

MATHEMATICS

FORMULAE BOOKLET

LAWS OF INDICES

Definitions

$$a^n = a \times a \times a \times \cdots \times a \quad (n \text{ times})$$

$$a^0 = 1 \quad (a \neq 0)$$

$$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$$

$$a^{m/n} = \sqrt[n]{a^m} \text{ or } (\sqrt[n]{a})^m \quad (a \geq 0, m \geq 0, n > 0)$$

Combining

Multiplication:

$$a^x a^y = a^{x+y}$$

Division:

$$\frac{a^x}{a^y} = a^{x-y} \quad (a \neq 0)$$

Powers:

$$(a^x)^y = a^{x \cdot y}$$

Distributing $(a \geq 0, b \geq 0)$

$$(a \cdot b)^x = a^x \cdot b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \quad (b \neq 0)$$

Careful!!

$$(a + b)^n \neq a^n + b^n$$

$$(a - b)^n \neq a^n - b^n$$

LOGARITHM RULES

For the following, assume that x, y, a , and b are all positive. Also assume that $a \neq 1, b \neq 1$.

Definitions

1. $\log_a x = N$ means that $a^N = x$.
2. $\log x$ means $\log_{10} x$. All $\log_a$ rules apply for $\log$. When a logarithm is written without a base it means common logarithm.
3. $\ln x$ means $\log_e x$, where e is about 2.718. All $\log_a$ rules apply for $\ln$. When a logarithm is written "ln" it means natural logarithm.

Rules

1. Inverse properties:
 $\log_a a^x = x$ and $a^{(\log_a x)} = x$
2. Product:
 $\log_a (x \cdot y) = \log_a x + \log_a y$
3. Quotient:
 $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
4. Power:
 $\log_a (x^p) = p \cdot \log_a x$
5. Change of base formula:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Careful!!

$$\log_a (x + y) \neq \log_a x + \log_a y$$

$$\log_a (x - y) \neq \log_a x - \log_a y$$

FACTORING RULES

DIFFERENCE OF SQUARES

$$a^2 - b^2 = (a + b)(a - b)$$

SUM OF SQUARES

$$a^2 + b^2 = (a + b + \sqrt{2ab})(a + b - \sqrt{2ab})$$

This identity is always true, but it's useful for factoring only **when 2AB is a perfect square**.

FORM $a^2 - (r + s)a + r.s$

$$a^2 - (r + s)a + r.s = (a - r)(a - s)$$

FORM $a^2 \pm 2ab + b^2$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

SUM AND DIFFERENCE OF CUBES

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

QUADRATIC FORMULA

If we consider a typical quadratic equation:

$$ax^2 + bx + c = 0$$

The solution of such an equation is given by the following **quadratic formula**:

$$x = \frac{-b \pm \sqrt{\text{discriminant}}}{2a} \quad \text{where the discriminant } \Delta = b^2 - 4ac$$

❖ If the discriminant is positive there are two real roots (i.e. solutions) x_1 and x_2 :

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Thus the quadratic equation can be factorised as follows:

$$ax^2 + bx + c = a \cdot \left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \cdot \left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right)$$

❖ If the discriminant is zero, there is exactly one distinct root:

$$x = \frac{-b}{2a}$$

Thus the quadratic equation can be factorised as follows:

$$ax^2 + bx + c = a \cdot \left(x + \frac{b}{2a} \right)^2$$

❖ And if the discriminant is negative, there are no real roots.

PARTIAL FRACTIONS

Provided that the **numerator** $f(x)$ is of **less degree than the denominator**, the following illustrate the form of partial fractions.

Linear Factors:

$$\frac{f(x)}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$$

Repeated Linear Factors:

$$\frac{f(x)}{(x+a)^n} = \sum_{i=1}^n \frac{A_i}{(x+a)^i}$$

Quadratic Factor:

$$\frac{f(x)}{ax^2+bx+c} = \frac{Ax+B}{ax^2+bx+c}$$

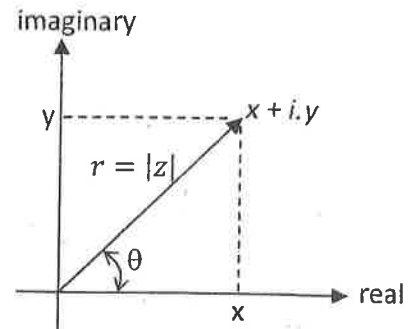
COMPLEX NUMBERS

The complex number z has the form

$$z = x + i.y$$

Where:

- $i^2 = -1$
- x is called the real part Re
- y is called the imaginary part Im



In terms of polar coordinates:

- $x = r \cos \theta$
- $y = r \sin \theta$

Therefore:

$$z = x + i.y = r(\cos \theta + i. \sin \theta)$$

Using Euler's formula

- $e^{i\theta} = \cos \theta + i. \sin \theta$
- $e^{-i\theta} = \cos \theta - i. \sin \theta$

The complex number z can be written:

$$z = x + i.y = r(\cos \theta + i. \sin \theta) = re^{i\theta}$$

Where:

- Modulus $r = |z| = \sqrt{x^2 + y^2}$
- Argument $\theta = \arg(z) = \tan^{-1} \frac{y}{x}$

Multiplication

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

Division

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

MATRICES

Basic Operations

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, and k a constant number

Then:

$$A + B = \begin{bmatrix} a + e & b + f \\ c + g & d + h \end{bmatrix}$$

$$A - B = \begin{bmatrix} a - e & b - f \\ c - g & d - h \end{bmatrix}$$

$$k \times A = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

$$A \times B = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

Transpose

If M is the general $m \times n$ matrix then the $n \times m$ matrix obtained from M by interchanging the rows and columns is called the transpose of M , written M^T (in some books M').

Determinants

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Inverse

$$A^{-1} = \left(\frac{1}{\det A} \right) \text{adj } A$$

Where the adjoint of A , denoted by $\text{adj } A$, is the transpose of the matrix of cofactors.

$$\therefore A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ for } a 2 \times 2$$

Differentiation Formulae

$y = f(x)$	dy/dx
c	0
x^n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
$\sin(x)$	$\cos x$
$\sin(kx)$	$k \cos(kx)$
$\cos(x)$	$-\sin(x)$
$\cos(kx)$	$-k \sin(kx)$
$\tan(kx)$	$k \sec^2(kx)$
$\ln x$	$\frac{1}{x}$

The Product Rule.

$$\text{If } y = u \cdot v, \quad \frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

The Quotient Rule.

$$y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

The Chain Rule

$$\text{If } y = f[g(x)]$$

$$\frac{dy}{dx} = f'[g(x)] \cdot g'(x)$$

$$f' \cdot g' \text{ (The Derivatives w.r.t } x)$$

Integration Formulae

$f(x)$	$\int f(x) dx$
x^n	$\frac{x^{n+1}}{n+1} + C \quad \forall n \neq -1$
$\frac{1}{x}$	$\ln x + C$
e^{kx}	$\frac{e^{kx}}{k} + C$
$\cos(kx)$	$\frac{\sin(kx)}{k} + C$
$\sin(kx)$	$-\frac{\cos(kx)}{k} + C$
$\sec^2(kx)$	$\frac{\tan(kx)}{k} + C$

Integration by Parts

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx}$$

Integration by Substitution

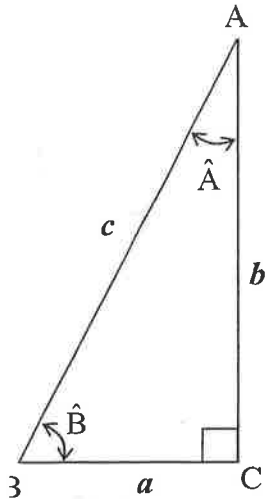
$$\int f[g(x)] \cdot g'(x) dx = \int f(u) \cdot du \quad \text{letting } u = g(x)$$

$$\begin{aligned} \int_a^b f(x) dx &= [F(x)]_a^b \\ &= \underline{F(b) - F(a)} \end{aligned}$$

TRIGONOMETRY

Trigonometric Ratios in Right-Angled Triangles

Given the right-angled triangle ABC as shown in the opposite figure, we define the trigonometric ratios as:



S O C A T O
H H A

$$\sin \hat{A} = \frac{\text{side opposite to angle } \hat{A}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos \hat{A} = \frac{\text{side adjacent to angle } \hat{A}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan \hat{A} = \frac{\text{side opposite to angle } \hat{A}}{\text{side adjacent to angle } \hat{A}} = \frac{a}{b}$$

$$\sin \hat{B} = \frac{\text{side opposite to angle } \hat{B}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\cos \hat{B} = \frac{\text{side adjacent to angle } \hat{B}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\tan \hat{B} = \frac{\text{side opposite to angle } \hat{B}}{\text{side adjacent to angle } \hat{B}} = \frac{b}{a}$$

$$\sin \hat{A} = \sin(180^\circ - \hat{A})$$

and

$$\cos \hat{A} = \cos(360^\circ - \hat{A})$$

Solution of Non-Right-Angled Triangles

The Sine Rule:

The sine rule is used when given:

1. Two angles and any side.

2. Two sides and an angle opposite to one of the given sides.

THE AMBIGUOUS CASE

The sine rule states:

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

The Cosine Rule:

The cosine rule is used when given:

- Two sides and the angle between them.
- All three sides.

The cosine rule states:

$$\begin{aligned} a^2 &= b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \hat{A} \\ b^2 &= a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \hat{B} \\ c^2 &= a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \hat{C} \end{aligned}$$

