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School of Engineering



PLC Assignment 1

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Electrical and Electronic Engineering HND

2023 / 2024

PLCs – GOEE5002

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Introduction

To properly specify a feedback control system, it is necessary to understand the basic underlying principles that govern the functioning of such systems. This report will demonstrate the understanding of said principles.

The report will contain information on PID (Proportional Integral Differential) control in a PLC system, creating a signal flow diagram from the block diagram of a system and the Mason's Gain formula of said signal flow diagram, State Space Modelling with an example, Root Locus analysis with an example, and information on Bode diagrams and Nicholls Charts.

Main Body

Task 1

PID Control

Programmable Logic Controllers (PLCs) are industrial computers with multiple inputs and outputs that control and monitor industrial equipment based on specific programming (inductive automation, 2020). Certain PLCs have only basic inputs and outputs (I/O) and are therefore more modular in nature, but they can be tailored to match various industrial applications by adding more back planes and functional modules.

A PID (Proportional Integral Derivative) controller is different to a PLC (Eurotherm, 2024). It requires inputs and outputs to receive information and send signals like a PLC, but it contains specialist algorithms designed to control a process with one or multiple control loops. When implemented in a PLC system, it provides precise and automated control.

The proportional component compares the feedback signal to the Setpoint value, and the resulting error value is then multiplied with a proportional constant to get an output that's proportional to the instantaneous error, such that if the magnitude of the current error is zero, then the output of the Proportional component of the PID will be zero (PDF Supply, 2024).

The integral component is required to eliminate the steady-state error that exists with the proportional component and integrates the steady-state error over a duration of time till the magnitude of the error signal reaches zero.

The derivative component overcomes the problem of the integral component reacting whenever the Setpoint is changed. Its output depends on the instantaneous rate of change of the system error in relation to time, which is then multiplied by the derivative constant.

By combining the outputs of the three components, the PID algorithm outputs a controlled output that's applied to the final control devices to alter the output of the system as desired.

Ziegler-Nichols Tuning Methods

The Ziegler-Nichols tuning methods are widely used techniques for tuning the parameters of a PID controller. These methods were developed by John G. Ziegler and Nathaniel B. Nichols in the 1940s. There are two primary Ziegler-Nichols tuning methods: the Ziegler-Nichols step response method and the Ziegler-Nichols frequency response method.

The Ziegler-Nichols step response method provides a practical and intuitive approach to tuning PID controllers (Ellis, 2012). By observing the system's behaviour directly, it allows for quick adjustment of controller parameters to achieve desired performance characteristics. However, like any tuning method, it has limitations and may not be suitable for all types of systems.

The Ziegler-Nichols frequency response method, unlike the Ziegler-Nichols step response method, which observes the system's step response, analyses the system's frequency response characteristics.

The Ziegler-Nichols tuning methods do however have their limitations. These limitations include higher overshoot, rise time, and settling time in the system operation (inca tools, 2024)

Task 2

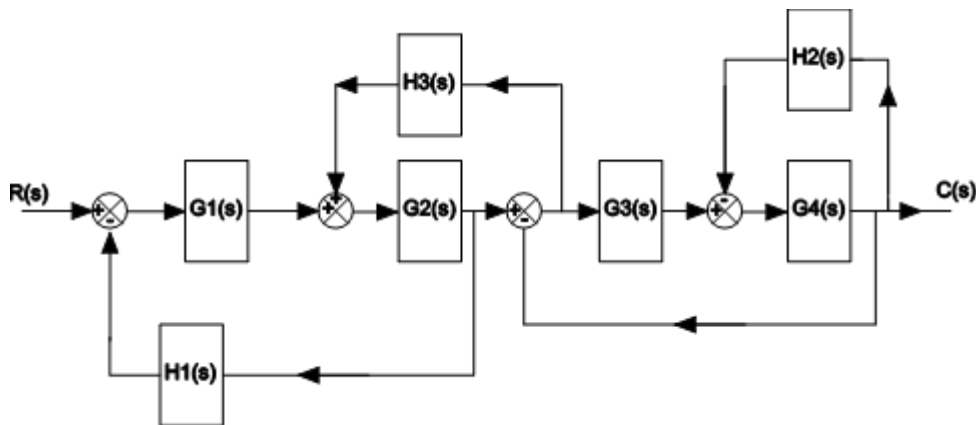
Signal Flow Diagram and Mason's Gain

Control systems are often represented by signal flow diagrams. A signal flow diagram is produced that corresponds with the system block diagram provided and then Mason Gain formula will be applied to derive a transfer function for the system.

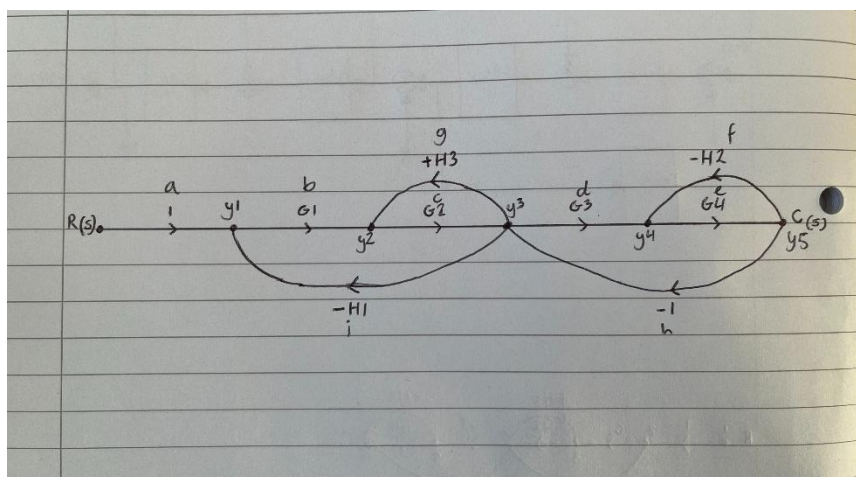
A signal flow diagram is a graphical representation of the relationships between different variables in a system (Tutorialspoint, 2024). It is a useful way to visualize the flow of signals through a system and how various components interact with each other. Block diagrams can be converted into signal flow diagrams by transforming the signals, variables, summing points and take-off points of block diagram into nodes, blocks into branches, and transfer functions as gains.

Mason's Gain Formula is a method used to determine the overall transfer function of a system represented by a signal flow graph (Khan, 2024). It simplifies the analysis of complex systems by accounting for the interactions between various paths and loops within the graph.

Below is the block diagram provided.



A signal flow diagram was created for this block diagram, which can be seen below.



As seen in the signal flow diagram, the summing points been changed to nodes, and the blocks have been made into branches. Each of the nodes have been labelled y_1 , y_2 , y_3 and so on in order to more easily refer to them when calculating Masson's gain for the signal flow diagram.

With the block diagram being turned into a signal flow diagram, the Mason's gain can be calculated. The Mason's gain formula is represented as follows:

$$T = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^N P_i \Delta_i}{\Delta}$$

Where:

- $C(s)$ = output node
- $R(s)$ = input node
- T = the transfer function or gain between $C(s)$ and $R(s)$.
- P_i = the i^{th} forward path gain
- N = number of forward paths
- $\Delta = 1 - (\text{sum of all individual loop gains}) + (\text{sum of gain products of all possible non-touching loops}) - (\text{sum of gain products of all possible three non-touching loops}) + \dots$
- Δ_i = path factor for the i^{th} path, this is obtained from Δ by removing the loops which are touching the i^{th} forward path

In the case of this signal flow graph, the calculations are as follows:

$$N = 1$$

$$P_1 = y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4$$

$$\text{Number of Individual Loops} = L = 4$$

$$L_1 = y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_1, \quad \text{loop gain} = bci$$

$$L_2 = y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_3, \quad \text{loop gain} = deh$$

$$L_3 = y_2 \rightarrow y_3 \rightarrow y_2, \quad \text{loop gain} = cg$$

$$L_4 = y_4 \rightarrow y_5 \rightarrow y_4, \quad \text{loop gain} = ef$$

$$\text{Number of two non-touching loops} = 2$$

$$L_1 L_4 = y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_1, \quad y_4 \rightarrow y_5 \rightarrow y_4, \quad \text{loop gain} = bcief$$

$$L_3 L_4 = y_2 \rightarrow y_3 \rightarrow y_2, \quad y_4 \rightarrow y_5 \rightarrow y_4, \quad \text{loop gain} = cgef$$

A higher number of non-touching loops is not present in this signal flow diagram.

$$\Delta = 1 - (bci + deh + cg + ef) + (bcief + cgef) - (0)$$

$$= 1 - (bci + deh + cg + ef) + (bcief + cgef)$$

As there are no loops that are not touching the first forward path,

$$\Delta_1 = 1$$

As $N=1$, the formula will be substituted as follows:

$$T = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^N P_i \Delta_i}{\Delta}$$

$$= \frac{P1\Delta1}{\Delta}$$

When substituting the necessary values, th formula will look like this:

$$T = \frac{C(s)}{R(s)} = \frac{(abcde)1}{1 - (bci + deh + cg + ef) + bcief + cgef}$$

Therefore, the transfer function is

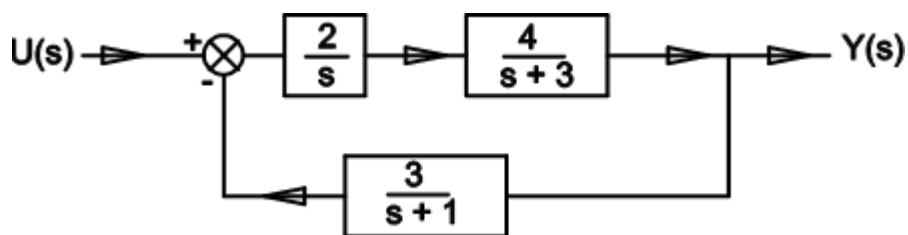
$$T = \frac{C(s)}{R(s)} = \frac{abcde}{1 - (bci + deh + cg + ef) + bcief + cgef}$$

Task 3

State Space Modelling

State space modelling is a mathematical framework used to describe and analyse dynamic systems.. It represents a system's state and evolution over time through state variables, described as unobservable quantities that define the internal state of the system, and equations, enabling the study and control of systems across various disciplines such as engineering, economics, and environmental science (Kotzé, n.a.).

A state space model has been obtained for the system below.



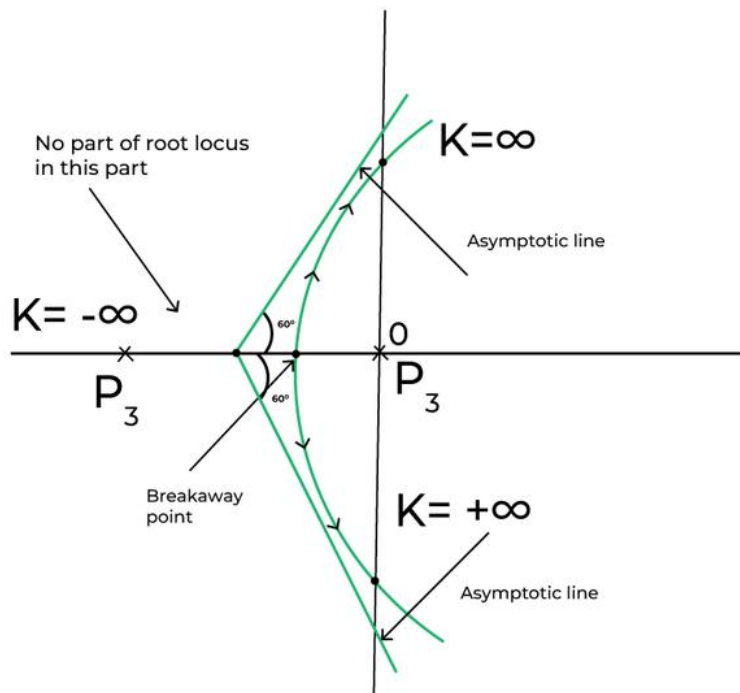
A state space model has also been obtained for the system shown below.

$$\begin{aligned} \dot{X} &= \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ Y &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad D = 0 \end{aligned}$$

Task 4

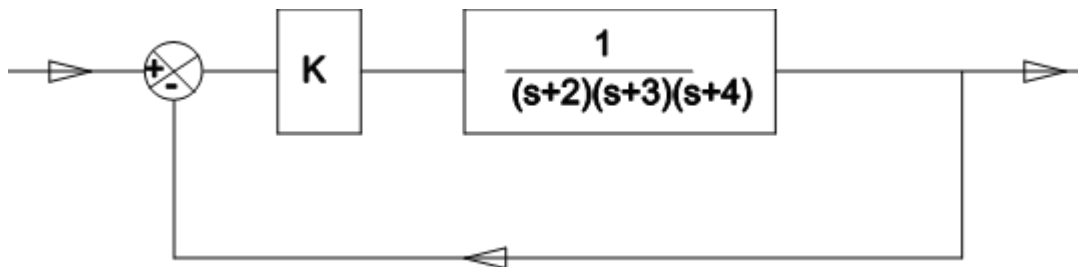
Root Locus Analysis

Time domain analysis can be examined using root locus analysis. Root locus analysis is a graphical technique used in control systems to investigate the changes in the roots of the characteristic equation of a system as a particular parameter (Electronics Coach, 2024), typically the gain K , is altered (Iqbal, n.a.). This method is very helpful for examining a feedback control system's stability and transient response.



Root Locus Diagram: (Geeks for Geeks, 2023)

A root locus diagram for the system below can be created in order to determine the value of K that will cause the system to become unstable.



The root locus equation is as follows:

$$\text{Root Locus} = \frac{C(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)}$$

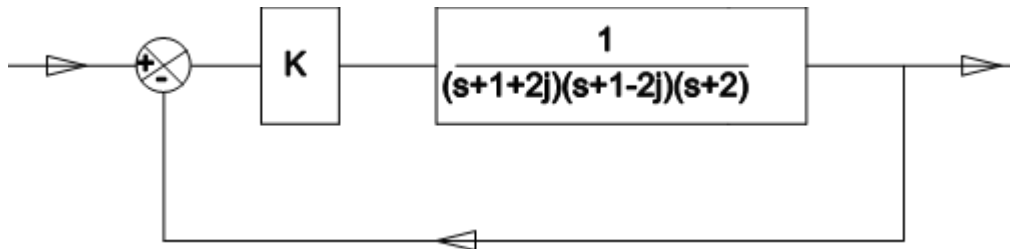
$$\frac{1}{(s+2)(s+3)(s+4)} = G(s)$$

$$\begin{aligned}
\frac{KG(s)}{1+KG(s)} &= \frac{K/(s+2)(s+3)(s+4)}{(1+K(s+2)(s+3)(s+4)) \times 1} \\
&= \frac{K/(s+2)(s+3)(s+4) \times (s+2)(s+3)(s+4)}{(1+K(s+2)(s+3)(s+4)) \times (s+2)(s+3)(s+4)} \\
&= \frac{K}{(s+2)(s+3)(s+4) + K} \\
&= \frac{K}{(s^2 + 3s + 2s + 6)(s+4) + K} \\
&= \frac{K}{(s^2 + 5s + 6)(s+4) + K} \\
&= \frac{K}{s^3 + 4s^2 + 5s^2 + 20s + 6s + 24 + K} \\
&= \frac{K}{s^3 + 9s^2 + 26s + 24 + K}
\end{aligned}$$

If:

$$s^3 + 9s^2 + 26s + 24 + K = 0$$

A root locus diagram can also be created for the system below.

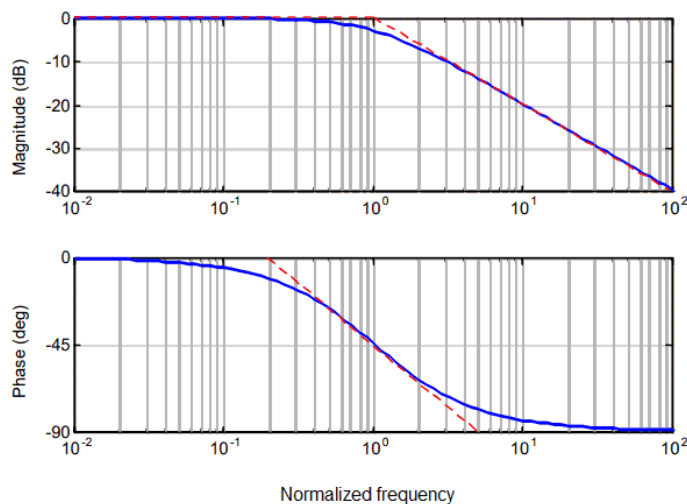


Task 5

Frequency domain can be analysed using Bode diagrams and Nichols charts, which are explained in-depth below.

Bode Diagram

The bode plot is named after its founder Hendrik Wade Bode while he was working for Bell Labs in the United States in the 1930s (Rhode & Schwarz, 2024). A Bode plot is a graph that depicts the frequency dependence of a system's gain and phase. It is frequently utilized in control systems analysis and design to aid in comprehending a system's behaviour across various frequencies. The bode plot consists of two graphs, called the magnitude plot and the phase plot.



Bode diagram: (Cadence, n.a.)

In a magnitude plot, the vertical axis typically represents gain while the horizontal axis represents frequency. This plot allows you to see how the gain of the system varies with frequency (Electrical4U, 2024).

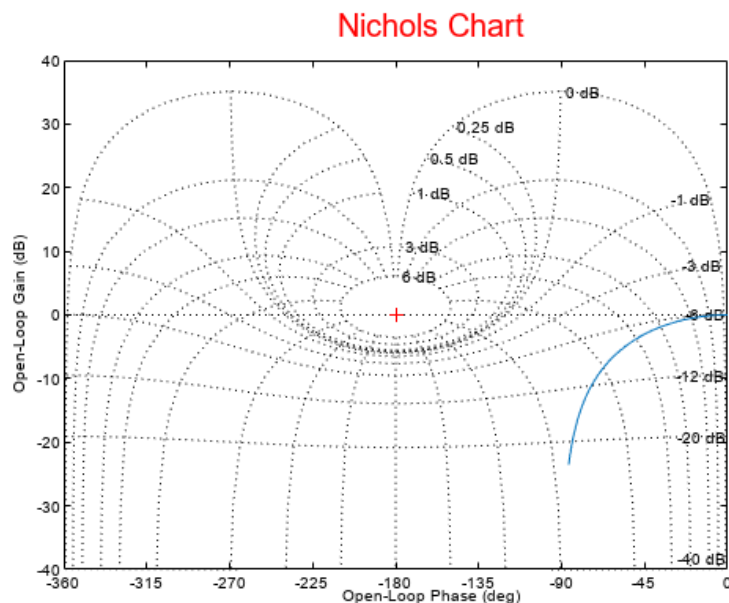
In a phase plot, the vertical axis represents phase angle and the horizontal axis yet again represents frequency. This plot allows you to see how the phase shift of the system changes with frequency.

The reasons to use the bode plot are as follows (Rhode & Schwarz, 2024):

- Bode plots provide a detailed understanding of a system's behaviour across a wide range of frequencies.
- They enable engineers to ensure stable system operation.
- Bode diagrams aid in designing and tuning control systems by providing insights into parameters such as bandwidth, resonance, and transient response.
- They help in identifying key system properties like poles, zeros, and resonant frequencies, crucial for accurate system modelling and analysis.
- Bode diagrams are effective educational tools for teaching concepts related to frequency domain analysis and control system design.

Nichols Chart

The Nichols plot is named after its founder, Nathaniel B. Nichols in 1947, as the superior practical method for feedback loop design and test (Seibt, 2014). The Nichols plot is a graph used in control system engineering to assess stability and performance of a feedback system in frequency domain. It shows the transfer function's phase and gain in relation to frequency of a system. It is helpful in conjunction with Bode graphs to evaluate stability margins and system performance.



Nichols Chart: (MathWorks, n.a.)

In a Nichols chart, the horizontal axis represents the phase angle, and the vertical axis represents the gain. The plot typically includes contours of constant gain and constant phase.

The reasons to use the Nichols chart are as follows (Ellis, 2012):

- Nichols charts offer a concise representation of a system's gain and phase behaviour across frequencies.
- They facilitate the determination of stability margins, providing insights into system stability and robustness.
- Nichols charts allow for easy comparison between different systems or design variations.
- They are useful for designing control systems in the frequency domain and helping engineers achieve desired performance specifications.

Conclusion

In conclusion, when it comes to feedback control systems and PLCs, there are a multitude of principles that exist that govern the functioning of the systems.

These principles include PID control, system representation with signal flow diagrams, Mason's gain, and state space modelling, time domain analysis with root locus, and frequency domain analysis with Bode diagrams and Nichols charts. In order to understand the specifications of a feedback system it is necessary to have an understanding of such principles and be able to apply them to various different systems.

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