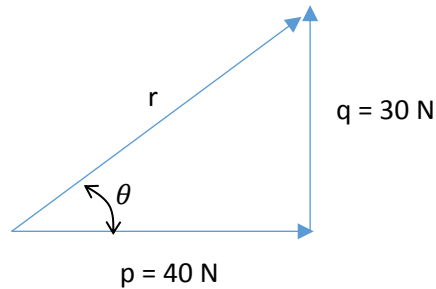


EXERCISES

I. Vector quantities

1. If $\vec{p}$ is a force of 40 N acting in the direction due east and $\vec{q}$ is a force of 30 N acting in the direction due north, what is the magnitude $|\vec{r}|$ and the direction of the vector sum $\vec{r} = \vec{p} + \vec{q}$?

Solution:

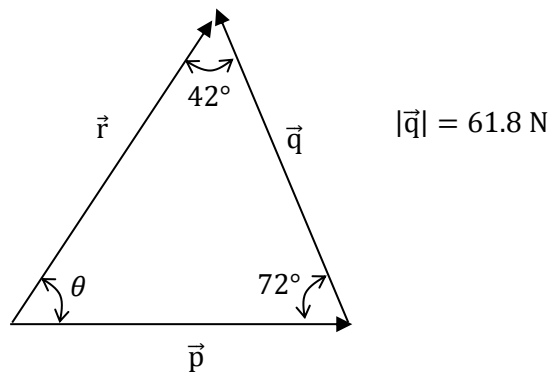


$$r = \sqrt{30^2 + 40^2} = 50 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{30}{40}\right) = 36.87^\circ$$

Therefore $\vec{r}$ is a force of 50 N acting in the direction due north-east making an angle of 36.87° with the horizontal.

2. If $\vec{p}$ is an horizontal force acting to the right and $\vec{q}$ is a force of 61.8 N acting in the direction north-west so it makes an angle of 72° with $\vec{q}$.
What is the magnitude $|\vec{r}|$ of the vector sum $\vec{r} = \vec{p} + \vec{q}$ given the following direction for $\vec{r}$?



Solution:

$$\theta = 180 - (42 + 72) = 66^\circ$$

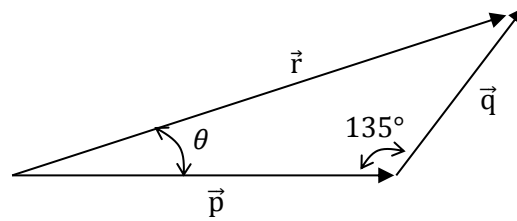
According to sine rule,

$$\frac{r}{\sin 72^\circ} = \frac{61.6}{\sin 66^\circ}$$

$$\therefore r = \frac{61.6 \sin 72^\circ}{\sin 66^\circ} = 64.13 \text{ N}$$

Therefore $\vec{r}$ is a force of 64.13 N acting in the direction due north-east making an angle of 66° with the horizontal.

3. If $\vec{p}$ is an horizontal force of 40 N acting to the right and $\vec{q}$ is a force of 30 N acting in the direction north-east so it makes an angle of 135° with $\vec{p}$ such as:



What is the magnitude $|\vec{r}|$ of the vector sum $\vec{r} = \vec{p} + \vec{q}$?

Solution:

According to cosine rule,

$$r^2 = 40^2 + 30^2 - 2 \times 40 \times 30 \times \cos 135^\circ$$

$$\therefore r = \sqrt{4197.06} = 64.78 \text{ N}$$

According to sine rule,

$$\frac{\sin \theta}{30} = \frac{\sin 135^\circ}{64.78}$$

$$\therefore \theta = \sin^{-1} \left(\frac{30 \sin 135^\circ}{64.78} \right) = 19.11^\circ$$

Therefore $\vec{r}$ is a force of 64.78 N acting in the direction due north-east making an angle of 19.11° with the horizontal.

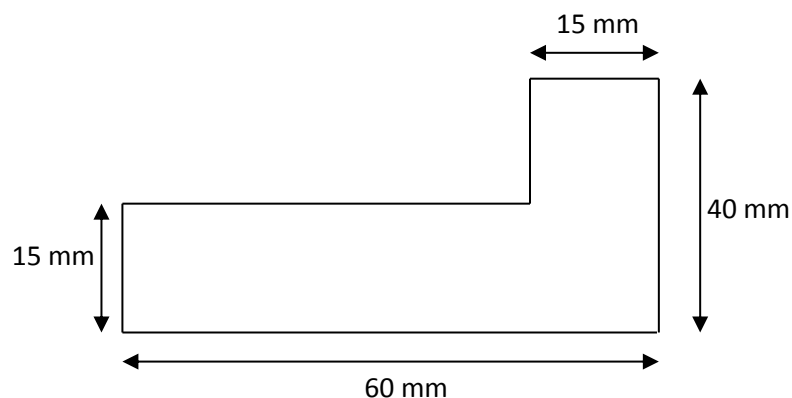
II. Stress and Strain

1. A wrought-iron bar with a diameter of 4 cm is used in a roof truss. If the tensile force supported is 90 kN, what is the direct stress?

Solution:

$$\sigma = \frac{P}{A} = \frac{90 \times 10^3}{\pi \frac{0.04^2}{4}} = \frac{90 \times 10^3}{1.257 \times 10^{-3}} = 71.62 \text{ MPa}$$

2. A steel girder having the cross-section shown below is used to support a compressive force of 500 kN. What is the compressive stress in the steel girder?



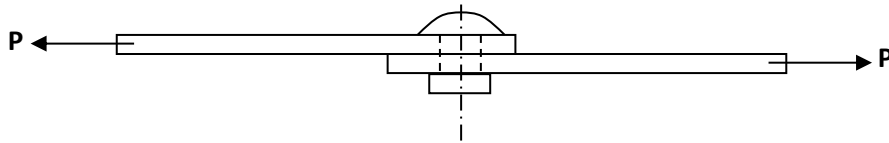
Solution:

$$A = (15 \times 10^{-3}) \times (60 \times 10^{-3} - 15 \times 10^{-3}) + (15 \times 10^{-3}) \times (40 \times 10^{-3})$$

$$\therefore A = 1.275 \times 10^{-3}$$

$$\sigma = \frac{P}{A} = \frac{500 \times 10^3}{1.275 \times 10^{-3}} = 392.16 \text{ MPa}$$

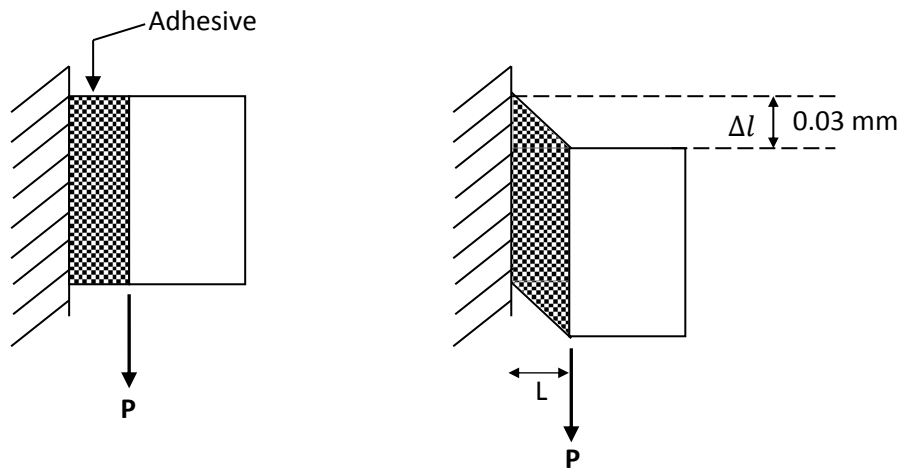
3. The forces P, acting on the plates shown in the following figure are 25 kN each and the rivet has a diameter of 1.8 cm. What is the shear stress in the rivet?



Solution:

$$\tau = \frac{P}{A} = \frac{25 \times 10^3}{\pi \frac{0.018^2}{4}} = \frac{25 \times 10^3}{2.545 \times 10^{-4}} = 98.24 \text{ MPa}$$

4. The following figure shows a component which is attached to a vertical surface by means of adhesive. The component sags by 0.03 mm under the action of a shear force P of 30 N. If the strain is found to be 2×10^{-3} , what is the initial thickness of the adhesive?



Solution:

$$\gamma = \frac{\Delta l}{L} = \frac{0.03 \times 10^{-3}}{L} = 2 \times 10^{-3}$$

$$\therefore L = \frac{\Delta l}{\gamma} = \frac{0.03 \times 10^{-3}}{2 \times 10^{-3}} = 1.5 \text{ cm}$$

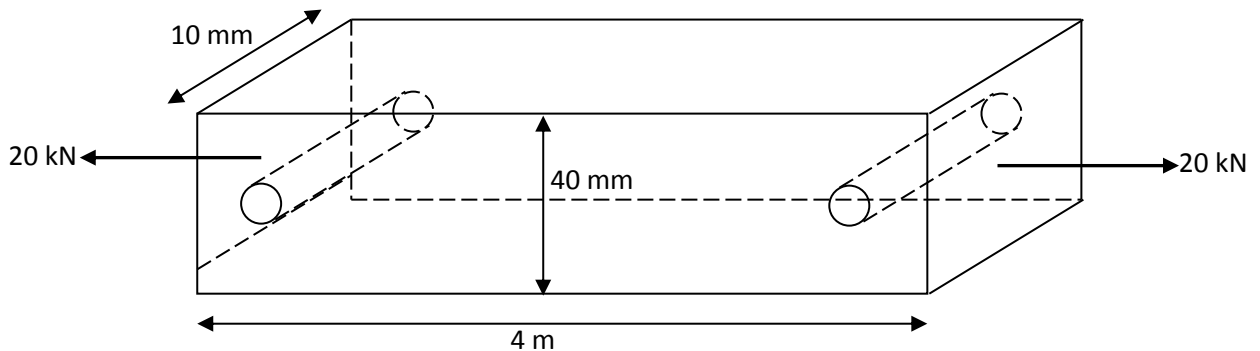
III. Elasticity, Plasticity and Hooke's Law

1. Calculate the strain that will be produced by a tensile stress of 10 N/mm^2 extending a bar of aluminium alloy with a Young's modulus of 70 kN/mm^2 .

Solution:

$$E = \frac{\sigma}{\varepsilon} \quad \therefore \text{strain } \varepsilon = \frac{\sigma}{E} = \frac{10}{70 \times 10^3} = 1.43 \times 10^{-4}$$

2. A tie bar has 2 holes, a distance of 4 m apart. By how much does this distance increase when a tensile load of 20 kN is applied to the tie bar? The tie bar has a rectangular section of $40 \text{ mm} \times 10 \text{ mm}$ and the material of which the tie bar is made has a modulus of Elasticity of 210 kN/mm^2 .



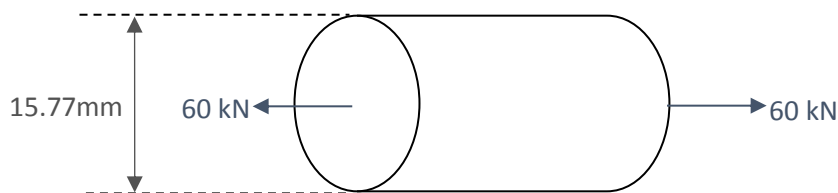
Solution:

$$\text{stress } \sigma = \frac{P}{A} = \frac{20}{40 \times 10} = 0.05 \text{ kN/mm}^2$$

$$\text{strain } \varepsilon = \frac{\delta L}{L} \quad \text{but } E = \frac{\sigma}{\varepsilon} \quad \text{hence } \varepsilon = \frac{\sigma}{E} = \frac{0.05}{210} = 2.38 \times 10^{-4}$$

$$\text{Therefore } \delta L = \varepsilon \cdot L = 2.38 \times 10^{-4} \times 4 = 9.52 \times 10^{-4} \text{ m} = 0.95 \text{ mm}$$

3. In a tensile test on a steel sample, an extensometer recorded an increase in length of 0.117 mm for an applied load of 60 kN. If the diameter and the original gauge length of the sample were 15.77 mm and 80 mm respectively, determine the modulus of elasticity for the steel.



Solution:

$$\text{modulus of elasticity } E = \frac{\text{stress}}{\text{strain}} \quad \therefore E = \frac{\sigma}{\varepsilon} \quad \text{where } \varepsilon = \frac{\delta L}{L}$$

Calculation of strain ε :

$$\varepsilon = \frac{\delta L}{L} = \frac{0.117}{80} = 1.4625 \times 10^{-3}$$

Calculation of stress σ :

$$\sigma = \frac{P}{A} = \frac{60}{\pi \frac{15.77^2}{4}} = 0.3072 \text{ kN/mm}^2$$

Therefore:

$$E = \frac{\sigma}{\varepsilon} = \frac{0.3072}{1.4625 \times 10^{-3}} = 210.04 \text{ kN/mm}^2 = 210.04 \text{ GN/m}^2$$

4. A steel bar $ABCD$ consists of three sections: AB is of 20 mm diameter and 200 mm long, BC is 25 mm square and 400 mm long, and CD is of 12 mm diameter and 200 mm long. The bar is subjected to an axial compressive load which induces a stress of 30 MN/m² on the largest cross-section.

When the load is applied, determine:

- a) The stress in the other two sections of the bar.
- b) The total decrease in the length of the bar.

For steel $E = 210 \text{ GN/m}^2$.

Solution:

• **BC: section 2**

Stress

$$\sigma_2 = \frac{P}{A_2} = \frac{P}{(25 \times 10^{-3})^2} = 30 \text{ MN/m}^2$$

$$\therefore P = 30 \times 10^6 \times (25 \times 10^{-3})^2 \quad \therefore P = 18750 \text{ N}$$

Decrease in length

$$E = \frac{\sigma}{\varepsilon} \quad \text{and} \quad \varepsilon = \frac{\delta L}{L} \quad \text{therefore} \quad E = \frac{\sigma}{\delta L/L} = \frac{\sigma L}{\delta L} \quad \therefore \delta L = \frac{\sigma L}{E}$$

$$\delta L_2 = \frac{\sigma_2 L_2}{E} = \frac{30 \times 10^6 \times 0.4}{210 \times 10^9} = 57.1 \times 10^{-6} \text{ m}$$

- **AB: section 1**

Stress

$$\sigma_1 = \frac{P}{A_1} = \frac{18750}{\frac{\pi}{4} \times (20 \times 10^{-3})^2} = 59.68 \text{ MPa}$$

Decrease in length

$$\delta L_1 = \frac{\sigma_1 L_1}{E} = \frac{59.68 \times 10^6 \times 0.2}{210 \times 10^9} = 56.8 \times 10^{-6} \text{ m}$$

- **CD: section 3**

Stress

$$\sigma_3 = \frac{P}{A_3} = \frac{18750}{\frac{\pi}{4} \times (12 \times 10^{-3})^2} = 165.79 \text{ MPa}$$

Decrease in length

$$\delta L_3 = \frac{\sigma_3 L_3}{E} = \frac{165.79 \times 10^6 \times 0.2}{210 \times 10^9} = 157.9 \times 10^{-6} \text{ m}$$

$$\text{Total decrease in length} = \delta L_1 + \delta L_2 + \delta L_3$$

$$= (56.8 + 57.1 + 157.9) \times 10^{-6}$$

$$= 0.272 \text{ mm}$$

