



University of Wales Trinity Saint David

School of Engineering

Calculation on Piston, Rod, Crankshaft
Assembles

Advanced Engine Design M6X00410

Zhangwei Guo

1805078

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Notation of Engine Data

Quantity	Notation	Production Engine	Unit
Mass of Piston	M pis	0.350	kg
Radius of the Crank	R	0.047	m
Length of Rod	L	0.134	m
Length of Shank	L Shank	0.100	m
Diameter of Bore	D	0.079	m
Cross-Sectional Area	CSA	195	mm ²
Youngs Modules of Bolt	E Bolt	210000	N/mm ²
Youngs Modules of Cap	E Cap	170000	N/mm ²

1 (a)

First of all, the load applied to the gudgeon pin needs to be calculated from gas force and inertia force.

Which gas force could be obtained from equation (1) and (2) as inertia force coming from equation (3) (McDonald, 2020).

$$Force = Pressure \times Area \quad (1)$$

$$Max\ Gas\ Force = Peak\ Pressure \times Bore\ Area \quad (2)$$

From

$$F_{inertia} = M_{pin} \times Acc\ and\ Acceleration = R\omega^2 \cos\theta + \left(\frac{R}{l}\right) R\omega^2 \cos 2\theta$$

$$F_{inertia} = M_{pin} (R\omega^2 \cos\theta + \left(\frac{R}{l}\right) R\omega^2 \cos 2\theta) \quad (3)$$

$$1st\ Order\ Inertia = M_{piston} R\omega^2 \cos\theta$$

$$2nd\ Order\ Inertia = M_{piston} \left(\frac{R}{l}\right) R\omega^2 \cos 2\theta$$

In which

$$M_{pin} = length\ \pi\ (R_2^2 - R_1^2)\ density$$

Length equals to 25mm, R2=10mm, R1=6mm (Bosch, 2014).

After that, net force comes out as follows:

$$F_{Net} = F_{Gas} - F_{Inertia} \quad (4)$$

From last assignment, the maximum net force appears at 24 degrees when RPM is 5000 is 42180 N on the standard engine.

Named the right connecting point at the end of the gudgeon pin is A, so the bending moment at A is clockwise during firing from $F_{Net}/2$ which is 21090 N while inertia force has an effect of 580 N of anti-clockwise on the gudgeon pin.

Assume the distance from load of $F_{Net}/2$ to A as L1 equals to 15mm and distance from inertia force to A as L2 equals to 10mm.

So net bending moment at A can be calculated from equation (5) as 310555 N*mm.

$$Bending\ Moment_{Net} = F_{Net}/2 \times L1 - F_{Inertia} \times L2 \quad (5)$$

$$Bending\ Stress_A = M_A \times R_2 / I \quad (6)$$

Bending stress at A is 454 N/mm² when $I = \pi \times (R2^4 - R1^4) / 64 = 6836 \text{ mm}^4$.

$$Shear\ Stress_A = \frac{\frac{F_{Net} - F_{Inertia}}{2}}{\pi \times (R2^2 - R1^2)} \quad (7)$$

Shear stress at A is 102 N/mm² from formula (7) (McDonald, 2020).

$$\sigma_{Combined} = (\sigma_A^2 + (2T)^2)^{0.5} \quad (8)$$

Combined stress at A is 497 N/mm² from formula (8).

When it comes to TDC, $F_{Net}/2$ is 6521 N as inertia force is 911.

So bending stress is -156 N/mm² and shear stress is -37 N/mm². Combined stress could be obtained as -160 N/mm².

$$\sigma_{Max} = 497 \text{ N/mm}^2$$

$$\sigma_{Min} = -160 \text{ N/mm}^2$$

$$Mean\ Stress = \frac{\sigma_{Max} + \sigma_{Min}}{2} = 168.5 \text{ N/mm}^2$$

$$Range = \sigma_{Max} - \sigma_{Min} = 657 \text{ N/mm}^2$$

Tool steel endurance limit equals to 0.45 times UTS. So $EL = 0.45 \times 1000 = 450 \text{ N/mm}^2$ when $UTS = 1000 \text{ N/mm}^2$.

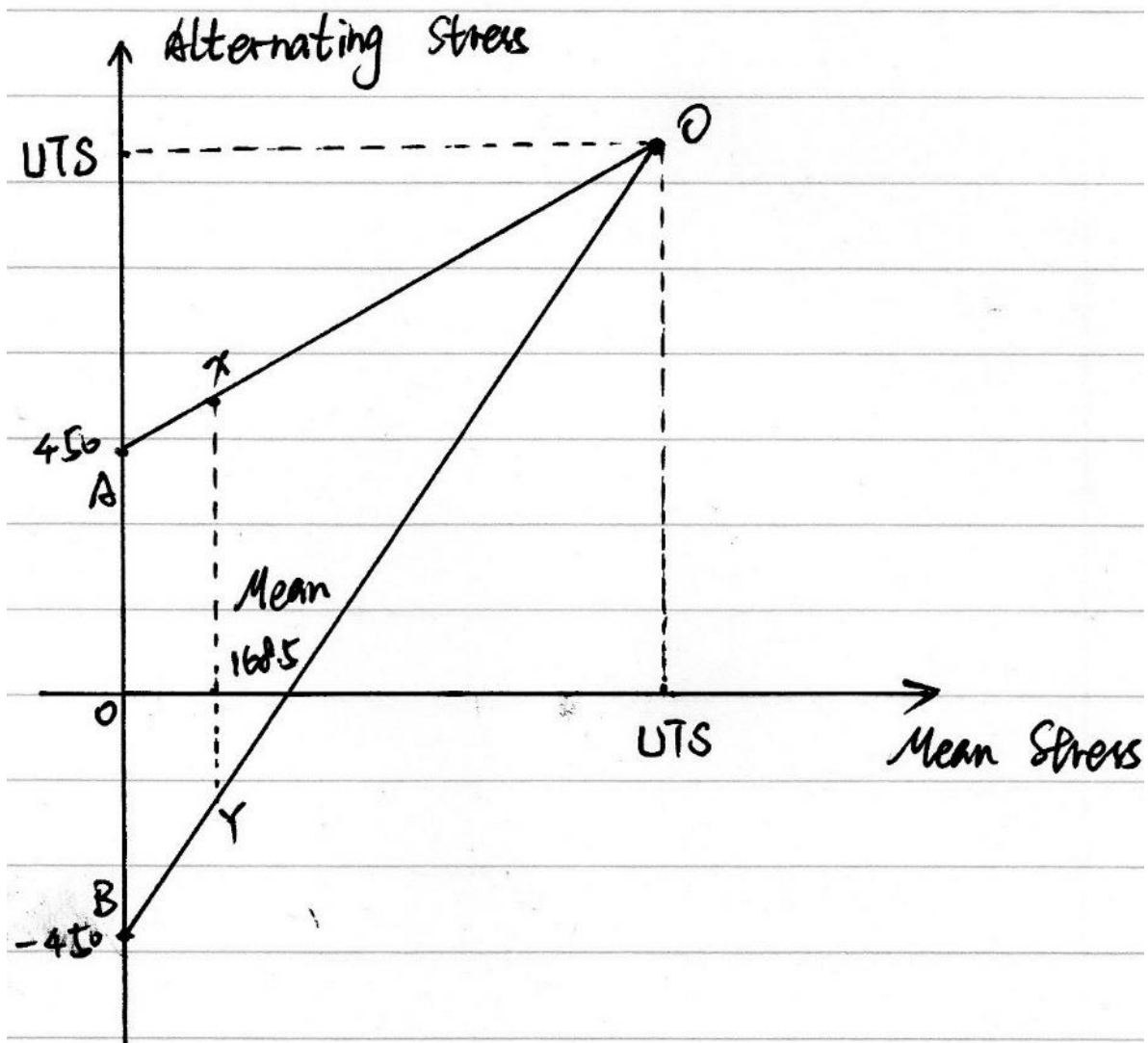


Figure.1 Goodman Diagram of Gudgeon Pin

As shown in figure 1,

$$XY/AB=OX/OA=(UTS-MEAN)/UTS$$

So allowance range at mean which is shown like XY is 748.35 N/mm².

$$\text{Fatigue Safety Factor} = \text{Allowance Range} / \text{Actual Range} \quad (9)$$

FSF of gudgeon pin is 1.14 from formula (9) (McDonald, 2020).

2 (a)

For the force within rod at 5000 RPM, from last assignment, the maximum net compressive force is 37968 N while the maximum net tensile force is 4952 N. Assume UTS as 750 N/mm² and stress concentration factors as 1.50 (Bosch, 2014).

2.1 Point A

For the section area of point A, area of the outside each side = 13x4 = 52mm² and the middle area of each side is 30 mm² (McDonald, 2020). So the total section area at point A is 164 mm².

$$\text{Reciprocating Mass} = \text{Piston} \times \text{density} \times w \times \text{area} \quad (10)$$

$$\sigma_{\text{Max nominal}} = \frac{4952}{164} = 30 \text{ N/mm}^2$$

$$\sigma_{\text{Min nominal}} = -\frac{37968}{164} = -231 \text{ N/mm}^2$$

$$\text{Mean Strss} = \frac{\sigma_{\text{Max nominal}} + \sigma_{\text{Min nominal}}}{2} = -100.5 \text{ N/mm}^2$$

$$\text{Range} = \sigma_{\text{Max nominal}} - \sigma_{\text{Min nominal}} = 261 \text{ N/mm}^2$$

The endurance limit without radiuses = UTS x 0.45 = 750 x 0.45 = 337.5 N/mm².

Final endurance limit with radiuses = UTS x 0.45 / K = 337.5 / 1.5 = 225 N/mm².

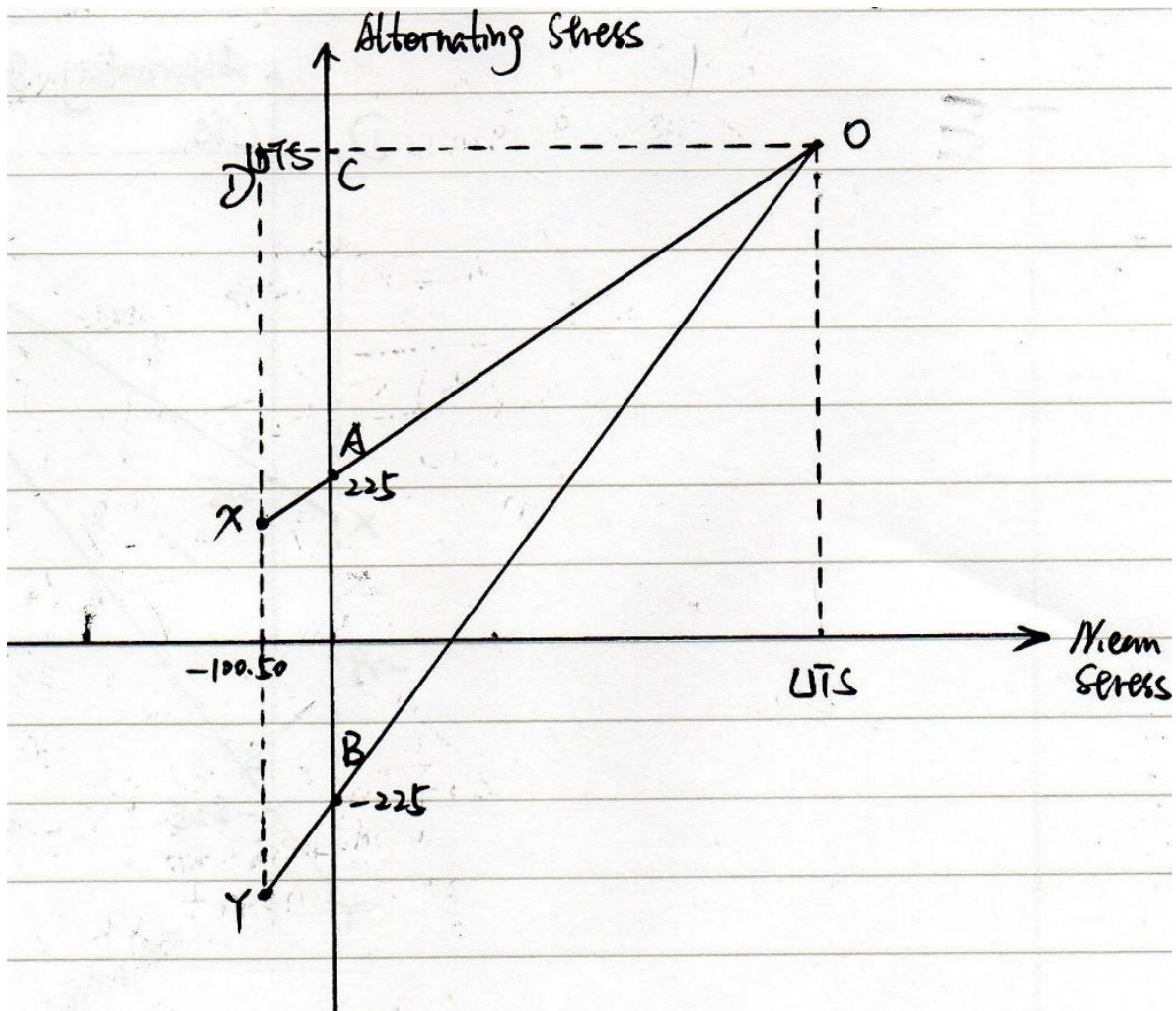


Figure.2 Goodman Diagram of Point A

As shown in figure 2,

$$XY/AB = OX/OA = OD/OC = (UTS-MEAN)/UTS$$

So allowance range at mean which is shown like XY is 510.3 N/mm².

FSF of point A could be calculated with formula (9) as 1.9 with actual range of 261 N/mm².

2.2 Point B

The sectional area of point B is 279 mm². The maximum compressive force is reduced by 1621 N to 36347 N which is similar to inertia force going up on account of the maximum compressive force is at lower RPM while the maximum tensile force is 11650 N.

$$\text{Reciprocating Mass} = \text{Piston(Pin)} + \frac{1}{3}\text{Rod} \quad (11)$$

$$\sigma_{Max} = \frac{11650}{279} = 42 \text{ N/mm}^2$$

$$\sigma_{Min} = -\frac{36347}{279} = -130 \text{ N/mm}^2$$

$$\text{Mean Strss} = \frac{\sigma_{Max} + \sigma_{Min}}{2} = -44 \text{ N/mm}^2$$

$$\text{Range} = \sigma_{Max} - \sigma_{Min} = 172 \text{ N/mm}^2$$

The endurance limit without radiuses = UTS x 0.45 = 750 x 0.45 = 337.5 N/mm².

Final endurance limit with radiuses = UTS x 0.45 / K = 337.5 / 1.5 = 225 N/mm².

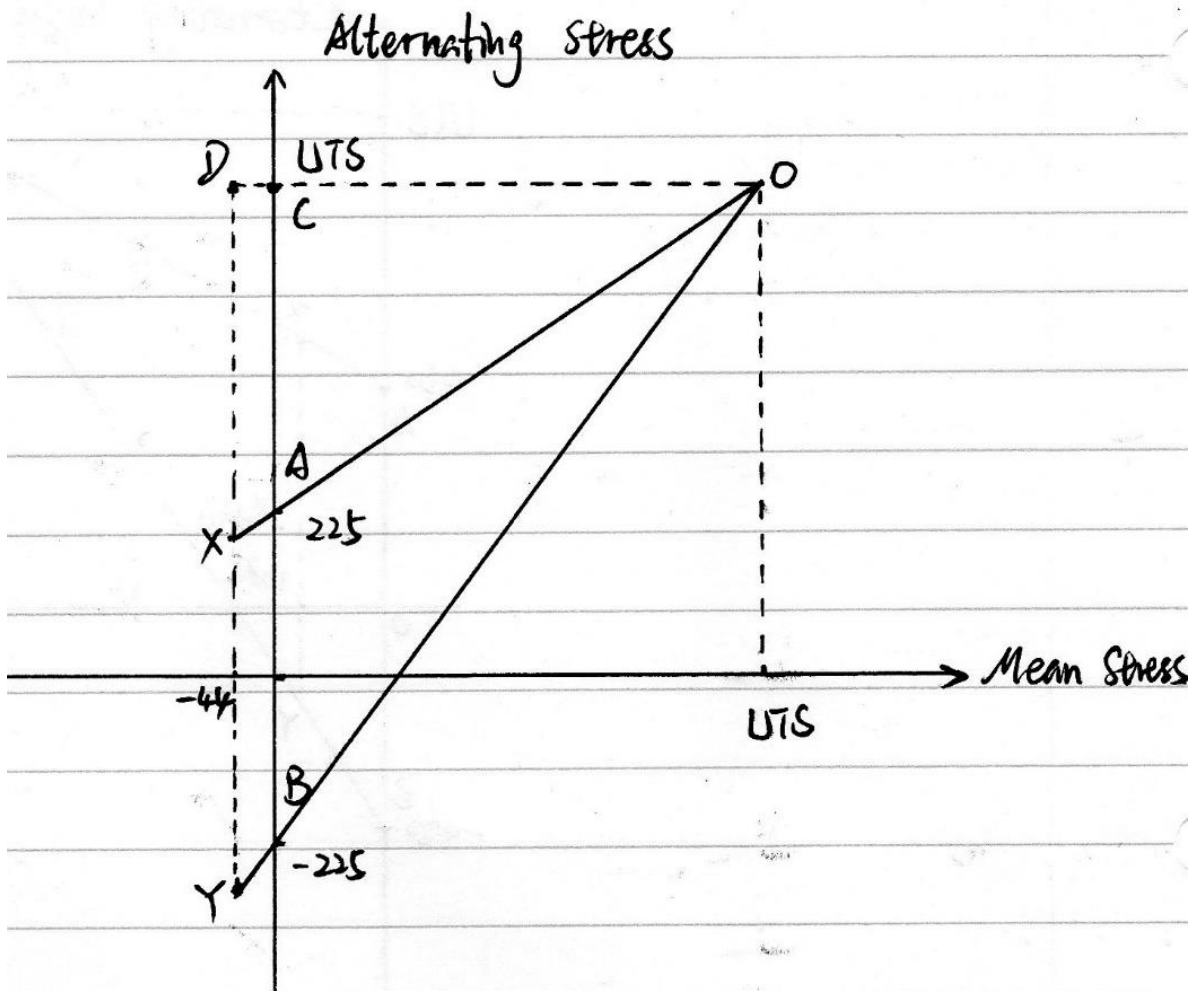


Figure.3 Goodman Diagram of Point B

As shown in figure 3,

$$XY/AB = OX/OA = OD/OC = (UTS - MEAN)/UTS$$

So allowance range at mean which is shown like XY is 476.4 N/mm².

FSF of point A could be calculated with formula (9) as 2.7 with actual range of 172 N/mm².

2.3 Point C

The cap cross sectional area of point C = $16 \times 9 = 144 \text{ mm}^2$. For each side, the maximum compressive force = $36347/2 = 17173.5 \text{ N}$ and the maximum tensile force is $11650/2 = 5825 \text{ N}$.

$$\text{Reciprocating Mass} = \text{Piston} + \frac{1}{3} \text{Rod} + \text{Gudgeon Pin} \quad (12)$$

$$\text{Rotate Mass} = \frac{2}{3} \text{Rod} \quad (13)$$

$$\text{Max Inertia Force at TDC} = \text{RotataeMass } R\omega^2 + \text{ReciprocatingMass } R\omega^2 \left(1 + \frac{R}{L}\right) \quad (14)$$

$$\sigma_{Max} = \frac{5825}{144} = 40 \text{ N/mm}^2$$

$$\sigma_{Min} = -\frac{17173.5}{144} = -119 \text{ N/mm}^2$$

$$\text{Mean Strss} = \frac{\sigma_{Max} + \sigma_{Min}}{2} = -79 \text{ N/mm}^2$$

$$\text{Range} = \sigma_{Max} - \sigma_{Min} = 159 \text{ N/mm}^2$$

The endurance limit without radiuses = $\text{UTS} \times 0.45 = 750 \times 0.45 = 337.5 \text{ N/mm}^2$.

Final endurance limit with radiuses = $\text{UTS} \times 0.45 / K = 337.5 / 1.5 = 225 \text{ N/mm}^2$.

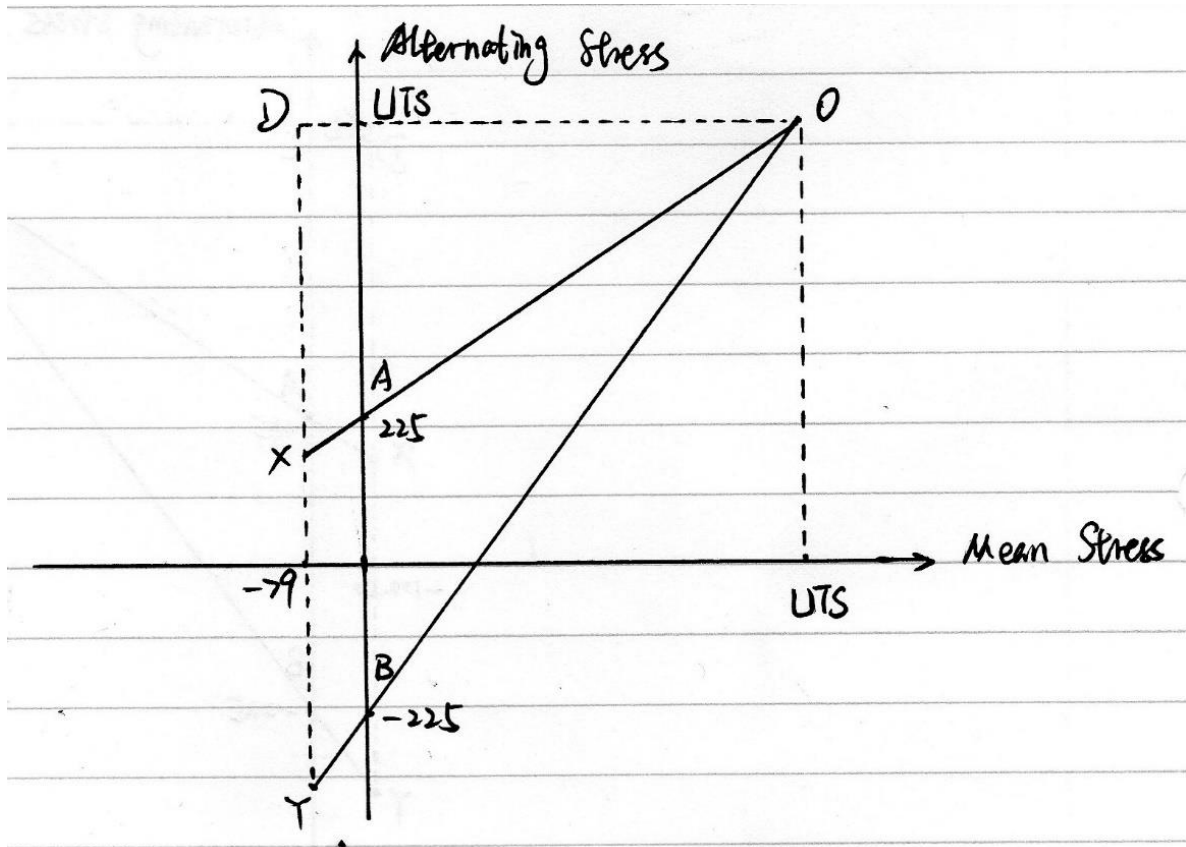


Figure.4 Goodman Diagram of Point C

As shown in figure 4,

$$XY/AB = OX/OA = OD/OC = (UTS-MEAN)/UTS$$

So allowance range at mean which is shown like XY is 497.4 N/mm².

FSF of point A could be calculated with formula (9) as 3.0 with actual range of 159 N/mm².

3 Calculation about con-rod and the standard bolt

3.1 (a)

For the stiffness of the bolt and cap-rod

$$\text{Bolt Stiffness} = K_B = E_B \frac{A_B}{L} \quad (15)$$

$$\text{Cap Stiffness} = K_{Cap} = E_{Cap} \frac{A_{Cap}}{L} \quad (16)$$

For the 8mm metric fine bolt, both of the inner and outer diameters could be found from Bosch Handbook which is 7mm and 8mm.

$$\text{Average Diameter} = \frac{\text{inner diameter} + \text{outer diameter}}{2} = \frac{7 + 8}{2} = 7.5\text{mm}$$

$$A_{Bolt} = \frac{\pi(\text{Average Diameter})^2}{4} = \frac{\pi \times 7.5^2}{4} = 44.2 \text{ mm}^2$$

$$A_{Cap} = \text{Length} \times \text{Width} = 16 \times 9 = 144 \text{ mm}^2$$

$$E_{Bolt} = 210 \times 10^3 \text{ N/mm}^2$$

$$E_{Cap} = 170 \times 10^3 \text{ N/mm}^2$$

$$\frac{\text{Bolt Stiffness}}{\text{Cap Stiffness}} = \frac{K_B}{K_{Cap}} = \frac{E_B A_B}{E_{Cap} A_{Cap}} = \frac{210 \times 44.2}{170 \times 144} = 0.38$$

The approximate ratio of the bolt stiffness (K_B) to the cap-rod stiffness (K_{Cap}) is 0.38.

3.2 (b)

As bolt stiffness (K_B) has a relationship with the E_{Bolt} , Area and length. From the elastic limit, length should be at least 28mm.

$$Bolt\ Stiffness\ (K_B) = E_B \frac{A_B}{L} = 210 \times 10^3 \times \frac{44.2}{28} = 331500 \frac{N}{mm} = 331.5\ N/m$$

3.3 (c)

For the maximum value of the alternating load, it equals to the minimum bolt stress plus the extra force coming from the maximum inertia load as the minimum bolt stress comes from initial tightening.

For black plated ford bolt, UTS=1000 N/mm². For these re-usable bolt, tighten to 90% yield and the axial stress comes with 60% yield.

$$Yield = 0.9 \times UTS = 0.9 \times 1000 = 900\ N/mm^2$$

$$Axial\ Stress = 0.6 \times yield = 0.6 \times 900 = 540\ N/mm^2$$

Inertia force on cap at TDC comes with 4952 N from follows

$$F_{inertia} = (M_{Rot} - M_{Cap}) \times R\omega^2 + M_{Roc}R\omega^2 \times (1 + \frac{R}{L}) \quad (17)$$

Extra force is 2736 N calculated from formula below

$$Extra\ force = \frac{K_B}{K_B + \frac{K_{Cap}}{K_B}} \times F_{Inertia} = \frac{210}{210 + \frac{170}{210}} \times 4952 = 2736\ N$$

Extra nominal stress at root could be obtained from equation as follows

$$Extra\ Stress = \frac{Extra\ Force}{\frac{\pi d^2}{4}} = \frac{2736}{\frac{\pi \times 7^2}{4}} = 71\ N$$

$$\sigma_{Min} = Axial\ stress = 540\ N/mm^2$$

$$\sigma_{Max} = \sigma_{Min} + Extra\ Stress = 540 + 71 = 611\ N$$

The maximum value of the alternating load is 611 N.

3.4 (d)

To calculate the required torque, there are two areas of friction. But first of all, the clamping force should be obtained from formula as follows

$$\text{Clamping Force} = \text{Area} \times \text{Axial Stress} = \pi \frac{7.5^2}{4} \times 540 = 23856 \text{ N}$$

One of them is circumferential friction force at threads which is 4294 N as $\mu_1=0.18$.

$$\text{Friction Force at Threads} = \mu_1 \times \text{clamping Force} = 0.18 \times 23856 = 4294 \text{ N}$$

The resisting torque at threads equals to the moment of circumferential force.

$$\text{Resisting Torque} = \text{Average Radius} \times \text{Friction Force} = \frac{7.5}{2} \times 4294 = 16102 \text{ Nmm} = 16 \text{ Nm}$$

The other circumferential friction force comes from seating face which is 2862 N as $\mu_2=0.12$.

$$\text{Friction Force at Face} = \mu_2 \times \text{clamping Force} = 0.12 \times 23856 = 2862 \text{ N}$$

Average seating face diameter is 10.5mm as the inner diameter is 8mm and outer diameter is 13mm.

Torque needed to overcome the friction on the seating face could be obtained from the equation below.

$$\text{Torque Needed} = \text{Average Radius} \times \text{Friction Force} = \frac{10.5}{2} \times 2862 = 15025 \text{ Nmm} = 15 \text{ Nm}$$

Total torque needed when tightening = 16+15 Nm=31Nm.

3.5 (e)

The Ultimate Tensile Strength of bolt is $UTS=1000\text{N/mm}^2$ and the stress concentration factors is $K_t=2.0$.

$$\sigma_{Min} = 0.9 \times \text{yield} = 0.9 \times 0.9 \times UTS = 0.9^2 \times 1000 = 810 \text{ N/mm}^2$$

$$\text{Extra Stress} = \frac{\text{Extra Force}}{\frac{\pi d^2}{4}} = \frac{2736}{\frac{\pi \times 7^2}{4}} = 71 \text{ N}$$

$$\sigma_{Max} = \sigma_{Min} + \text{Extra Force} = 810 + 71 = 881 \text{ N/mm}^2$$

$$\text{Mean Strss} = \frac{\sigma_{Max} + \sigma_{Min}}{2} = 845.5 \text{ N/mm}^2$$

$$\text{Range} = \sigma_{Max} - \sigma_{Min} = 71 \text{ N/mm}^2$$

3.6 (f)

The endurance limit = $UTS \times 0.45 = 1000 \times 0.45 = 450 \text{ N/mm}^2$.

Final endurance limit = $UTS \times 0.45 / K = 450 / 2.0 = 225 \text{ N/mm}^2$.

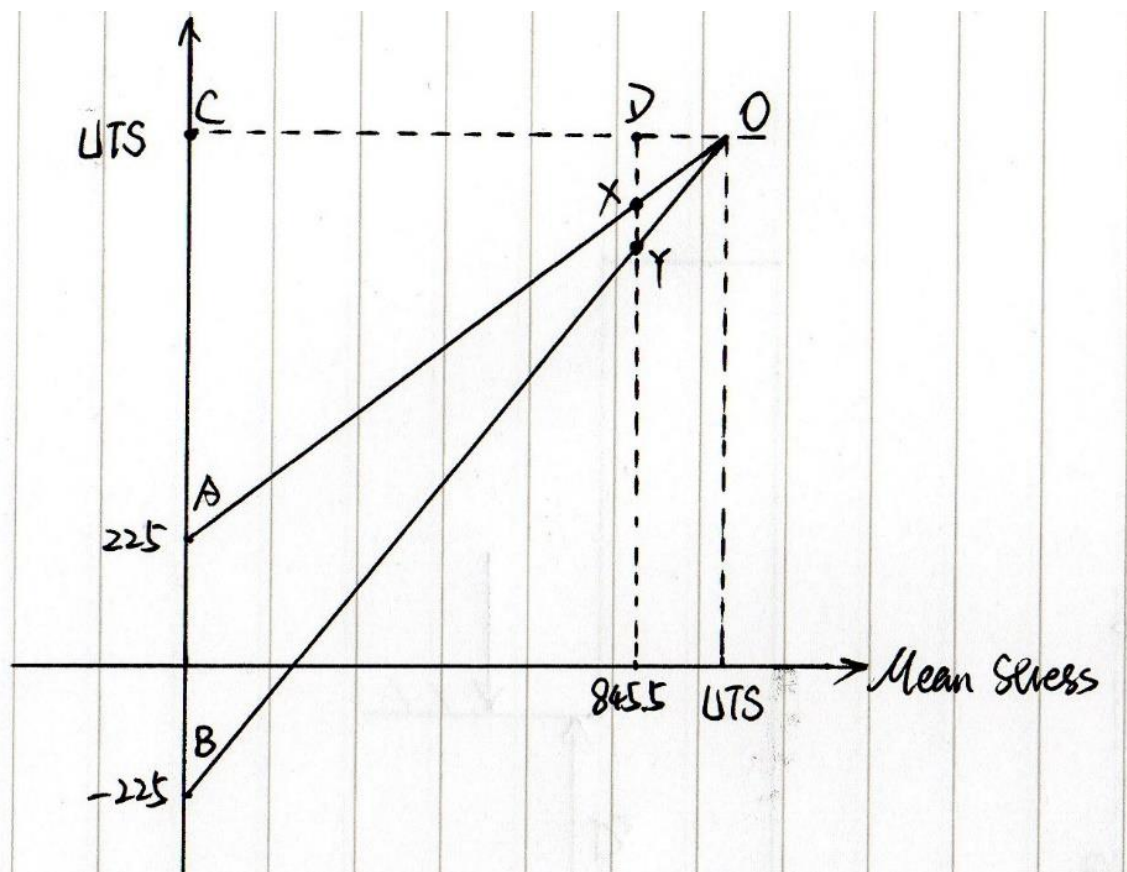


Figure.5 Goodman Diagram of Standard Bolt

As shown in figure 5,

$$XY/AB = OX/OA = OD/OC = (UTS-MEAN)/UTS$$

So allowance range at mean which is shown like XY is 69.5 N/mm².

FSF could be calculated with formula (9) as 0.98 with actual range of 71 N/mm².

3.7 (g)

Upgrade the Ultimate Tensile Strength of bolt from 1000 N/mm² to 1100 N/mm².

$$\sigma_{Min} = 0.9 \times yield = 0.9 \times 0.9 \times UTS = 0.9^2 \times 1100 = 891 \text{ N/mm}^2$$

$$Extra \text{ Stress} = \frac{Extra \text{ Force}}{\frac{\pi d^2}{4}} = \frac{2736}{\frac{\pi \times 7^2}{4}} = 71 \text{ N}$$

$$\sigma_{Max} = \sigma_{Min} + Extra \text{ Force} = 891 + 71 = 962 \text{ N/mm}^2$$

$$Mean \text{ Strss} = \frac{\sigma_{Max} + \sigma_{Min}}{2} = 926.5 \text{ N/mm}^2$$

$$Range = \sigma_{Max} - \sigma_{Min} = 71 \text{ N/mm}^2$$

The endurance limit = UTS x 0.45 = 1100 x 0.45 = 495 N/mm².

Final endurance limit = UTS x 0.45 / K = 495 / 2.0 = 247.5 N/mm².

Allowance range at mean is improved to 78 N/mm².

FSF is also improved to 1.1 with actual range of 71 N/mm².

3.8 (h)

Increasing the ultimate tensile strength of the bolts used can bring many benefits in terms of durability. The most intuitive aspect is the improvement of the fatigue safety factor, which helps increase the number of times the bolt can be used in the maximum strength range.

4 Compressive Force.

For the critical fillet sections, it means A and B shown in figure 6 which is the weakest section for the engine crankshaft. UTS equals to 750 N/mm².

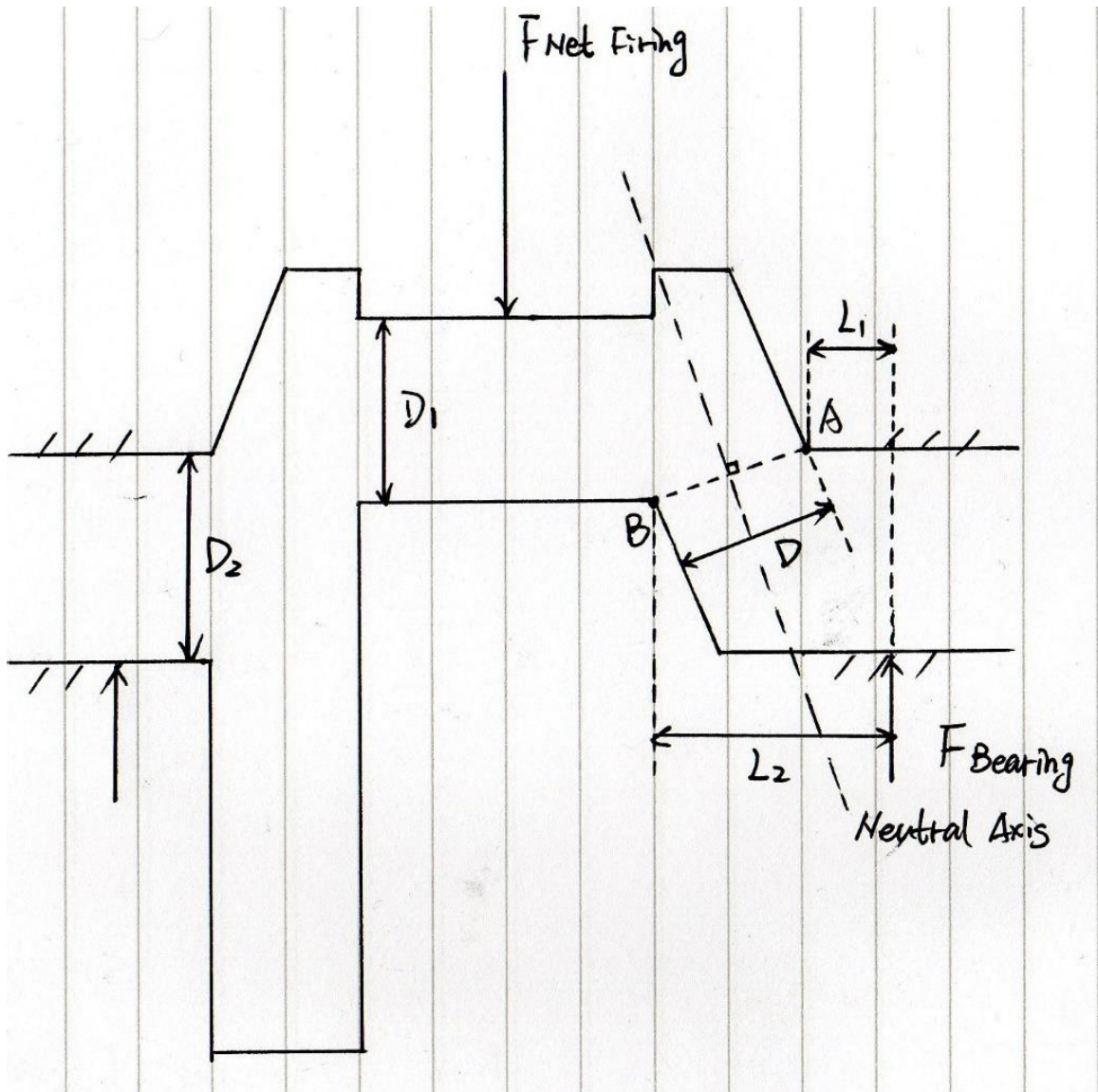


Figure.6 Sketch of Crankshaft

$$F_{Net\ Firing} = F_{Gas} - F_{Inertia} = 37968\ N$$

$$F_{Bearing} = \frac{F_{Net\ Firing}}{2} = 18984\ N$$

D = 20mm, D1 = 40mm, D2 = 45mm, L1 = 12mm, L2 = 40mm.

The effective width of critical beam which is b could be calculated as 42.5mm from

$$b = \frac{D1 + D2}{2} = \frac{40 + 45}{2} = 42.5mm$$

The second moment of effective beam I could be obtained from

$$I = b \times \frac{D^3}{12} = 42.5 \times \frac{20^3}{12} = 28333 \text{ mm}^4$$

Bending moment at A could be calculated from right of A

$$M_{A \text{ Bending}} = F_{\text{Bearing}} \times L_1 = 18984 \times 12 = 227808 \text{ Nmm}$$

Point A goes to compressive while point B goes to tensile at firing and the bending moment at point B is bigger than point A as the distance from B to the centre of bearing.

$$M_{B \text{ Bending}} = F_{\text{Bearing}} \times L_2 = 18984 \times 40 = 759360 \text{ Nmm}$$

Bending stress at B equals to bending moment times distance to neutral axis over second moment of area.

$$\sigma_B = M_{B \text{ Bending}} \times \frac{D}{2I} = 759360 \times \frac{10}{28333} = 268 \text{ N/mm}^2$$

Shear force equals to bearing force which is 18984 N.

$$\text{Shear Stress}_B = \frac{\text{Shear Force}}{\pi \times \left(\frac{D^2}{4}\right)} = \frac{18984}{\pi \times 20^2} = 15 \text{ N/mm}^2$$

$$\sigma_{\text{Max}} = \sigma_{\text{Combined}} = (\sigma_A^2 + (2T)^2)^{0.5} = (268^2 + (2 \times 15)^2)^{0.5} = 269 \text{ N/mm}^2.$$

$$\sigma_{\text{Min}} = -\frac{4952}{18984} \times \sigma_{\text{Max}} = -\frac{4952}{18984} \times 269 = -70 \text{ N/mm}^2$$

$$\text{Mean Strss} = \frac{\sigma_{\text{Max}} + \sigma_{\text{Min}}}{2} = 99.5 \text{ N/mm}^2$$

$$\text{Range} = \sigma_{\text{Max}} - \sigma_{\text{Min}} = 339 \text{ N/mm}^2$$

Polished endurance limit for crank = UTS x 0.45 = 750 x 0.45 = 337.5 N/mm².

Final endurance limit at B = UTS x 0.45 / 2 = 750 x 0.45 / 2 = 168.75 N/mm² while the stress concentration factors Kt = 2.

5 References

Bosch, 2014. *Automotive Handbook*. Germany: s.n.

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