

The Development of a Model to Explain Group Decision-Making Processes: An Application of Fuzzy Methodology

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A model is developed, using fuzzy methodology, to explain group decision-making in Quality Function Deployment (QFD), where the members have optimistic attitudes towards the result of the decision-making process. An example is provided, as well as the appropriate computational procedures, to show the usefulness of the proposed model in explaining group-decision-making processes in QFD, when the information is imprecise and the members have optimistic attitudes toward the outcome of the processes.

Quality function deployment (QFD) has been shown to be an effective quality systems tools to 'capture' customer requirements and then fulfill those requirements by providing better products or services to meet customer requirements with regard to technology (Chan and Wu, 2002, 2002-03; Wu, 2002-03; Wu et al. 2005). At an early stage of product design, the views and opinions of the customer are translated into technical product specifications to improve customer satisfaction, from product planning and process design to product manufacturing and delivery (Sullivan, 1986; Gryna, 2001; Chan and Wu, 2002-03). Following QFD, customer requirements can be classified and weighted according to a variety of methods, including market surveys and focus groups (Griffin and Hauser, 1993; Gryna, 2001). One problem here is that frequently, due to limited experience, interviewees might not be able to clearly state their psychological needs. Another is that translating customer needs into technical measures as well as determining their relationships are typically done by cross-functional teams (Clausing, 1994), in which team members frequently come to different conclusions because of their different experiences and a lack of precise information.

Arrow (1963) has argued that it is impossible to satisfy all decision makers' individual preferences in group decision-making processes. No voting method is perfect, but applying fuzzy set theory to group decision-making processes could improve the way in which decisions are reached. In practice, decision makers might have different preferences and attitudes towards risk, such as being optimistic or pessimistic towards the achievement of 'good outcomes' especially when their experiences are limited and information is inexact or ambiguous. In our view, there is a need for a model to explain group decision-making approach, using with fuzzy set theory specially developed to deal with ambiguous and imprecise—or fuzzy—data, when the members of the group have an optimistic attitudes towards risk. This is the objective of the present investigation.

This study is organized as follows: Section 2 reviews the basic concepts of fuzzy sets, fuzzy operations, linguistic variables, and weighted Min aggregation. A proposed model

for fuzzy group decision-making processes in QFD with optimistic attitudes on the part of group members is summarized in Section 3. In Section 4 there is an example to show the usefulness of this proposed model. Finally, a number of suggestive conclusions are drawn in Section 5.

2. Fuzzy Sets, Fuzzy Operations, Fuzzy Preference Relation, and Weighted Min Aggregation

In this section, the theories of fuzzy sets, fuzzy operations, linguistic variables, fuzzy preference relations, and weighted Min aggregation are briefly reviewed.

2.1 Fuzzy Sets

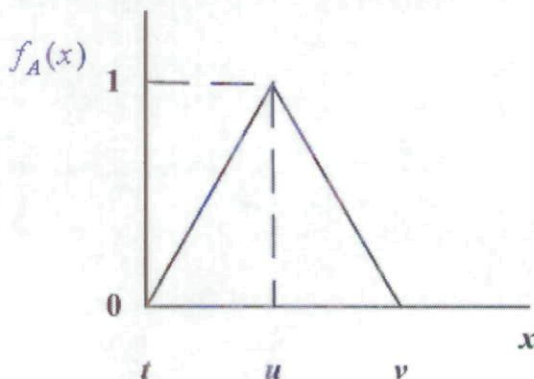
Zadeh (1965) proposed fuzzy sets be defined as follows: In a universe of discourse U_x , a fuzzy subset A of U_x is characterized by a membership function $f_A(x)$, where $f_A: U_x \rightarrow [0,1]$ the membership function associates with each member x of U_x and a number $f_A(x)$ is in the interval $[0,1]$, representing the grade of membership of x in A . The larger is $f_A(x)$ the stronger is the degree of membership of x in A . The height of the fuzzy set represents the maximum degree of membership. If there exists at least an element whose degree of the membership is one, then the fuzzy set is called normal.

The triangular fuzzy numbers proposed by Dubois and Prade (1978) are defined as follows: A fuzzy number A in R (real line) is a triangular fuzzy number if its membership function $f_A: R \rightarrow [0,1]$ is equal to

$$f_A(x) = \begin{cases} \frac{x-t}{u-t}, & t \leq x \leq u \\ \frac{x-v}{u-v}, & u \leq x \leq v, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

with $t \leq u \leq v$. The triangular fuzzy number can be denoted by $A = (t, u, v)$ as depicted in Figure 1.

Figure 1. The membership function of a triangular fuzzy number



2.2 Operations of triangular fuzzy numbers

The arithmetic operations of triangular fuzzy numbers can be explained as follows (Liang and Wang, 1993): If two triangular fuzzy numbers are $A_i = (t_i, u_i, v_i)$ and $A_j = (t_j, u_j, v_j)$, ($A_i > 0$ and $A_j > 0$), then those operators are defined as:

$$\text{Addition } \oplus : A_i \oplus A_j = (t_i + t_j, u_i + u_j, v_i + v_j) \quad (i, j \in N); \quad (2)$$

$$\text{Negation of } A_i : \ominus A_i = (-v_i, -u_i, -t_i); \quad (3)$$

$$\text{Subtraction } \ominus : A_i \ominus A_j = (t_i - v_j, u_i - u_j, v_i - t_j); \quad (4)$$

$$\text{Multiplication } \otimes : h \otimes (t, u, v) = (ht, hu, hv), \quad (5)$$

$$A_i \otimes A_j = (t_i t_j, u_i u_j, v_i v_j), \quad (v_i v_j \geq 0);$$

$$\text{Inverse of } A_i : A_i^{-1} = (v_i^{-1}, u_i^{-1}, t_i^{-1}), \quad (v_i > 0); \quad (6)$$

$$\text{Division } \oslash : A_i \oslash A_j = (t_i / v_j, u_i / u_j, v_i / t_j), \quad (v_i \geq 0, v_j > 0). \quad (7)$$

2.3 Linguistic variables

Zadeh (1975a, 1975b, 1976) proposed linguistic variables, variables whose values are words or sentences in an artificial or natural language; e.g., very low, low, fair, high, very high. Basic variables can be defined as $[0, 1]$, where "1" can be regarded as the best or most suitable situation, and "0" as the worst or least suitable situation.

2.4 Weighted Min Aggregation

The topic of weighted aggregation in fuzzy set theory has been studied by a number of researchers, such as Yager (1978, 1981, 1985, 1987), Dubois and Prade (1986), Dubois et al. (1988), and Sanchez (1989). Many applications of multi-criteria decision-making, pattern recognition, diagnosis and logic control in fuzzy sets encounter the weighted aggregation problem. Yager (1994) discussed issues involved in employing the so-called Min operation to reduce the effect of elements of low importance, of little value. Elements with low values play the most significant role in the Min aggregation. The method proposed reduces the effect of such elements by 'transforming' their values into larger values that are closer to one.

The operation of weighted Min aggregation is summarized by Yager (1993a, 1993b, 1993c, 1994) as follows: Min aggregation is anti-monotone in cardinality, and has these features, as an operator

$$\text{Min}[a_1, a_2, \dots, a_n] \geq \text{Min}[a_1, a_2, \dots, a_n, a_{n+1}]$$

Suppose associated with each fuzzy set A_i (characterizing the i -th attribute or criterion) is a weight $w_i \in [0, 1]$ representing its importance in the aggregation procedure, $i = 1, 2, 3, \dots, n$. The general process for the inclusion of importance in the aggregation involves the transformation of fuzzy sets, as follows:

Let Agg be an aggregation operator, and Min , the weighted aggregation. Let x be an alternative and let $(a_1, a_2, \dots, a_n)$ denote the degrees to which x satisfies the criteria, i.e.,

$$a_i = A_i(x), \quad i = 1, 2, 3, \dots, n.$$

We first transform each of the membership grades using the weights

$$g(w_i, a_i) = \hat{a}_i, \quad i = 1, 2, 3, \dots, n$$

to obtain the weighted aggregate

$$\hat{a} = \text{Agg}(\hat{a}_1, \dots, \hat{a}_n)$$

The form of g depends on the mode of aggregation being performed, the operation Agg .

A class of functions that can be used for the inclusion of importance in the Min aggregation is described by Yager (1994) as follows:

$$g(w_i, a_i) = S(1 - w_i, a_i)$$

where S is a t-conorm. Two significant manifestations of this operation are

$$g(w_i, a_i) = \text{Max}\{1 - w_i, a_i\} \quad (\text{Max t-conorm})$$

$$g(w_i, a_i) = 1 - w_i + a_i w_i$$

using $S(a, b) = a + b - ab$, which is $\text{Min}\{\hat{a}_1, \dots, \hat{a}_n\}$, where $g(w_i, a_i) = \hat{a}_i, i = 1, 2, 3, \dots, n$.

If $w_i = 0$, then from the fundamental character of t-conorms, it follows that

$$S(1 - w_i, a_i) = S(1, w_i) = 1$$

The result from this equation is one because of zero importance.

Garthwaite et al. (2005) examined the situation where the opinions of more than one person are involved in the group decision-making, and the majority preferences of group members are not transitive. Arrow (1963) showed that group decision-making usually leads to better solutions in politics (Garthwaite, 2005). In the present research we examine group decision-making processes in QFD by applying a model based on the algorithm of Lin (1999) and the weighted Min aggregation proposed by Yager (1994) to evaluate group decision-making risk when members have optimistic attitudes.

3. A Proposed Model for Group Decision-Making Based on Weighted Min Aggregation

A proposed model for group decision-making in QFD is summarized in this section, based upon Li's algorithm (1999) and the weighted Min aggregation proposed by Yager (1994). Assume there are m cross-functional team members; n interviewed customers; i technical measures, where $i = 1, 2, 3, \dots, p$, and j customer requirements, where $j = 1, 2, 3, \dots, q$. The notation of e_{ij} represents the 'crisp value' after aggregating fuzzy performance ratings (relationships between customer requirements and technical measures) and fuzzy weights (customer requirements) by means of the weighted Min aggregation. A proposed procedure for the model is as follows:

Step 1: Define the linguistic terms as triangular fuzzy numbers.

Step 2: Aggregate fuzzy weights of customer requirements through n interviewed customers by means of extended addition and scalar multiplication to form a comprehensive weight vector W , in which weight $w_j = (1/n) \otimes (w_{j1} \oplus w_{j2} \oplus \dots \oplus w_{jn})$ is a triangular fuzzy number of the form

$$(w_{1j}, w_{2j}, w_{3j}) = (\frac{1}{n} \sum_{h=1}^n w_{1jh}, \frac{1}{n} \sum_{h=1}^n w_{2jh}, \frac{1}{n} \sum_{h=1}^n w_{3jh}), \quad (8)$$

Step 3: Aggregate fuzzy performance ratings of customer requirements and technical measures in m cross-functional team members, by means of extended addition and scalar multiplication, to form a comprehensive performance matrix S , in which performance rating $s_{ij} = (1/m) \otimes (s_{ij}^1 \oplus s_{ij}^2 \oplus \dots \oplus s_{ij}^m)$ is a triangular fuzzy number of the form

$$(s_{1ij}, s_{2ij}, s_{3ij}) = (\frac{1}{m} \sum_{h=1}^m s_{1ij}^h, \frac{1}{m} \sum_{h=1}^m s_{2ij}^h, \frac{1}{m} \sum_{h=1}^m s_{3ij}^h), \quad (9)$$

Step 4: Aggregate these fuzzy performance ratings and technical measures with fuzzy weights by means of weighted Min and Max aggregations, to form an weighted, comprehensive aggregation matrix E , in which $E=[e_{ij}]$, where $e_{ij} = \text{Min}_t \{g(w_{ij}, s_{tij}), t=1,2,3\}$, based upon Yager (1994), and there exists one method which can be described as follows:

$$\begin{aligned} e_{ij} &= \text{Min}_t \{g(w_{ij}, s_{tij})\} \mid g(w_{ij}, s_{tij}) = \text{Max}\{1 - w_{ij}, s_{tij}\}, t=1,2,3\} \\ &= \text{Min}_t \{\text{Max}\{1 - w_{ij}, s_{tij}\}, t=1,2,3\}. \end{aligned} \quad (10)$$

Step 5: Calculate ranking values (average) $\bar{A}_i$, where $\bar{A}_i$ is defined as

$$\bar{A}_i = (1/q) \otimes (e_{i1} \oplus e_{i2} \oplus \dots \oplus e_{iq}). \quad (11)$$

Step 6: Rank the alternatives according to their ranking values and select the alternative with the maximum ranking value as the optimal alternative.

4. An Illustrated Example

To illustrate the proposed model, assume QFD for the sake of simplicity has three customer requirements, denoted CR, and five technical measures, denoted TM. Assume there are 5 customers involved in determining the weight for each CR, and there are 7 members in a cross-functional team to 'evaluate' the 'relationship' between CR and TM. To apply the proposed model summarized in Section 3, a number of steps need to be followed: In Step 1, the linguistic values of the importance weights for each CR and the 'relationship' between CR and TM are defined. In our example, five ranks are used and depicted in Tables 1 and 2, where $CR = \{VL, L, M, H, VH\}$ and the relationship between CR and TM is $\{VW, W, M, S, VS\}$. Normal fuzzy numbers with triangular membership functions for CR and the relationship between CR and TM are listed in Tables 3 and 4 respectively. Triangular fuzzy numbers for the linguistic values are depicted in Tables 3 and 4, and the membership functions of the triangular fuzzy numbers are displayed in Figures 2 and 3 respectively. Five customers were asked to evaluate the importance for each CR, and the results are summarized in Table 5, where the customers' opinions are quite different. Finally, the relationships between CR and TM as evaluated by the cross-functional team with 7 members are provided in Table 6.

In Step 2, aggregate the fuzzy importance weights for each CR from the five customers by applying Equation (8). The fuzzy importance weights of these three customer requirements are given in Table 7. In step 3 aggregate the fuzzy relationships between CR and TM by applying Equation (9), to the judgments of the cross-functional team

Table 1. Linguistic values of the importance weight for each CR

Five ranks of the importance weight for each CR
VL: Very low, L: Low, M: Middle, H: High, VH: Very high

Table 2. Linguistic values of the relationship between CR and TM

Five ranks of the relationship between CR and TM
VW: Very weak, W: Weak, M: Medium, S: Strong, VS: Very strong

Table 3. Triangular fuzzy numbers of linguistic values {VL, L, M, H, VH} for each CR

Linguistic value	VL	L	M	H	VH
Triangular fuzzy number	(0, 0, 0.25)	(0, 0.25, 0.5)	(0.25, 0.5, 0.75)	(0.5, 0.75, 1.0)	(0.75, 1.0, 1.0)

Table 4. Triangular fuzzy numbers of linguistic values {VW, W, M, S, VS} for the relationship between CR and TM

Linguistic value	VW	W	M	S	VS
Triangular fuzzy number	(0.2, 0.2, 0.4)	(0.2, 0.4, 0.6)	(0.4, 0.6, 0.8)	(0.6, 0.8, 1.0)	(0.8, 1.0, 1.0)

Figure 2. Membership functions of linguistic values {VL, L, M, H, VH} for each CR.

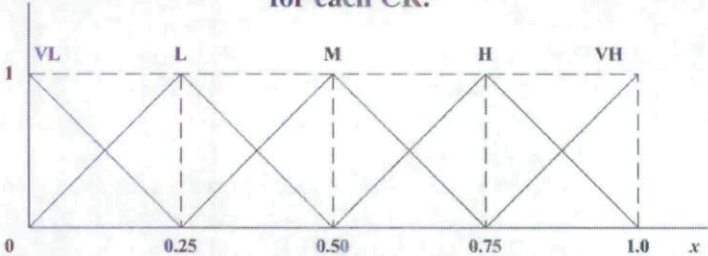


Figure 3. Membership functions of linguistic values {VW, W, M, S, VS} for the relationship between CR and TM.

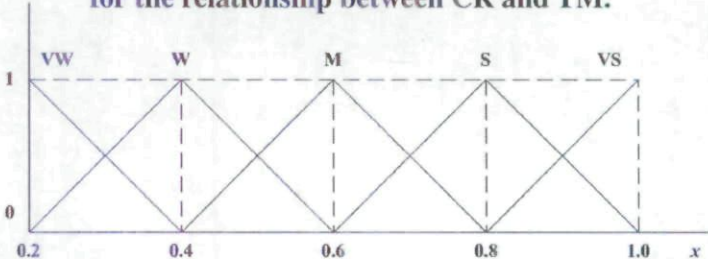


Table 5. Fuzzy importance weights by five customers for each CR

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5
CR1	VL	L	M	H	VH
CR2	VL	M	L	VL	H
CR3	VH	H	VH	M	L

Table 6. The relationship between CR and TM evaluated by a cross-functional team

		TM1	TM2	TM3	TM4	TM5
Member 1	CR1	VW	S	VW	S	W
Member 2		VW	M	VW	VS	VW
Member 3		W	VS	M	S	W
Member 4		M	S	W	VS	M
Member 5		S	VS	W	M	VW
Member 6		VS	M	VS	VW	VS
Member 7		VS	W	VW	VW	VS
Member 1	CR2	VS	S	VS	W	W
Member 2		VS	S	VW	W	W
Member 3		S	VS	S	VS	W
Member 4		M	M	W	W	VW
Member 5		M	M	M	W	S
Member 6		VW	VW	VW	W	W
Member 7		VS	VS	VW	VW	VS
Member 1	CR3	VW	W	M	VW	S
Member 2		S	M	S	S	VS
Member 3		W	W	W	VW	VS
Member 4		S	VS	VS	VS	M
Member 5		VS	VS	VS	W	W
Member 6		M	M	M	VS	S
Member 7		W	W	S	S	M

Table 7. The fuzzy importance weights for each CR

Customer Requirement	Fuzzy Importance Weight
CR1	(0.3,0.5,0.7)
CR2	(0.15,0.3,0.55)
CR3	(0.45,0.7,0.85)

members. The fuzzy relationships are summarized in Table 8. In Step 4, establish the weighted and comprehensive decision matrix, depicted in Table 9, by applying the weighted Min aggregation in Equation 10.

In step 5 calculate the ranking values (average) $\bar{A}_i$ with $i = 1, 2, 3, 4$, and 5 by means of Equation 11. The values of $\bar{A}_1, \bar{A}_2, \bar{A}_3, \bar{A}_4$, and $\bar{A}_5$ for these five technical measures where there are optimistic attitudes are 0.631, 0.655, 0.569, 0.612, and 0.593 respectively. The importance of these five technical measures can be ranked in accordance with their values. The technical measure with the maximum ranking value is the first priority. In this case the priorities for the optimistic attitudes are $TM_2 > TM_1 > TM_4 > TM_5 > TM_3$, with TM_2 being the most important technical measure.

To show how this proposed model works, we have the assumed QFD has only three customer requirements and five technical measures. The methodology developed in this research can be applied to more customer requirements and technical measures, as well as to more people in the decision-making group. Different approaches to deal with 'optimistic' or 'pessimistic' attitudes can be used to extend the model.

Using weighted Min aggregation in QFD group decision-making, the efficiency with which customer requirements can be collected and the relevant relationships analyzed can be improved, to allow groups to make decisions effectively, even with imprecise and vague information.

5. Conclusions

QFD involves two major group decision-making processes--those involved in collecting customer requirements and in analyzing the relationships that exist between these

Table 8. The fuzzy relationship between CR and TM

	TM1	TM2	TM3	TM4	TM5
CR1	(0.457,0.6,0.743)	(0.543,0.743,0.886)	(0.314,0.429,0.6)	(0.514,0.657,0.8)	(0.4,0.543,0.686)
CR2	(0.571,0.743,0.857)	(0.543,0.714,0.857)	(0.371,0.486,0.657)	(0.286,0.457,0.629)	(0.343,0.514,0.686)
CR3	(0.429,0.6,0.771)	(0.429,0.629,0.771)	(0.543,0.743,0.886)	(0.486,0.629,0.771)	(0.543,0.743,0.886)

Table 9. The weighted and comprehensive decision matrix formed by weighted Min aggregation (optimistic attitudes)

Customer Requirement	Technical Measure				
	TM1	TM2	TM3	TM4	TM5
CR1	0.6	0.7	0.5	0.657	0.543
CR2	0.743	0.714	0.657	0.629	0.686
CR3	0.55	0.55	0.55	0.55	0.55

customer requirements and technical measures in cross-functional teams. Different kinds of subjective opinions between members are found quite often in such teams due to the limitations of experience of members and/or a lack of precise information. Different members tend to hold different attitudes towards risk in making the kinds of decisions involved. Under such circumstances, a model is needed to explain group decision-making in QFD under conditions of imprecise information and optimistic risk attitudes on the part of group members. The present investigation has developed and applied one such possible model using fuzzy set theory.

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