

Notation, Useful Equations and Theory

This section only gives the basic information needed to do the experiments. For full theory, refer to the textbooks listed in **Maintenance, Spare Parts and Customer Care** on page 71.

Notation

Symbol	Definition	Units
A	Heat Transfer Area	m^2
A_O	Orifice Area	m^2
C	Heat capacity	J.K^{-1}
C_d	Coefficient of discharge (orifice)	-
c_p and c_{pA} and c_{pW}	Specific Heat Capacity, of air and water at Constant Pressure (atmospheric pressure for air)	$\text{J.kg}^{-1}\text{K}^{-1}$
$LMTD$	Logarithmic Mean Temperature Difference	K or $^{\circ}\text{C}$ where shown
ρ and ρ_A and ρ_W	Density, of air and water	kg.m^{-3}
η_o	Overall Efficiency	%
C_{EB}	Energy Balance Coefficient	-
$\bar{\eta}_A$ and $\bar{\eta}_W$	Mean Temperature Efficiencies	%
Δp	Orifice pressure drop	Pa
Q	Heat energy transferred for a unit mass	J.kg^{-1}
$\dot{Q}$	Heat energy transferred for a unit time (heat transfer rate - or power)	W
$\dot{Q}_A$ and $\dot{Q}_W$	Heat energy (power) absorbed by the air and emitted by the water in a unit time	W
R	Specific gas constant for dry air	287 J/kg.K
T, T_W, T_A	Temperature, of water and air	K or $^{\circ}\text{C}$ where shown
ΔT	Change in Temperature	K or $^{\circ}\text{C}$ where shown
$\bar{T}$	Mean Temperature	K or $^{\circ}\text{C}$ where shown
U	Heat Transfer Coefficient	$\text{W.m}^{-2}\text{K}^{-1}$
$\dot{m}_W$ and $\dot{m}_A$	Mass Flow Rate (hot water circuit and air circuit)	kg.s^{-1}

Water Flow Conversion

The equipment measures volumetric water flow in litres per minute ($\text{L}\cdot\text{min}^{-1}$), but your calculations need mass flow in kilograms per second ($\text{kg}\cdot\text{s}^{-1}$).

$$\text{Mass flow (kg}\cdot\text{s}^{-1}) \dot{m}_W = \text{Volume flow (L}\cdot\text{min}^{-1})/60000 \times \text{Water density (kg}\cdot\text{m}^{-3})$$

Airflow

The equipment measures the pressure drop across the inlet orifice. If you use this pressure drop with the orifice discharge coefficient (C_d), its area and local air conditions, you can calculate the air flow through the duct.

$$\dot{m}_A = C_d \times A_O \sqrt{\Delta p \times 2 \times \rho_A} \quad (1)$$

Average Temperatures in Heat Exchangers

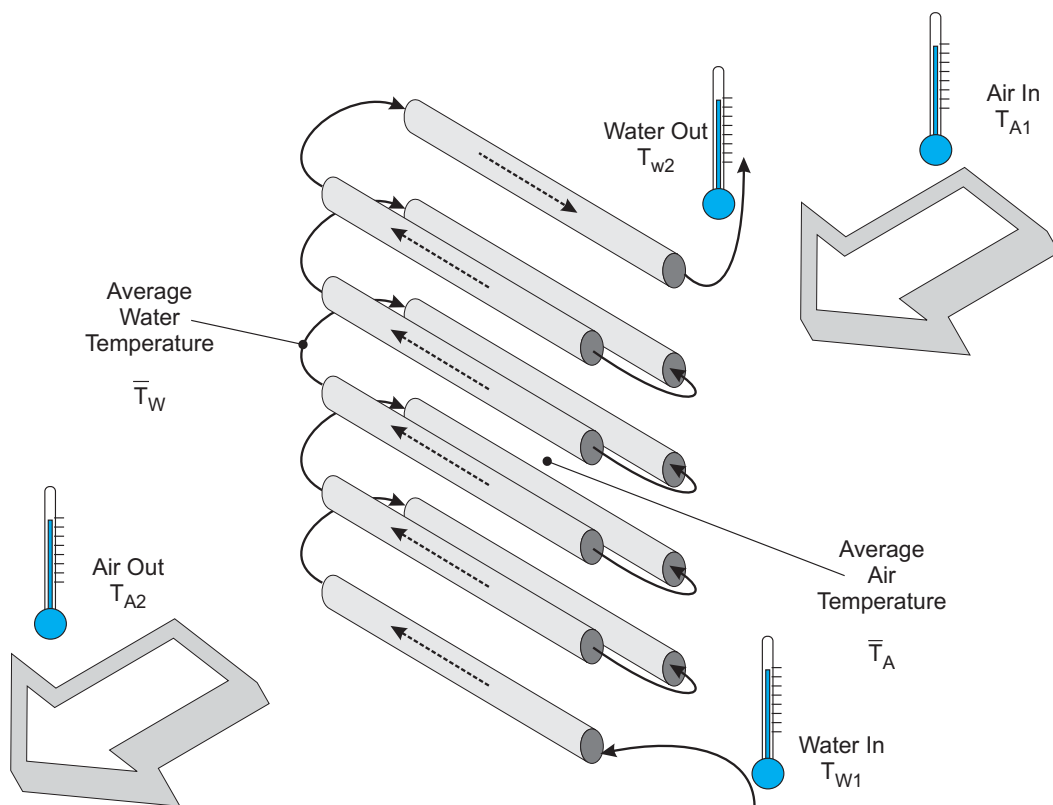


Figure 24 Average Temperatures Around the Heat Exchangers

For most heat transfer equations, and calculations of specific heat capacity and density of water, you must find the average temperature for the water and air circuits around your heat exchanger. This is the calculated value of the temperature at a mid-point between the inlet and outlet of the circuit (see Figure 24).

To find the average temperature of the **water** circuit:

$$\bar{T}_W = \frac{T_{W1} + T_{W2}}{2} \quad (2)$$

To find the average temperature of the **air** circuit:

$$\bar{T}_A = \frac{T_{A1} + T_{A2}}{2} \quad (3)$$

Useful Equations

Heat Capacity (C)

This is a measure of the heat needed to increase the temperature of a given mass of material by 1 degree (Kelvin or Celsius).

It is a product of the mass of a material (m) and its specific heat capacity (c).

$$C = m \times c \quad (4)$$

Specific Heat Capacity at Constant Pressure (c_p)

Specific Heat Capacity of Water at Constant Pressure

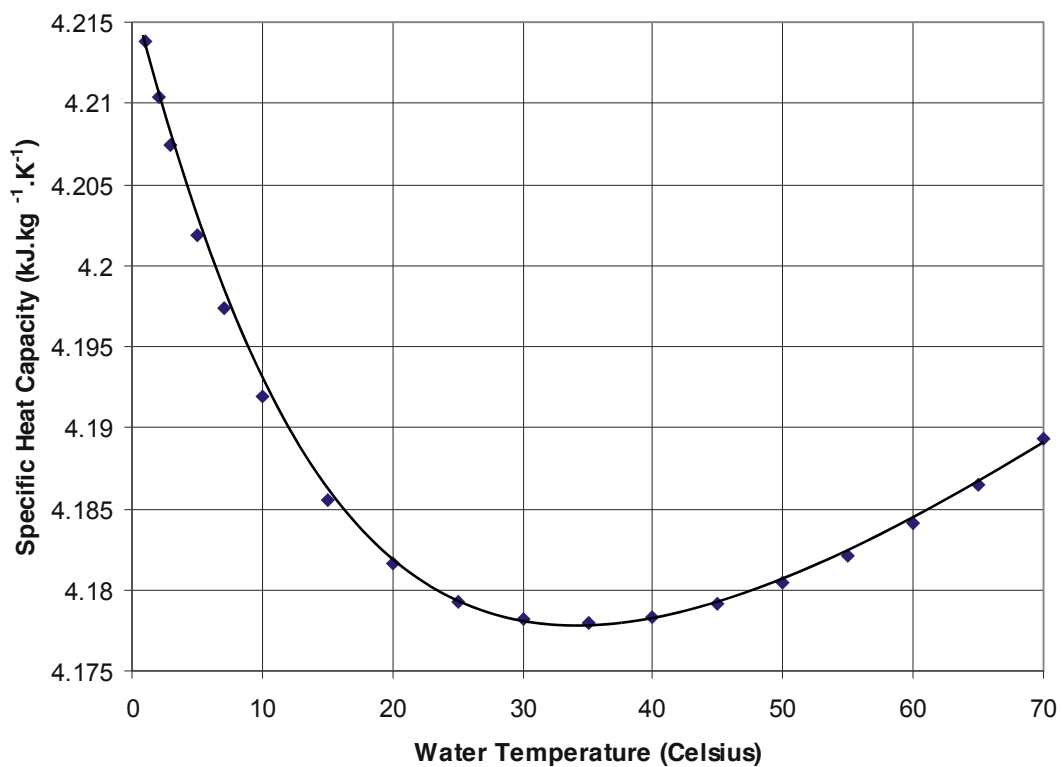


Figure 25 Specific Heat Capacity of Water at Constant Pressure

Specific heat capacity is the amount of heat energy needed to raise the temperature of exactly 1 kg of material by 1 degree (Celsius). It is a ratio of the change in heat energy and the change in temperature.

$$c = \frac{\Delta Q}{\Delta T} \quad (5)$$

Pressure and temperature affects specific heat capacity, so equation 6 assumes constant pressure over a known range of temperature. It is the 'isobaric' (equal pressure) specific heat capacity.

$$c_p = \frac{Q}{\Delta T} \quad (6)$$

For air, the specific heat capacity remains reasonably constant at around $1005 \text{ J.kg}^{-1}.\text{K}^{-1}$ at normal atmospheric pressure and for the temperature ranges used in the heat exchangers.

For water, you may use Figure 25 to find the specific heat capacity. Alternatively, Equation 7 (based on data from tests) will give you an accurate answer (where your temperature T is in Celsius):

$$\frac{c_{pw}(\text{at } T^{\circ}\text{C})}{c_{pw}(\text{at } 15^{\circ}\text{C})} = 0.996185 + 0.0002874\left(\frac{T + 100}{100}\right)^{5.26} + 0.011160 \times 10^{-0.036T} \quad (7)$$

From tests, the specific heat capacity of water at 15°C is $4185.5 \text{ J.kg}^{-1}.\text{K}^{-1}$



You must use the mean (or average) temperature of your flow circuit in this equation.

Density (ρ)

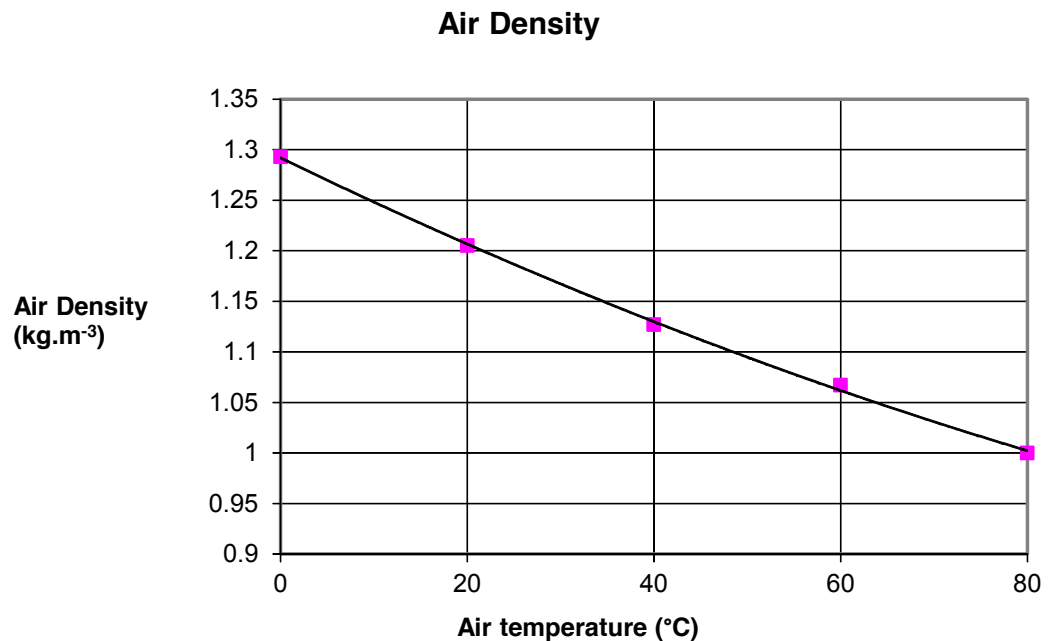


Figure 26 Air Density

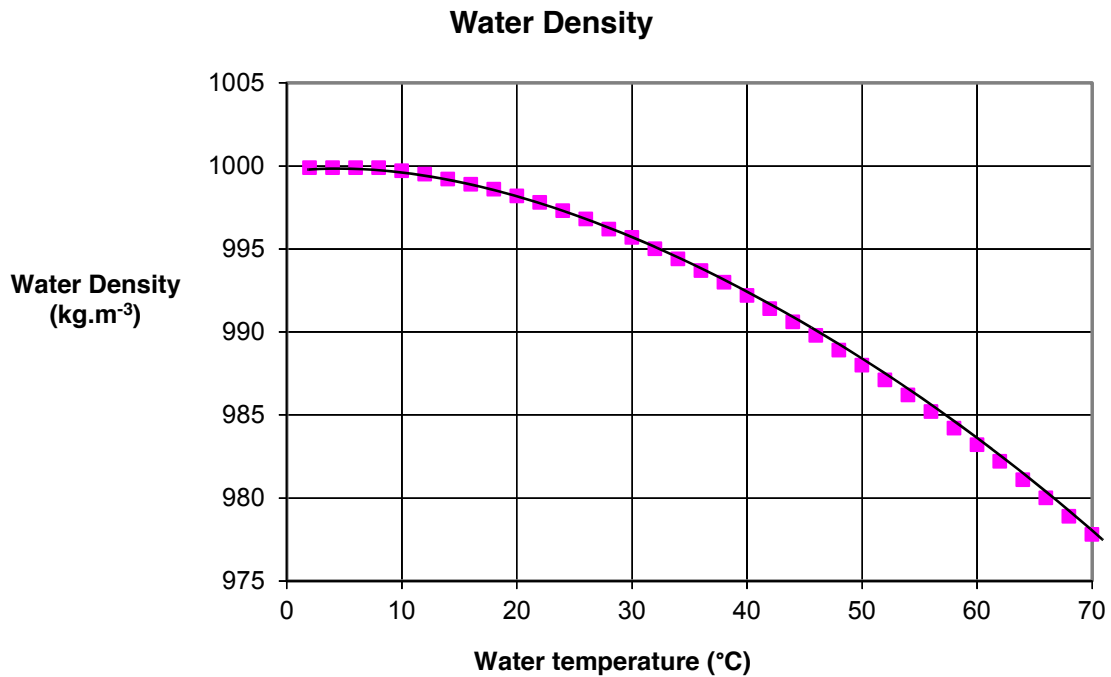


Figure 27 Water Density

Air has a maximum density at absolute zero, decreasing as temperature increases. You can use Figure 26 to determine its value over the range of temperatures used with the heat exchangers. Note that local air pressure affects air density, so the chart is relevant for normal atmospheric pressures. If you need more accuracy, Equation 8 allows you calculate the air density at a given pressure and temperature.

$$\rho_A = \frac{p}{RT} \quad (8)$$

Water has a maximum density at approximately 4 degrees celsius, decreasing as temperature increases or decreases from this value. This affects all your calculations. You can use Figure 27 to find water density over the range of temperatures used with the heat exchangers, for a unit mass and a constant pressure. If you need more accuracy, Equation 9 allows you calculate the water density (where T is your average water temperature in degrees centigrade).

$$\rho_W = \frac{\left(999.839 + 16.952T - 7.990 \times 10^{-3}T^2 - 46.241 \times 10^{-6}T^3 + 105.846 \times 10^{-9}T^4 - 281.030 \times 10^{-12}T^5\right)}{\left(1 + 16.887 \times 10^{-3}T\right)} \quad (9)$$

Heat Energy (Power) Transfer, Energy Balance and Efficiencies

In heat exchangers, heat energy transfers or ‘flows’ from the hotter circuit to the cooler circuit. Power is the rate of heat energy or heat energy transferred or used per unit time.

The heat transfer rate $\dot{Q}$ is a function of the fluid (or air) mass flow rate, the temperature change and the specific heat capacity of the fluid or air (at mean temperature). Because the calculation is based on a mass flow rate, the heat transfer is also a rate (or power).

$$\dot{Q} = \dot{m} \times c_p \times \Delta T \quad (10)$$

The heat (power) emitted from the hot water is:

$$\dot{Q}_W = \dot{m}_W \times c_{pW} \times \Delta T_W \quad (11)$$

NOTE  The temperature change of the water is negative, giving a negative heat transfer (emission).

The heat (power) absorbed by the air is:

$$\dot{Q}_A = \dot{m}_A \times c_{pA} \times \Delta T_A \quad (12)$$

In an ideal heat water to air heat exchanger, that does not lose or absorb heat from its surroundings, the cooler air absorbs all the heat from the hot fluid. So the heat transfer rate is:

$$\dot{Q} = \dot{Q}_W = \dot{Q}_A = \dot{m}_W \times c_{pW} \times \Delta T_W = \dot{m}_A \times c_{pA} \times \Delta T_A \quad (13)$$

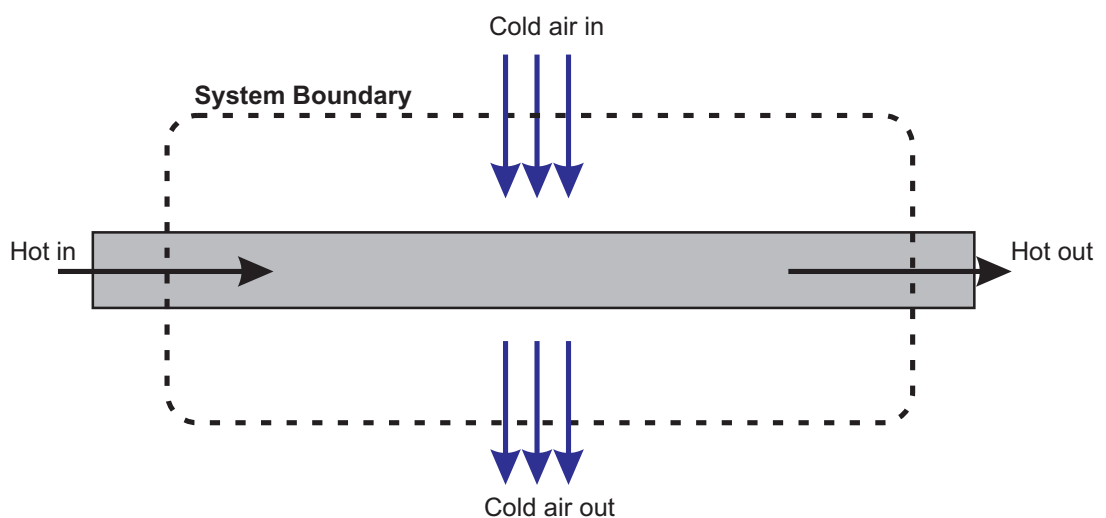


Figure 28 System Boundary

To better understand the heat transfer in a heat exchanger, you must assume it is a 'system' with the hot water and cooler air flows as its input and outputs (see Figure 28).

As mentioned earlier, there is no heat transfer across the 'system boundary' in an ideal heat exchanger. But, in reality the hot water is usually at a different temperature to the surroundings or duct work, so some heat transfers across the system boundary. For example, the heat from the warm water tubes or the heated air would transfer to the cooler air duct. Figure 29 shows the input, outputs and losses in diagram form as relative quantities.

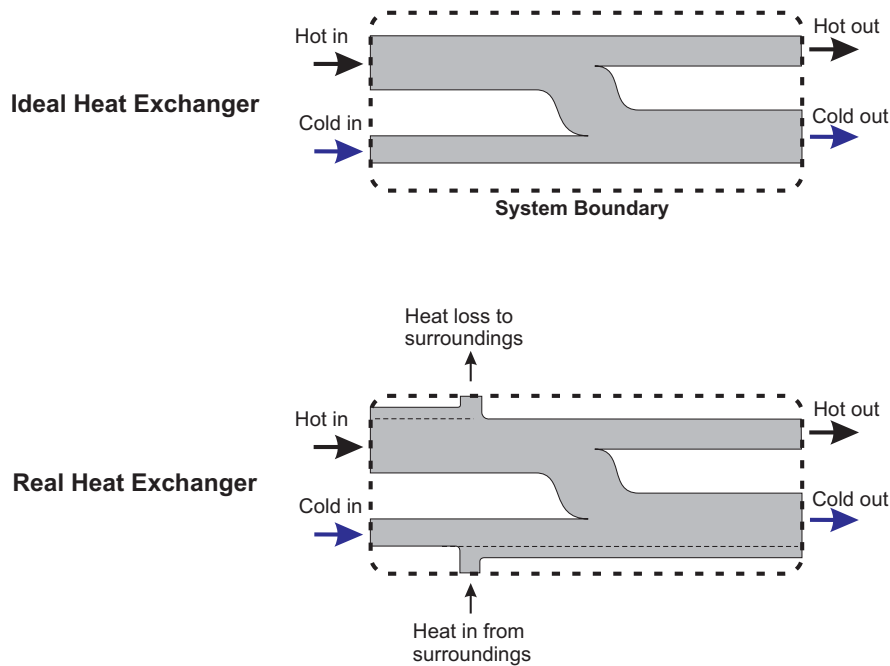


Figure 29 Ideal and Real Heat Exchangers

The energy balance coefficient (C_{EB}) shows the relationship between the energy absorbed and emitted, where:

$$C_{EB} = \frac{\dot{Q}_A}{\dot{Q}_W}$$

But, as shown, because of the possible heat flow into or out of the system, the energy balance coefficient can give answers of greater than 1 if the heat exchanger absorbs energy from its surroundings. So, it is only for guidance, because, in reality:

$$\dot{Q}_W = \dot{Q}_A \pm \text{Losses or gains from surroundings}$$

The mean temperature efficiency and heat transfer coefficient give more useful results for comparison between heat exchangers.

The **temperature efficiency** of the **hot water** circuit of the Heat Exchanger is the ratio of the temperature change in the hot water, divided by the difference between the maximum and minimum temperatures of the hot and cold circuits:

$$\eta_w = \frac{T_{W1} - T_{W2}}{T_{W1} - T_{A1}} \times 100 \quad (14)$$

The **temperature efficiency** of the **cooler air** circuit of the Heat Exchanger is the ratio of the temperature change in the cold circuit, divided by the difference between the maximum and minimum temperatures of the hot and cold circuits:

$$\eta_A = \frac{T_{A2} - T_{A1}}{T_{W1} - T_{A1}} \times 100 \quad (15)$$

The **mean temperature efficiency** of the two circuits is the average efficiency of them both:

$$\bar{\eta} = \frac{\eta_W + \eta_A}{2} \quad (16)$$

Logarithmic Mean Temperature Difference (LMTD)

This is a measure of the heat driving force that creates the heat transfer. It is a logarithmic average of the temperature difference between the hot and cold circuits at each end of the heat exchanger:

$$LMTD = \frac{(T_{W2} - T_{A2}) - (T_{W1} - T_{A1})}{\ln\left(\frac{T_{W2} - T_{A2}}{T_{W1} - T_{A1}}\right)} \quad (17)$$

Heat Transfer Coefficient (U)

This is the heat transfer coefficient for the wall and boundary layers. It is a measure of how well the heat exchanger works. A good heat exchanger will give a high coefficient, therefore this value is important to engineers.

$$U = \frac{\dot{Q}_W}{A \times LMTD} \quad (18)$$

Experiments

Safety

WARNING



Always clean up any water spills immediately. Never use any electrical switches when you have wet hands. Disconnect electrical supply before cleaning up water spills.

Keep hands and paperwork away from the air duct inlet and exhaust.

Never exceed a heater temperature of 60°C on the temperature controller.

If you do not use the equipment as described in these instructions, its protective parts may not operate correctly.

WARNING



Wear eye protection when using this equipment.

Useful Notes

CAUTION



The water connectors 'self-seal' but you will spill small amounts of water when you connect and disconnect your heat exchangers. Make sure your workbench can resist water and that you have some paper towels or clean cloth around to wipe up any spills.

Local Temperatures

Local ambient temperatures have a large affect on your results. Make sure that you do your experiments in a place that has a constant ambient temperature. TecQuipment's results are based on a standard room temperature of approximately 20°C.

Fan, Heater and Pump Performance

The fan, heater and pump may not reach the maximum values suggested in the experiment if you use lower voltages than recommended (see **Technical Details** on page 15). The equipment will still work but you may need to reduce the recommended experiment values.

At Least Two People

This equipment is simple to use, but you must record several readings while you do experiments. Unless you have TecQuipment's VDAS, TecQuipment recommend that you have at least one person to help you do experiments. One person can record the results, while the other controls the experiment.

Thermal Equilibrium for Accurate Results

Your results will be more accurate if you wait for the temperatures to stabilize before you take results. This is when the heat exchanger system reaches 'thermal equilibrium' and can take several minutes.