

ACB 4003 Exam 2024

Q1: [10 marks]

$$(i) \quad \cancel{3x} + \cancel{4y} - 2z + \cancel{2x} - \cancel{9y} + 3z + \cancel{5x} - \cancel{6y} + 10z \\ = 10x - 11y + 11z \quad [2]$$

$$(ii) \quad x(x+5) - (x-4) = x^2 + 5x - x + 4 \\ = x^2 + 4x + 4 \quad [2]$$

$$(iii) \quad 3q^1r^1 \times 4p^1q^3 \times 5q^1r^2 = 60p^1q^5r^3 \quad [2]$$

$$(iv) \quad \frac{12p^2q^3r}{6p^3q^3r^{-3}} = 2p^{-1}q^0r^4 = \frac{2r^4}{p} \quad \text{if } q \neq 0 \quad [2]$$

$$(v) \quad \frac{\cancel{2}fg}{\cancel{8}16h} \times \frac{\cancel{8}gh^2}{\cancel{4}fh} \times \frac{\cancel{32}fh^3}{\cancel{64}f^3} \\ = \frac{1}{8} \frac{f^2g^2h^5}{f^4h^2} \\ = \frac{1}{8} \frac{g^2h^3}{f^2} \quad [2]$$

Q2: [10 MARKS]

(i) $x^2 + 10x + 25 = 0$

$$(x + 5)(x + 5) = 0$$

$\Rightarrow$ either or

$$x + 5 = 0$$

$$x + 5 = 0$$

$$x = -5$$

$$x = -5$$

$x = -5$ (twice)

[5]

(ii) $\frac{(x-3)}{4} = x + \frac{(2x-4)}{8}$

Mult by 8 $\Rightarrow$

$$2(x-3) = 8x + 2x - 4$$

$$2x - 6 = 8x + 2x - 4$$

$$-6 + 4 = 8x$$

$$-2 = 8x$$

$$-\frac{1}{4} = x$$

[5]

Q.3 [15 marks]

$$(i) \quad 4^{2x-1} = 3^{x+1}$$
$$\log_{10} 4^{2x-1} = \log_{10} 3^{x+1}$$

$$(2x-1) \log_{10} 4 = (x+1) \log_{10} 3$$

$$2x \log_{10} 4 - \log_{10} 4 = x \log_{10} 3 + \log_{10} 3$$

$$2x \log_{10} 4 - x \log_{10} 3 = \log_{10} 4 + \log_{10} 3$$

$$x [2 \log_{10} 4 - \log_{10} 3] = \log_{10} 4 + \log_{10} 3$$

$$x = \frac{[\log_{10} 4 + \log_{10} 3]}{[2 \log_{10} 4 - \log_{10} 3]}$$

$$= \underline{1.48} \quad \text{2dp's.} \quad [6]$$

or

$$x = \frac{\log_{10}(4 \times 3)}{\log_{10}(16/3)}$$

$$x = \frac{\log_{10} 12}{\log_{10}(16/3)}$$

$$x = 1.48 \text{ (2dp's)}$$

$$(ii) \log(x^2+4) - \log(x+8) = \log x$$

$$\Rightarrow \log \left[\frac{x^2+4}{x+8} \right] = \log x$$

$$\Rightarrow \frac{x^2+4}{x+8} = x$$

$$\Rightarrow x^2+4 = x(x+8)$$

$$\Rightarrow \cancel{x^2}+4 = \cancel{x^2}+8x$$

$$\Rightarrow 4 = 8x$$

$$\underline{\underline{\frac{1}{2} = x}}$$

[5]

$$\underline{b} \quad i) \log(x^2 y z) = \log x^2 + \log y + \log z$$

$$= 2 \log x + \log y + \log z \quad [2]$$

$$ii) \log \left(\frac{x^4 y^2}{z} \right) = \log x^4 + \log y^2 - \log z$$

$$= 4 \log x + 2 \log y - \log z. \quad [2]$$

Q.4: [10 marks]

$$\underline{a} = 2\underline{i} - 3\underline{j} + 4\underline{k} \quad \underline{b} = \underline{i} + 3\underline{j} - \underline{k}$$

$$\begin{aligned} \text{(i)} \quad \underline{a} \cdot \underline{b} &= (2\underline{i} - 3\underline{j} + 4\underline{k}) \cdot (\underline{i} + 3\underline{j} - \underline{k}) \\ &= (2 \times 1) + (-3 \times 3) + (4 \times -1) \\ &= 2 - 9 - 4 \\ &= -11 \end{aligned} \quad [2]$$

$$\begin{aligned} \text{(ii)} \quad \underline{a} \cdot \underline{b} &= |\underline{a}| |\underline{b}| \cos \theta \\ -11 &= \sqrt{2^2 + (-3)^2 + 4^2} \cdot \sqrt{1^2 + 3^2 + (-1)^2} \cos \theta \\ -11 &= \sqrt{4 + 9 + 16} \cdot \sqrt{1 + 9 + 1} \cos \theta \\ -11 &= \sqrt{29} \sqrt{11} \cos \theta \end{aligned} \quad [3]$$

$$\frac{-11}{\sqrt{29 \times 11}} = \cos \theta$$

$$\Rightarrow \theta = \cos^{-1} \left[\frac{-11}{\sqrt{29 \times 11}} \right]$$

$$\underline{\theta = 128.02^\circ \text{ (2dp's)}}$$

$$\begin{aligned} \text{(iii)} \quad \underline{c} &= -3\underline{i} + \underline{j} + 0\underline{k} \\ \underline{b} &= \underline{i} + 3\underline{j} - \underline{k} \end{aligned}$$

$$\begin{aligned} \underline{c} \cdot \underline{b} &= -3 + 3 + 0 \\ &= 0 \end{aligned}$$

$\therefore \underline{c}, \underline{b}$ are tr.

[2]

Q11 If $\underline{d} = \underline{a} \times \underline{b}$ Then

$$\underline{d} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -3 & 4 \\ 1 & 3 & -1 \end{vmatrix}$$

$$\underline{d} = +\underline{i} \begin{vmatrix} -3 & 4 \\ 3 & -1 \end{vmatrix} - \underline{j} \begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix} + \underline{k} \begin{vmatrix} 2 & -3 \\ 1 & 3 \end{vmatrix}$$

$$\underline{d} = +\underline{i} \{3 - 12\} - \underline{j} \{-2 - 4\} + \underline{k} \{6 - (-3)\}$$

$$\underline{d} = \underline{i} \{-9\} - \underline{j} \{-6\} + \underline{k} \{9\}$$

$$\underline{d} = -9\underline{i} + 6\underline{j} + 9\underline{k}$$

$$\underline{\hat{n}} = \frac{\underline{d}}{|\underline{d}|} = \frac{-9\underline{i} + 6\underline{j} + 9\underline{k}}{\sqrt{(-9)^2 + (6)^2 + (9)^2}}$$

$$\underline{\hat{n}} = \frac{-9\underline{i} + 6\underline{j} + 9\underline{k}}{\sqrt{81 + 36 + 81}}$$

$$\underline{\hat{n}} = \frac{-9\underline{i} + 6\underline{j} + 9\underline{k}}{\sqrt{198}}$$

[3]

Q5: [15 marks]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{bmatrix}$$

$$|A| = +1 \begin{vmatrix} -1 & 1 \\ 4 & -2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} + 3 \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

$$= +1 [2 - 4] - 2 [-4 - 3] + 3 [8 - (-3)]$$

$$= 1(-2) - 2(-7) + 3(11)$$

$$= -2 + 14 + 33$$

$$\underline{|A| = 45}$$

Matrix of Co-factors

$$C = \begin{bmatrix} + \begin{vmatrix} -1 & 1 \\ 4 & -2 \end{vmatrix} & - \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} & + \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} \\ - \begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix} & + \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} & - \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \\ + \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} & - \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} & + \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 7 & 11 \\ +16 & -11 & 2 \\ 5 & 5 & -5 \end{bmatrix}$$

Then The Transposed matrix is

$$C^T = \begin{bmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{bmatrix}$$

$$\text{So } A^{-1} = \frac{1}{|A|} \cdot C^T$$

$$A^{-1} = \frac{1}{45} \begin{bmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{bmatrix}$$

(15)

Q6: [10 marks]

$$x + 2y + 3z = 5$$

$$2x - y + z = 0$$

$$3x + 4y - 2z = 2$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$$
$$\underline{A} \quad \underline{x} = \underline{b}$$

$$\therefore \underline{A}^{-1} \cdot \underline{A} \cdot \underline{x} = \underline{A}^{-1} \underline{b}$$
$$\underline{x} = \underline{A}^{-1} \underline{b}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{45} \begin{bmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{45} \begin{bmatrix} -10 + 0 + 10 \\ 35 + 0 + 10 \\ 55 + 0 - 10 \end{bmatrix}$$

$$\Rightarrow \underline{x = 0 \quad y = 1, \quad z = 1.}$$

Q.7 [20 MARKS]

(a) (i) $y = 5x - 3$

$$\frac{dy}{dx} = 5 \quad (2)$$

(ii) $y = 4x + 10x^{1/2} + 5/x$

$$\begin{aligned} \frac{dy}{dx} &= 4 + 10 \cdot \frac{1}{2} x^{-1/2} \\ &= 4 + 5x^{-1/2} \\ &= 4 + \frac{5}{\sqrt{x}} \end{aligned} \quad (2)$$

(iii) $y = 2x^{-5} + 9x^3$

$$\begin{aligned} \frac{dy}{dx} &= -10x^{-6} + 27x^2 \\ \frac{dy}{dx} &= \frac{-10}{x^6} + 27x^2 \end{aligned} \quad (2)$$

(iv) $y = \cos(20x)$

$$\frac{dy}{dx} = -20\sin(20x) \quad (2)$$

(v) $y = x^2 \cdot \sin(x)$ Product Rule

$$\begin{aligned} \frac{dy}{dx} &= x^2 \cdot \cos(x) + \sin(x) \cdot 2x \\ &= x[x \cos(x) + 2 \sin(x)] \end{aligned} \quad (2)$$

(vi) $y = (8x+3)^5$

$$\begin{aligned} \frac{dy}{dx} &= 5(8x+3)^4 \cdot 8 \\ &= 40(8x+3)^4 \end{aligned}$$

Chain Rule (2)

(vii) $y = 2e^x + \ln(x)$

$$\frac{dy}{dx} = 2e^x + \frac{1}{x} \quad (2)$$

(b) $y = \frac{(x+3)}{(x^2+1)}$ (Quotient Rule)

$$\frac{dy}{dx} = \frac{(x^2+1) \cdot 1 - (x+3) \cdot 2x}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2-6x}{(x^2+1)^2}$$

$$= \frac{-x^2-6x+1}{(x^2+1)^2}$$

when $x=2$,

$$\frac{dy}{dx} = \frac{-(2)^2-6(2)+1}{(2^2+1)^2}$$

$$\frac{dy}{dx} = \frac{-15}{25} = -\frac{3}{5} \quad (2)$$

Q 8: [10 marks]

$$\begin{aligned} \text{(i)} \int (4x + x^3) dx &= \frac{4x^2}{2} + \frac{x^4}{4} + C \\ &= \underline{2x^2 + \frac{x^4}{4} + C} \quad [2] \end{aligned}$$

$$\begin{aligned} \text{(ii)} \int (2x + 3e^x) dx &= \frac{2x^2}{2} + 3e^x + C \\ &= \underline{x^2 + 3e^x + C} \quad [2] \end{aligned}$$

$$\text{(iii)} \int \left(\frac{1}{x} + 7 \right) dx = \ln x + 7x + C \quad [2]$$

$$\begin{aligned} \text{(iv)} \int (x^2 - 1)(3x + 4) dx &= \int (3x^3 - 3x + 4x^2 - 4) dx \\ &= \underline{\frac{3x^4}{4} - \frac{3x^2}{2} + \frac{4x^3}{3} - 4x + C} \quad [2] \end{aligned}$$

$$\begin{aligned} \text{(v)} \int_3^6 (3x^2 + 2x) dx &= \left[\frac{3x^3}{3} + \frac{2x^2}{2} \right]_3^6 \\ &= [x^3 + x^2]_3^6 \\ &= [(6)^3 + (6)^2] - [(3)^3 + (3)^2] \\ &= 216 + 36 - 27 - 9 \\ &= \underline{216} \quad [2] \end{aligned}$$

