



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 6

LEFEL 6

EXAMINATION PERIOD
MAY 2016

CYFNOD ARHOLIADAU
MAI 2016

MODULE CODE: M3X7200
ADVANCED STRESS ANALYSIS

TIME: 2 HOURS

ANSWER THREE OUT OF FOUR QUESTIONS

Equipment or materials required – None

Adnoddau neu ddefnyddiau sydd eu hangen – Dim

A formula sheet is provided for reference.

Calculators are permitted.

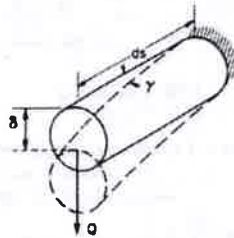
Assume 'g' = 9.81 m/s².

ONE

a)

- (i) Consider the below bar subjected to shear, prove from first principles that the shear strain γ , due to the load is given by; $\gamma = \frac{Q}{AG}$, where Q is the shear load, ds is the length of the bar, A is the cross sectional area and G is the Shear Modulus.

[2 Marks]



- (ii) Hence prove that the strain energy, U, for a bar in Shear is given by;

$$U = \frac{Q^2 L}{2AG}$$

[4 Marks]

- (iii) What is meant by the term Resilience? Hence from prove from the standard relationship for a bar in tension, $U = \frac{P^2 L}{2AE}$, where P is the load and E is the

Youngs Modulus, that resilience may be defined as $Re\ silience = \frac{\sigma^2}{2E}$

[4 Marks]

b)

- (i) A steel rod has a square cross section 15mm x 15mm and a length of 1m. Calculate the strain energy when a stress of 300MPa is produced by stretching it. (E=200GPa) **(50.6J)**

[3 Marks]

- (ii) The rod then has a metal collar fitted at the lower end, and a mass, m, is dropped onto the collar. Show, by considering the energy transferred from the falling weight to the rod, that the height, h, is related to the stress, σ , by the following relationship;

$$h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$$

Where V is the volume of the bar, and L is the length. Assume that the extension of the bar is significant in the energy transfer.

[4 Marks]

- (iii) If the stress must not exceed 75MPa, from what height could 200kg fall onto the collar, without exceeding this? **(1.2 mm)**

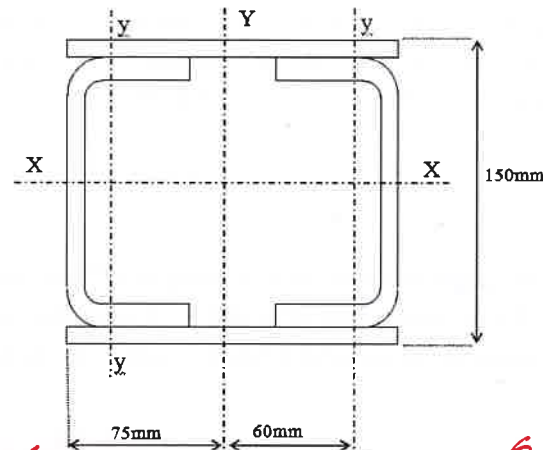
[3 Marks]

TWO

- a) Given the fact that Aluminium is approximately one third the density of steel but also one third the stiffness, briefly discuss why, in terms of vehicle stiffness, structural panels made from Aluminium can be beneficial over steel panels.

[2 Marks]

- b) The engine longitudinal of an integral vehicle chassis is fabricated from steel and is 1m long. The fabrication is symmetrical about the YY axis and is constructed from two flat 2mm plates and two 'C section' folded steel sheets as shown below. Given that the Young's Modulus, E , is 200GPa the compressive yield stress, σ_c , is 320MPa and a constant a of 1/7500, assuming the longitudinal is fixed at one end and free at the other, determine the following;



Properties of ONE folded sheet steel section;

$$I_{xx} = 1.5 \times 10^6 \text{ mm}^4$$

$$I_{yy} = 5.5 \times 10^5 \text{ mm}^4$$

$$A = 384 \text{ mm}^2$$

$$I_{xx} = 6.29 \times 10^6 \text{ mm}^4; I_{yy} = 4.99 \times 10^6 \text{ mm}^4$$

- Which axis will the longitudinal buckle about? (y -axis) [5 Marks]
- What is the radius of gyration about this axis? (60.4mm) [1 Mark]
- What is the slenderness ratio of this longitudinal? (16.6) [1 Mark]
- According to Euler theory, what load will this longitudinal buckle at? (2.46 MN) [3 Marks]
- According to Rankine-Gordon theory, what load will this longitudinal buckle at? (0.42 MN) [3 Marks]
- Comment on the results of (iv) and (v), and with the use of a diagram explain the difference between experimental results and buckling loads according to Euler theory. [5 Marks]

THREE

In general terms, contact mechanics is the study of the deformation of solids that are in contact with each other at one or more points.

- (a) Describe in more specific terms the concept of *Hertzian* contact stresses in solid objects that are in contact with each other. Suggest examples where these stresses are of importance in engineering components.

[5 marks]

- (b) With regard to a problem involving *Hertzian* contact stresses, state the general assumptions that are made in order to analytically solve the problem.

[3 marks]

- (c) Sketch appropriate diagrams of two cylinders in contact with one another, held along their length, by a force F . Your sketches should include the recognised nomenclature applied to contact mechanics for this situation, and show that the contact area will be $2a \times L$.

[4 marks]

- (d) Two cylindrical rollers of length 450mm, one of diameter 25mm and the other 50mm, are brought into contact by an applied force of 45kN. Given that the rollers are both manufactured from the same material, which has a Young's Modulus of 210GN/m², determine;

- (i) The contact area between the two rollers.

- (ii) The maximum contact pressure (i.e. stress) experienced.

$(0.0863 \times 10^{-3} \text{ m}^2)$
 (664.13 MN/m^2)

[8 marks]

FOUR

Residual stresses are created by almost every manufacturing process, and despite their widespread occurrence, these stresses occur without any external loads which make them easy to ignore.

- (a) State three notable classifications of manufacturing processes that are known to give rise to residual stresses, and the significant influences they can have on an engineering component.

[6 marks]

- (b) Residual stresses may be categorised by their cause (e.g. thermal or elastic mismatch), or according to the method by which they are measured. Discuss the third method by which these self-equilibrating stresses may be categorised. Your discussion should include appropriate terminology that describes the different classes.

[6 marks]

- (c) The static loading performance of brittle materials can be dramatically improved by the intelligent use of residual stress. Suggest two common examples where residual stresses are induced in a material for beneficial reasons. Also discuss the mechanisms involved in improving the loading performance.

[6 marks]

- (d) State two methods by which compressive in-plane residual stresses may be induced in the near surface region of an engineering component, in order to reduce the possible initiation of a fatigue crack.

[2 marks]

FORMULA SHEET

$$U = \frac{P^2 L}{2AE}$$

Strain energy in tension

$$U = \frac{Q^2 L}{2AG}$$

Strain energy in shear

$$U = \frac{M^2 L}{2EI}$$

Strain energy in bending

$$U = \frac{T^2 L}{2GJ}$$

Strain energy in torsion

$$\text{P.E.} = mgh$$

Potential energy

$$\text{K.E.} = \frac{1}{2} mv^2$$

Kinetic energy

$$I_{NA} = \frac{BD^3}{12}$$

Second moment of area of a rectangle

$$I_{NA} = I_G + Ah^2$$

Parallel axis theorem

$$\text{slenderness ratio} = \frac{L}{k}$$

$$k = \sqrt{\frac{I}{A}}$$

Radius of gyration

$$P_e = \frac{\pi^2 EI}{L^2}$$

Buckling: pinned-pinned

$$P_e = \frac{\pi^2 EI}{4L^2}$$

Buckling: fixed-free

$$P_e = \frac{4\pi^2 EI}{L^2}$$

Buckling: fixed-fixed

$$P_e = \frac{2\pi^2 EI}{L^2}$$

Buckling: fixed-pinned

$$P_R = \frac{\sigma_y A}{1 + a(L/k)^2}$$

Rankine Load

Diwedd/End