in that the projectile slows, stops, then travels back along its initial pathway (Figure 6).

4.3 Job 3 – 400ms⁻¹

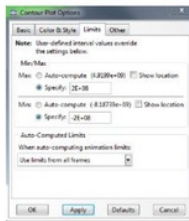


Figure 19 - Stress Parameters – Job 3 – Max Principle view

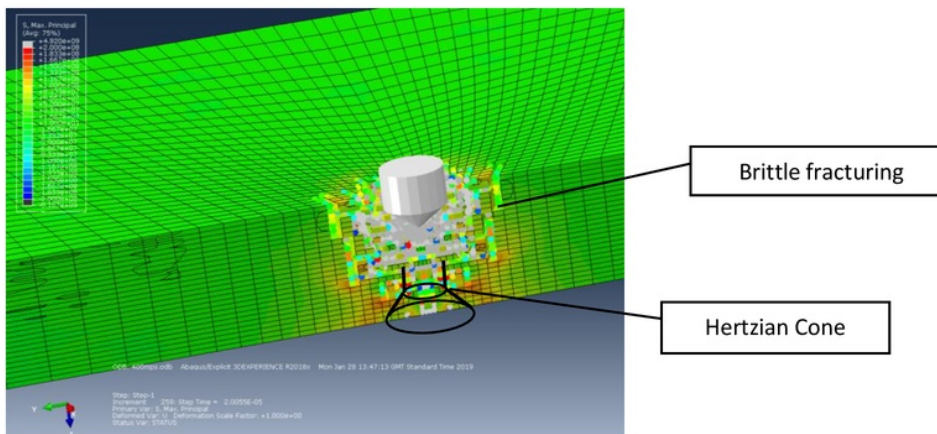


Figure 20 - Job 3 Time Step 2.0055e-05 seconds – Max Principle view

At 400ms⁻¹ it can be noted that the behaviour of the plate has apparently changed. In Figure 20, it can be seen that there is a catastrophic failure in the form of brittle fracture within the plate. This is

evidence of strain hardening throughout the affected area, causing an increase in brittleness hence the change in behaviour. It can be seen that there is still evidence of a Hertzian Cone on the bottom face of the plate.

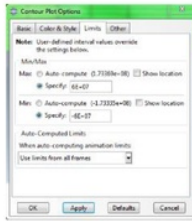
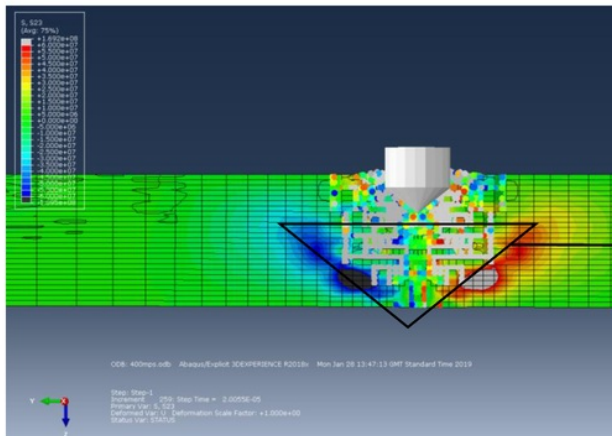
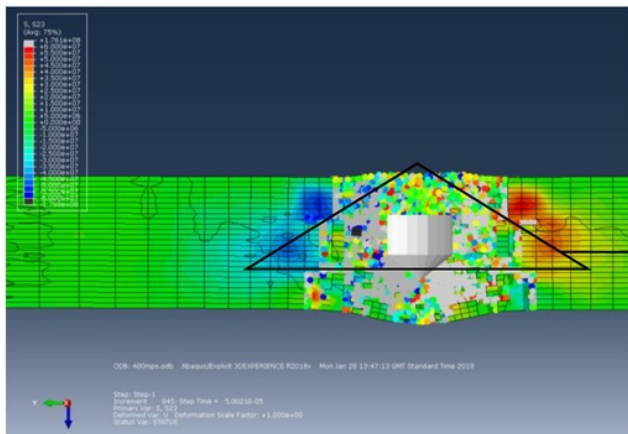


Figure 21 - Stress Parameters – Job 3 – S23 view



Shear stress wave
propagating downwards

Figure 22 - Job 3 Time Step 2.0055e-05 seconds – S23 view



Shear stress wave
reflected upwards

Figure 23 - Job 3 Time Step 3.0008e-05 seconds – S23 view

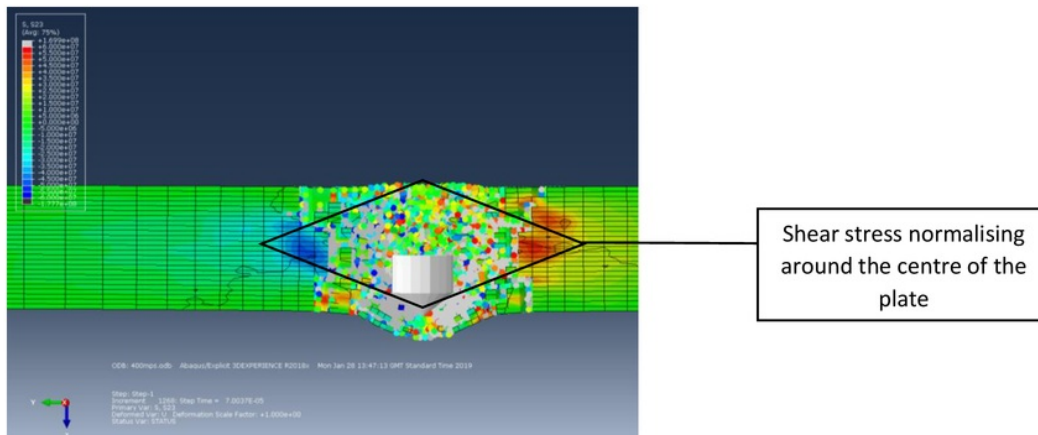


Figure 24 - Job 3 Time Step 7.0037e-05 seconds – S23 view

In the same time step viewed in the S23 plane, it can be seen that the shear stress is still acting in a similar way to the previous jobs. See figures 22, 23 and 24 to see the shear stress waves reflecting off the bottom face, then the top face and down again. The shear force then normalises around the centre of the plate as the reflected forces interfere with each other.

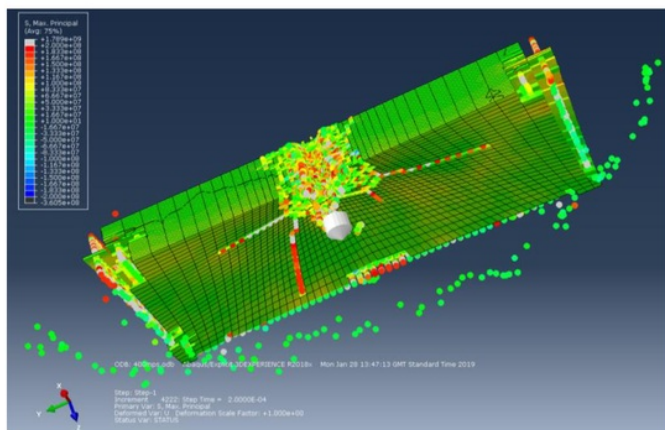


Figure 25 - Job 3 Time Step 2.0000e-04 seconds – Max Principle view

As can be seen in Figure 25, the projectile fully perforates the plate at this velocity. There is failure visible at the boundary of the model as when the stress waves rebound off the fixed edges, the forces are effectively doubled as they reflect back into the plate.

As seen in Figure 8, the velocity of the projectile is observed to increase post perforation. This could be an effect of the forces reflecting from the outer boundaries and transferring back into the projectile before it completes the perforation, but further experimentation would be required to explain this completely. I equally cannot account for the drop in velocity in the final step of the simulation.

5 Conclusions

Within this study, there were several limitations that affected the accuracy of the experiment.

When the program was initially set up, the plate was set as deformable, while the projectile was set to be non-deformable. Realistically, the projectile could have also been given appropriate material properties to observe the effects of materials interacting. The model was meshed in such a way that it was a finer mesh at the point of impact, with a coarser structure away from this point. If there was a finer mesh throughout, with more nodes to observe, this would assist in the analysis. This would also allow greater analysis of the forces acting on the boundaries of the plate.

Further to this, it was observed throughout the experiment that the stress waves were reflecting off the boundary and interfering with the view of the forces at play. A bigger plate would allow the stress waves further to travel, which would give more time to analyse the tension/compression at the point of impact. Another effect of an increased plate area would be a potential for greater flexing of the plate throughout the simulation at lower velocities. Another way to mitigate some of the interference would be to change the shape of the plate to a disc. If there was a more uniform spread of stress, and a uniform distance for the stresses to travel this might show a more easily readable stress pattern for longer.

Another aspect of this experiment is other external forces which are absent in a simulation. Friction from the air or the dissipation of kinetic energy into heat or sound energy would all play a part in a 'real world' scenario. As was previously mentioned, there was a change in velocity post perforation in Job 3 which cannot currently be explained. It was expected that as there were no external forces on the projectile post perforation, the projectile would have reached a constant velocity and continued as there were no other forces acting upon it. This was not the case as it was observed to have decreased in velocity without any input that could be seen. This may have been an artefact of the program causing it to behave in such a way. Further investigation could be made into this in future experiments.

Temperature also plays a part in stress and strain. During the research for this study, it was found that elastic and plastic properties are affected depending on the temperature of the material. [9] [10] For example, metals generally exhibit more brittle behaviour at lower temperatures, and more ductile properties at higher temperatures. There were no temperature parameters set within the simulation, so this would also affect the experiment.

Further research may also be made into the behaviour of materials at higher projectile velocity, the stress wave shapes supplied by different shaped projectiles, or the effect of multiple layers of target material (alternating differing materials or leaving spaces between).

In short, the scope for research revealed within this study is massive. Given the limits supplied, we can conclude that projectile impact on a plate mostly consists of direct tensile stress.

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6 References

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7 Appendix

	Aluminium 6061-91
Density	$2.7 \times 10^3 \text{ kg/m}^3$
Young's Modulus	69.0 GPa
Poisson's Ratio	0.33
Yield Tensile Stress	395 MPa
Ultimate Yield Stress	405 MPa
Plastic Strain (Elongation at break)	12%

Table 1 - Material Properties [9]

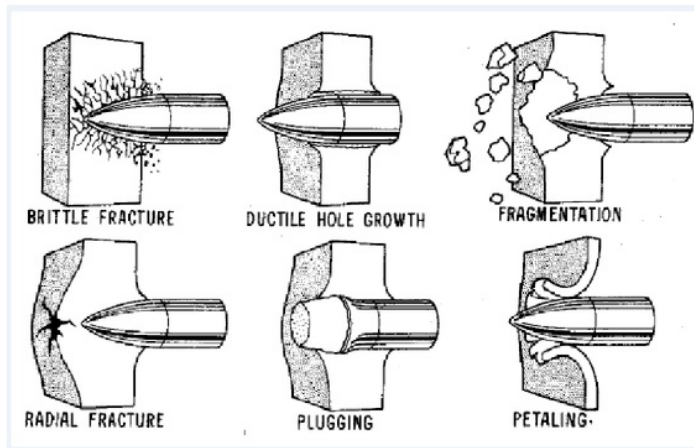


Figure 26 - Common Failure Modes on different types of materials [11]