



12 Pages

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1	Student Number	EVA18843717
2	Title of Examination Paper	Electrical Principles Theory
3	Module Code	GOEE5001
4	Date and Time of Examination	2/5/23 1:20
5	Examination Venue	Gower college Swansea
6	Seat Number	1
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
1	27
2	8
3	24
4	21
Total	80/100

Complete, fold and seal
(see note 2 below)

SURNAME: Evans

FORENAME(S): Dafydd John
Llewelyn

80% ✓
G.M.

1) $V = 60 e^{-t} \text{ V}$, $t=0$, $R=12$, $L=6$

(a)

$$L \frac{di}{dt} + Ri = V$$

$$6 \frac{di}{dt} + 12i = 60 e^{-t} \quad \checkmark$$

$$6 \mathcal{L}\left[\frac{di}{dt}\right] + 12 \mathcal{L}[i] = 60 \mathcal{L}[e^{-t}] \quad \checkmark$$

$$6 [sI(s) - i(0)] + 12 I(s) = \frac{60}{s+1} \quad \checkmark$$

$$6sI(s) + 12I(s) = \frac{60}{s+1} \quad \checkmark$$

$$I(s) (6s + 12) = \frac{60}{s+1}$$

$$I(s) = \frac{60}{(s+1)(6s+12)} \quad \checkmark \quad A$$

$$I(s) = \frac{10}{(s+1)(s+2)} \quad \checkmark \quad A$$

$$\frac{10}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \quad \checkmark$$

$$10 = A(s+2) \quad \checkmark + B(s+1) \quad \checkmark$$

$$(s = -2) \quad 10 = A(0) + B(-2+1) \quad \checkmark$$

$$10 = -1B$$

$$B = \frac{10}{-1} = -10 \quad \checkmark$$

$$(s = -1) \quad 10 = A(-1+2) + B(0) \quad \checkmark$$

$$10 = 1A$$

$$A = \frac{10}{1} = 10 \quad \checkmark$$

$$A = 10, \quad B = -10$$

$$I(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$= \frac{10}{s+1} + \frac{-10}{s+2}$$

$$L(t) = \frac{10}{s+1} - \frac{10}{s+2}$$

$$I(s) = 10e^{-t} - 10e^{-2t} \quad A$$

$$i) b) e^{3t} + 12 = \frac{1}{s-3} + \frac{12}{s} \quad \checkmark \quad 2 //$$

$$ii) \cos(5t) + 2 \sin(2t) = \frac{s}{s^2+5^2} + \frac{2(2)}{s^2+2^2} \quad \checkmark$$

$$= \frac{s}{s^2+5^2} + \frac{4}{s^2+2^2} \quad \checkmark \quad 2 //$$

$$= \frac{s}{s^2+25} + \frac{4}{s^2+4} \quad \checkmark$$

$$iii) (t^3 + 4t + 2) e^{3t}$$

$$= \left(\frac{3!}{s^{3+1}} + \frac{4}{s^2} + \frac{2}{s} \right) \frac{1}{s-3}$$

$$= \left(\frac{6}{s^4} + \frac{4}{s^2} + \frac{2}{s} \right) \frac{1}{s-3} \quad \checkmark \quad 2 //$$

$$iv) 5e^{5t} \cos(2t) - e^{3t}$$

$$= 5 \cdot \frac{s-5}{(s-5)^2 + 2^2} - \frac{1}{s-3}$$

$$= \frac{5s-25}{(s-5)^2 + 2^2} - \frac{1}{s-3} \quad \checkmark \quad 2 //$$

$$v) 2e^{-2t} \sin(4t)$$

$$= 2 \cdot \frac{4}{(s+2)^2 + 4^2} = \frac{8}{(s+2)^2 + 4^2} = \frac{8}{(s+2)^2 + 16} \quad \checkmark \quad 2 //$$

2

$$\begin{aligned}
 a) Z_T &= \frac{(0 - j10)(60 + j30)}{-j10 + 60 + j30} + 20 \\
 &= \frac{(10 \angle -90^\circ)(67.08 \angle 26.57^\circ)}{60 + j20} + 20 \\
 &= \frac{670.8 \angle -63.43^\circ}{63.25 \angle 18.43^\circ} + 20 \\
 &= 107.21 \angle -81.86^\circ + 20 \\
 &= 15.18 - j106.13 \quad \text{or} \quad 111.81 \angle -71.66^\circ \Omega
 \end{aligned}$$

Q28/20

$$\begin{aligned}
 b) I_T &= \frac{V}{Z} = \frac{240 \angle 0^\circ}{111.81 \angle -71.66^\circ} = 2.15 \angle 71.66^\circ \text{ A} \\
 &\quad \text{error carried forward} \\
 &\quad \text{or } 0.68 + j2.04 \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 c) V_{-j10} &= \frac{60 + j30}{60 + j30 - j10} \times 240 \angle 0^\circ \\
 &= \frac{60 + j30}{60 + j20} \times 240 \angle 0^\circ \\
 &= \frac{67.08 \angle 26.57^\circ}{63.25 \angle 18.43^\circ} \times 240 \angle 0^\circ \\
 &= 1.06 \angle 8.14^\circ \times 240 \angle 0^\circ \\
 &= 254.4 \angle 8.14^\circ \text{ V}
 \end{aligned}$$

$$V_{-j10} = \frac{j30}{-j10 + j30}$$

$$\begin{aligned}
 I_{-j10} &= \frac{60 + j30}{60 + j30 - j10} \times 2.15 \angle 71.66^\circ \\
 &= \frac{67.08 \angle 26.57^\circ}{60 + j20} \times 2.15 \angle 71.66^\circ \\
 &= \frac{67.08 \angle 26.57^\circ}{63.25 \angle 18.43^\circ} \times 2.15 \angle 71.66^\circ \\
 &= 1.06 \angle 8.14^\circ \times 2.15 \angle 71.66^\circ \\
 &= 2.279 \angle 79.8^\circ \text{ A}
 \end{aligned}$$

$$I_{-j10} = 2.279 \angle 79.8^\circ \text{ A}$$

$$V = IR = 2.279 \angle 79.8^\circ \times 0 - j10$$

$$= 2.279 \angle 79.8^\circ \times 10 \angle -90^\circ$$

$$= 22.79 \angle -10.2^\circ \text{ V}$$

$$\text{or } 22.43 - j4.04 \text{ V}$$

✓ ON

$$2 \text{ d) } P = I^2 R$$

$$= (2.15)^2 (35.18)$$

$$= 162.61955 \text{ W}$$

$$= 162.62 \text{ W}$$

✓

ON

✓

$$3) f(x) = \begin{cases} -3 & \text{if } -\pi < x < 0 \\ 3 & \text{if } 0 \leq x < \pi \end{cases} \text{ and } f(x+2\pi) = f(x)$$

Q3

24/25

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx \quad \checkmark$$

$$= \frac{1}{\pi} \int_{-\pi}^0 f(x) dx + \frac{1}{\pi} \int_0^{\pi} f(x) dx \quad \checkmark$$

$$= \frac{1}{\pi} \int_{-\pi}^0 -3 dx + \frac{1}{\pi} \int_0^{\pi} 3 dx$$

$$= \frac{-3}{\pi} [x]_{-\pi}^0 + \frac{3}{\pi} [x]_0^{\pi} \quad \checkmark \quad \checkmark$$

$$a_0 = 0 \quad \checkmark$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$= \frac{1}{\pi} \int_{-\pi}^0 f(x) \cos nx dx + \frac{1}{\pi} \int_0^{\pi} f(x) \cos nx dx$$

$$= \frac{1}{\pi} \int_{-\pi}^0 -3 \cos nx dx + \frac{1}{\pi} \int_0^{\pi} 3 \cos nx dx \quad \checkmark$$

$$= \frac{1}{\pi} \left[-3 \frac{\sin nx}{n} \right]_{-\pi}^0 + \frac{1}{\pi} \left[\frac{3 \sin nx}{n} \right]_0^{\pi} \quad \checkmark$$

$$= \frac{-3}{\pi n} [\sin nx]_{-\pi}^0 + \frac{3}{\pi n} [\sin nx]_0^{\pi} \quad \checkmark$$

$$= \frac{-3}{\pi n} [\sin 0 - \sin -\pi] + \frac{3}{\pi n} [\sin \pi - \sin 0]$$

$$= \frac{-3}{\pi n} [0 - 0] + \frac{3}{\pi n} [0 - 0] \quad \checkmark \quad \checkmark$$

$$a_n = 0 \quad \checkmark$$

$$\begin{aligned}
 b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx \\
 &= \frac{1}{\pi} \int_{-\pi}^0 f(x) \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx \, dx \\
 &= \frac{1}{\pi} \int_{-\pi}^0 -3 \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} 3 \sin nx \, dx \\
 &= \frac{1}{\pi} \left[-\frac{-3 \cos nx}{n} \right]_{-\pi}^0 + \frac{1}{\pi} \left[-\frac{3 \cos nx}{n} \right]_0^{\pi} \\
 &= \frac{3}{\pi n} [\cos nx]_{-\pi}^0 + \frac{-3}{\pi n} [\cos nx]_0^{\pi} \\
 &= \frac{3}{\pi n} [\cos(0) - \cos(-\pi)] + \frac{-3}{\pi n} [\cos(\pi) - \cos(0)] \\
 &= \frac{3}{\pi n} [1 - -1] + \frac{-3}{\pi n} [-1 - 1] \\
 &= \frac{3}{\pi n} [2] - \frac{3}{\pi n} [-2] \\
 &= \frac{6}{\pi n} + \frac{6}{\pi n}
 \end{aligned}$$

$$b_n = \frac{12}{\pi n} \quad \text{for odd}$$

$$b_n = 0 \quad \text{for even}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

$$a_0 = 0, \quad a_n = 0, \quad b_n = \frac{12}{\pi n} \quad \text{for odd, } 0 \text{ for even.}$$

$$f(x) = \left(\frac{12}{\pi} \sin x + \frac{12}{\pi 3} \sin 3x + \frac{12}{\pi 5} \sin 5x + \frac{12}{\pi 7} \sin 7x + \dots \right)$$

4) a Star network $Z_1 = 5\Omega$, $Z_2 = j10$, $Z_3 = 10$

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

$$= \frac{(5 \cdot j10) + (j10 \cdot 10) + (10 \cdot 5)}{5}$$

$$= \frac{j50 + j100 + 50}{5}$$

$$= \frac{50 + j150}{5} = 158.11 \angle 71.57^\circ$$

$$= 31.622 \angle 71.57^\circ \Omega$$

or ~~499~~ $10 + j30 \Omega$

Provide
Sketch

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

$$= \frac{(5 \cdot j10) + (j10 \cdot 10) + (10 \cdot 5)}{j10}$$

$$= \frac{50 + j150}{j10} = \frac{158.11 \angle 71.57^\circ}{10 \angle 90^\circ}$$

$$= 15.811 \angle -18.43^\circ \Omega$$

or $15 - j5 \Omega$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

$$= \frac{(5 \cdot j10) + (j10 \cdot 10) + (10 \cdot 5)}{10}$$

$$= \frac{50 + j150}{10} = \frac{158.11 \angle 71.57^\circ}{10}$$

$$= 15.811 \angle 71.57^\circ \Omega$$

or $5 + j15 \Omega$

Q4
21/25
Q4a

8/10

4 b

$$\begin{aligned}
 V_{TH} &= \frac{3 + j4}{3 + j4 + 5} \times 10 \angle 0^\circ \\
 &= \frac{3 + j4}{8 + j4} \times 10 \angle 0^\circ \\
 &= \frac{5 \angle 53.13^\circ}{8.944 \angle 26.57^\circ} \times 10 \angle 0^\circ \\
 &= 0.559 \angle 26.56^\circ \times 10 \angle 0^\circ \\
 &= 5.59 \angle 26.56^\circ \checkmark
 \end{aligned}$$

$$\begin{aligned}
 Z_{TH} &= \frac{(3 + j4)(5)}{3 + j4 + 5} + j5 \\
 &= \frac{(5 \angle 53.13^\circ)(5)}{8 + j4} + j5 \\
 &= \frac{25 \angle 53.13^\circ}{8.944 \angle 26.57^\circ} + j5 \\
 &= 2.7952 \angle 26.56^\circ + j5 \\
 &= 2.5 + j1.25 + j5 \\
 &= 2.5 + j6.25 \checkmark \\
 \text{or } 6.73 \angle 68.2^\circ \checkmark
 \end{aligned}$$

$$\begin{aligned}
 Z_L &= Z_{TH}^* = 2.5 - j6.25 \\
 \text{or } 6.73 \angle -68.2^\circ
 \end{aligned}$$

$$\begin{aligned}
 I_T &= \frac{V}{Z_{TH} + Z_{TH}^*} = \frac{5.59 \angle 26.56^\circ}{2.5 + j6.25 + 2.5 - j6.25} \\
 &= \frac{5.59 \angle 26.56^\circ}{5} \\
 &= 1.118 \angle 26.56^\circ \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 P &= I^2 R \\
 &= (1.118)^2 (2.5) = 3.12481 \text{ W}
 \end{aligned}$$

not requested

Q4
b
=

13/15