

Complete, fold and seal
(see note 2 below)

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

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FORENAME(S): Samir

1	Student Number	1804596/1 ~~ALT-1984254~~
2	Title of Examination Paper	Mathematics
3	Module Code	M4X00391
4	Date and Time of Examination	10/5/19 pm
5	Examination Venue	HALL
6	Seat Number	1
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
Q 1	
a	20
b	9
c	3
d	4
e	3
Q 2	
a	12
b	10
c	12
d	1
e	5
Total	79/80

✓
✓
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✓
✓

✓
✓
✓
✓
✓

✓ 98.75 % ✓

Question one

(a)

(i) $\int (x^3 + 2x)(x^2 - 4x) dx$

$$\int x^5 - 4x^4 + 2x^3 - 8x^2 dx \quad \checkmark \quad \textcircled{1}$$

$$= \frac{1}{6}x^6 - \frac{4}{5}x^5 + \frac{2}{4}x^4 - \frac{8}{3}x^3 + C \quad \checkmark$$

$$= \frac{1}{6}x^6 - \frac{4}{5}x^5 + \frac{1}{2}x^4 - \frac{8}{3}x^3 + C \quad \checkmark \quad \textcircled{5}$$

6

(ii) $\int (x^4 + 4x)^3 - 2x + 4 \cos 5x dx$

$$= \int (x^4 + 4x)^2 (x^4 + 4x) - 2x + 4 \cos 5x dx \quad \checkmark \quad \textcircled{1}$$

$$= \int (x^8 + 8(x^4 \times 4x) + 16x^2)(x^4 + 4x) - 2x + 4 \cos 5x dx$$

$$= \int (x^8 + 8x^5 + 16x^2)(x^4 + 4x) - 2x + 4 \cos 5x dx \quad \checkmark \quad \textcircled{1}$$

$$= \int (x^{12} + 8x^9 + 16x^6 + 4x^9 + 32x^6 + 64x^3 - 2x + 4 \cos 5x) dx$$

$$= \frac{1}{13}x^{13} + \frac{8}{10}x^{10} + \frac{16}{7}x^7 + \frac{4}{10}x^{10} + \frac{32}{7}x^7 + \frac{64}{4}x^4 - \frac{2}{2}x^2 + \frac{4}{5}\sin 5x + C \quad \checkmark \quad \textcircled{3}$$

6

(iii) $\int_0^5 (3x^2 - 1)(x + 5) dx$

$$= \int_0^5 3x^3 + 15x^2 - x - 5 dx \quad \checkmark \quad \textcircled{1}$$

$$= \int_0^5 \left[\frac{3}{4}x^4 + \frac{15}{3}x^3 - \frac{1}{2}x^2 - 5x \right] dx \quad \checkmark$$

$$= \left[\frac{3}{4}(5)^4 + \frac{15}{3}(5)^3 - \frac{1}{2}(5)^2 - 5(5) \right] - [0 + 0 - 0 - 0] \quad \checkmark \quad \textcircled{1}$$

$$= \left[\frac{1875}{4} + 625 - \frac{25}{2} - 25 \right] \quad \checkmark \quad \textcircled{1}$$

$$= \frac{4225}{4}$$

$$= 1056.25 \quad \checkmark \quad \textcircled{5}$$

4

$$\begin{aligned} \sin &\rightarrow \cos \\ \cos &\rightarrow -\sin \end{aligned}$$

$$(iv) \int_0^1 (e^x + 2)(e^{2x} + 1) dx$$

$$= \int_0^1 e^{3x} + e^x + 2e^{2x} + 2 dx \quad \checkmark \quad (1)$$

$$= \frac{1}{3} e^{3x} + e^x + \frac{2}{2} e^{2x} + 2x + c$$

$$= \left[\frac{1}{3} e^{3x} + e^x + e^{2x} + 2x \right] \quad \checkmark \quad (1)$$

$$= \left[\frac{1}{3} e^3 + e + e^2 + 2 \right] - \left[\frac{1}{3} e + e + e + 0 \right] \quad \checkmark$$

$$= \frac{1}{3} e^3 + e + e^2 + 2 - \frac{1}{3} - 1 - 1$$

$$= \frac{1}{3} e^3 + e + e^2 + \frac{1}{3} = ? \quad = \frac{1}{3}$$

(b)

$$(i) \int 5x \cos 2x dx$$

$$u = 5x \quad \checkmark$$

$$\int dv = \int \cos 2x dx$$

$$\frac{du}{dx} = 5 \quad \checkmark \quad (1)$$

$$v = \frac{1}{2} \sin 2x \quad \checkmark \quad (1)$$

$$\int u dv = uv - \int v du$$

$$= 5x \left(\frac{1}{2} \sin 2x \right) - \int \left(\frac{1}{2} \sin 2x \right) (5) dx \quad (1)$$

$$= \frac{5}{2} x \sin 2x + \frac{5}{4} \cos 2x + c \quad \checkmark \quad (2)$$

$$(ii) \int 2x \ln x dx = \int \ln x (2x) dx$$

$$u = \ln x \quad \checkmark$$

$$\int dv = \int 2x dx \quad \checkmark$$

$$\frac{du}{dx} = \frac{1}{x} \quad \checkmark \quad (1)$$

$$v = \frac{2}{2} x^2 = x^2 \quad \checkmark \quad (1)$$

$$\int u dv = uv - \int v du$$

$$= \ln(x^2) - \int x^2 \left(\frac{1}{x}\right)$$

$$= x^2 \ln x - \int \frac{x^2}{x} \quad \checkmark$$

$$= x^2 \ln x - \int x \quad \checkmark$$

$$= x^2 \ln x - \frac{1}{2} x^2 + C \quad \checkmark \textcircled{2}$$

(C)

$$(i) \int \frac{21x^2 + 4x}{7x^3 + 2x^2 - 1} dx$$

$$= \ln(7x^3 + 2x^2 - 1) + C \quad \checkmark \textcircled{1}$$

$$(ii) \int \frac{5}{7x+2} dx$$

$$5 \times \frac{7}{7} \int \frac{1}{7x+2} dx \quad \checkmark$$

$$= \frac{5}{7} \int \frac{7}{7x+2} dx \quad \checkmark$$

$$= \frac{5}{7} \ln(7x+2) + C \quad \checkmark \textcircled{1}$$

$$(iii) \int \frac{x^2}{3x^3+1} dx \quad \checkmark$$

$$\int \frac{9}{9} \frac{x^2}{3x^3+1} dx \quad \checkmark \textcircled{1}$$

$$= \frac{1}{9} \int \frac{9x^2}{3x^3+1} dx \quad \checkmark$$

$$= \frac{1}{9} \ln(3x^3+1) + C \quad \checkmark$$

$$(d) \int \frac{\ln x}{x(1+\ln x)^2} dx$$

$$u = 1 + \ln x \quad \checkmark \quad \Rightarrow \quad \ln x = u - 1 \quad \checkmark$$

$$\frac{du}{dx} = \frac{1}{x} \quad \checkmark$$

$$dx = x du \quad \checkmark$$

$$= \int \frac{u-1}{x(u)^2} x du \quad \checkmark$$

$$= \int \frac{u-1}{u^2} du$$

$$= \int \frac{u}{u^2} - \frac{1}{u^2} du \quad \checkmark$$

$$= \int \frac{1}{u} - \frac{1}{u^2} du$$

$$= \ln u - \left(\frac{1}{-1} u^{-1}\right) + C \quad \checkmark$$

$$= \ln u + \frac{1}{u} + C$$

$$= \ln(1+\ln x) + \frac{1}{1+\ln x} + C \quad \checkmark$$

$$(e) \int \frac{1}{2} \sin^5 x \cdot \cos x \, dx$$

$$u = \sin x \quad \checkmark$$

$$\frac{du}{dx} = +\cos x \quad \checkmark \quad \textcircled{1}$$

$$dx = \frac{+du}{\cos x} \quad \checkmark \quad \textcircled{1}$$

$$\int \left(\frac{1}{2} u^5 \cdot \cos x \right) \frac{+du}{\cos x}$$

$$= \int +\frac{1}{2} u^5 \, du$$

$$= +\frac{1}{2} \int u^5 \, du \quad \checkmark$$

$$= +\frac{1}{2} \left(\frac{1}{6} u^6 \right) + C \quad \checkmark$$

$$= +\frac{1}{12} (\sin^6 x) + C \quad \checkmark$$

$$= +\frac{1}{12} \sin^6 x + C \quad \checkmark \quad \textcircled{2}$$

3

Question two

$$\begin{aligned}
 \text{(i)} \quad z_1 + z_2 &= (2+3i) + (5-2i) \\
 &= 2+3i+5-2i \\
 &= \underline{7+i} \quad \checkmark \quad \textcircled{1}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad z_1 - z_3 &= (2+3i) - (4+i) \quad \textcircled{1} \\
 &= 2+3i-4-i \\
 &= \underline{-2+2i} \quad \checkmark \quad \textcircled{1}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{z_1}{z_2} &= \frac{2+3i}{5-2i} \times \frac{(5+2i)}{(5+2i)} \quad \textcircled{1} = \frac{10+4i+15i-6}{25+4} \quad \textcircled{1} \\
 &= \frac{4+19i}{29} \quad \textcircled{1} = \frac{4}{29} + \frac{19}{29}i \quad \checkmark \quad \textcircled{1}
 \end{aligned}$$

$$= 0.137 + 0.655i$$

i v and
✓
solved
in last
page

$$\begin{aligned}
 \text{(b)} \quad 14x^2 &= -3x-9 \\
 14x^2 + 3x + 9 &= 0 \quad \checkmark \quad \textcircled{1} \\
 a=14 \quad b=3 \quad c=9 \quad \checkmark
 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - (4 \times 14 \times 9)}}{2(14)} \quad \checkmark \textcircled{1}$$

$$x = \frac{-3 \pm \sqrt{9 - 504}}{28}$$

$$x = \frac{-3 \pm \sqrt{-495}}{28} \quad \checkmark \textcircled{1}$$

$$x = \frac{-3 \pm 22.24i}{28} \quad \checkmark \textcircled{1}$$

$$x = \frac{-3}{28} \pm 0.794i$$

$$x = -0.107 \pm 0.794i$$

$$x_1 = -0.107 + 0.794i \quad \checkmark \textcircled{1}$$

$$x_2 = -0.107 - 0.794i \quad \checkmark \textcircled{1}$$

x_1 in polar form :-

$$r = \sqrt{x^2 + y^2} =$$

$$= \sqrt{(0.107)^2 + (0.794)^2} \quad \checkmark$$

$$= \sqrt{0.011449 + 0.630} \quad \checkmark$$

$$= \sqrt{0.64}$$

$$r = 0.8011 \quad \checkmark \textcircled{1}$$

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$$\begin{aligned}\theta_{\text{ref}} &= \tan^{-1} \frac{y}{x} \\ &= \tan^{-1} \frac{0.794}{0.447}\end{aligned}$$

$$= \tan^{-1} 7.42$$

$$\theta_{\text{ref}} = 82.32$$

$$\theta = 180 - 82.32 = 97.67^\circ$$

$$x_1 = r \cos \theta = 0.8011 \cos 97.67^\circ$$

x_2 in polar form

$$\begin{aligned}\theta_{\text{ref}} &= \tan^{-1} \frac{y}{x} \\ &= \tan^{-1} \frac{0.794}{0.107}\end{aligned}$$

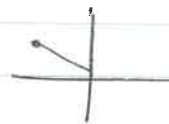
$$\theta_{\text{ref}} = \tan^{-1} 82.32$$

$$\theta = 180 + 82.32$$

$$\theta = 262.32^\circ$$

$$x_2 = r \cos \theta$$

$$= 0.8011 \cos 262.32^\circ$$



(c)
 (i) $(6 + 5i)^5$

$$r = \sqrt{6^2 + 5^2}$$

$$r = 7.81 \quad \checkmark \quad \textcircled{1}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$= \tan^{-1} \frac{5}{6}$$

$$= 39.80$$



$$\theta = 39.80 \quad \checkmark \quad \textcircled{1}$$

$$\text{the number} = [7.81 \angle 39.80]^5$$

$$= r^n [\cos n\theta + i \sin n\theta]$$

$$= (7.81)^5 [\cos 5 \times 39.8 + i \sin 5 \times 39.8] \quad \checkmark \quad \textcircled{1}$$

$$= 29057.29 [-0.945 + i 0.3255] \quad \checkmark \quad \textcircled{1}$$

$$= -27473.66 + i 9460.128 \quad \checkmark$$

$\textcircled{1}$

$\textcircled{1}$

(ii) $(-2 + 3i)^7$

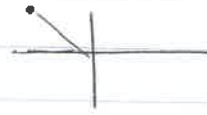
$$r = \sqrt{2^2 + 3^2}$$

$$r = 3.6 \quad \checkmark \quad \textcircled{1}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$\theta_{\text{ref}} = \tan^{-1} \frac{3}{2}$$

$$= 56.30$$



$$\theta = 180 - 56.30$$

$$= 123.69^\circ \quad \checkmark \quad \textcircled{1}$$

$$(3 - j6)^7 = 3.6^7 [\cos n\theta + j \sin n\theta]$$

$$= 3.6^7 [\cos 7 \times 123.69 + j \sin 7 \times 123.69] \quad \checkmark \quad \textcircled{1}$$

$$7836.4 [\cos 865.83 + j \sin 865.83] \quad \checkmark \quad \textcircled{1}$$

$$7836.4 [-0.827 + j 0.5616] \quad \checkmark$$

$$-6480.71 + j 4400.931 \quad \checkmark$$

$$\textcircled{1} \quad \textcircled{1}$$

d) $1 - 8j$ Conjugate for $(1 + 8j)$ $\checkmark$

e) $z_1 z_2 z_3$

$$= (6 + j)(2 - 5j)(1 - j)$$

$$(6 + j)(2 - 5j) = 12 + 30j + 2j + 0.5$$

$$= (17 - 28j) \quad \checkmark \quad \textcircled{1}$$

$$(17 - 28j)(1 - j) =$$

$$= 17 - 17j - 28j - 28$$

$$= -11 - 45j$$

$$\checkmark \quad \textcircled{1} \quad \textcircled{1}$$

$-11 - 45i$ in polar form

$$r = \sqrt{11^2 + 45^2} \quad \checkmark$$

$$r = 46.32 \quad \checkmark$$

$$\begin{aligned} \theta_{\text{ref}} &= \tan^{-1} \frac{y}{x} \\ &= \tan^{-1} \frac{45}{11} \end{aligned}$$

$$\tan^{-1} 4.09$$

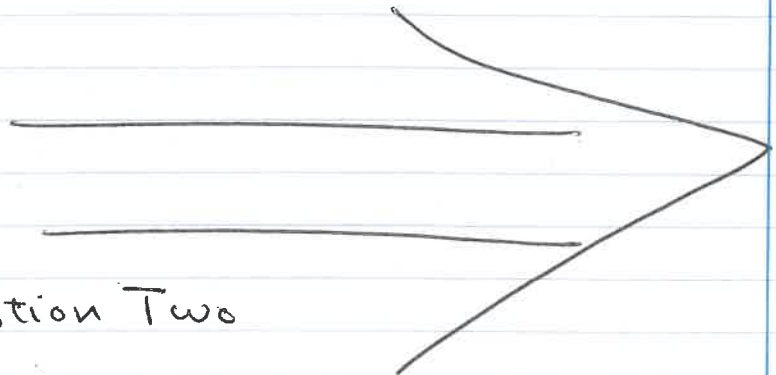
$$\theta_{\text{ref}} = 76.26$$

$$\theta = 180 + 76.26 \quad (1)$$

$$= 256.26 \quad \checkmark$$

$$(-11 - 45i) = 46.32 \angle 256.26 \quad (1)$$

$$= 2.223$$



Question Two

(i)

(iv)

(v)

Question Two

(a)

(iv) $\bar{z}_1 + \bar{z}_2$

$$\overline{(2+3i)} + \overline{(4+i)}$$

$$= 2-3i + (4-i)$$

$$= 2-3i+4-i$$

$$= \underline{6-4i}$$

(v) $(z_1)^2 + z_2 + z_3$

$$(z_1)^2 = (2+3i)^2$$

$$= (2+3i)(2+3i)$$

$$4+6i+6i-9$$

$$= -5+12i$$

$$(-5+12i) + (5-2i) + (4+i)$$

$$= -5+12i+5-2i+4+i$$

$$= 4+11i$$

$$= \underline{4+11i}$$