



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

Yr Athrofa
Centre for Education
Addysg i Gymru | Education for Wales



Partneriaeth Dysgu Proffesiynol **Yr Athrofa**
Yr Athrofa Professional Learning Partnership

Trawsnewid Addysg Gyda'n Gilydd Transforming Teaching Together



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ECAD 6015

Arwain y Dysgu (gyda SAC): Beth, sut a pham ydw i'n addysgu?

Leading the Learning (with QTS): what, how and why am I teaching?

Sesiwn 2 - Addysgeg Datrys Problemau

Session 2 - The Pedagogy of Problem Solving

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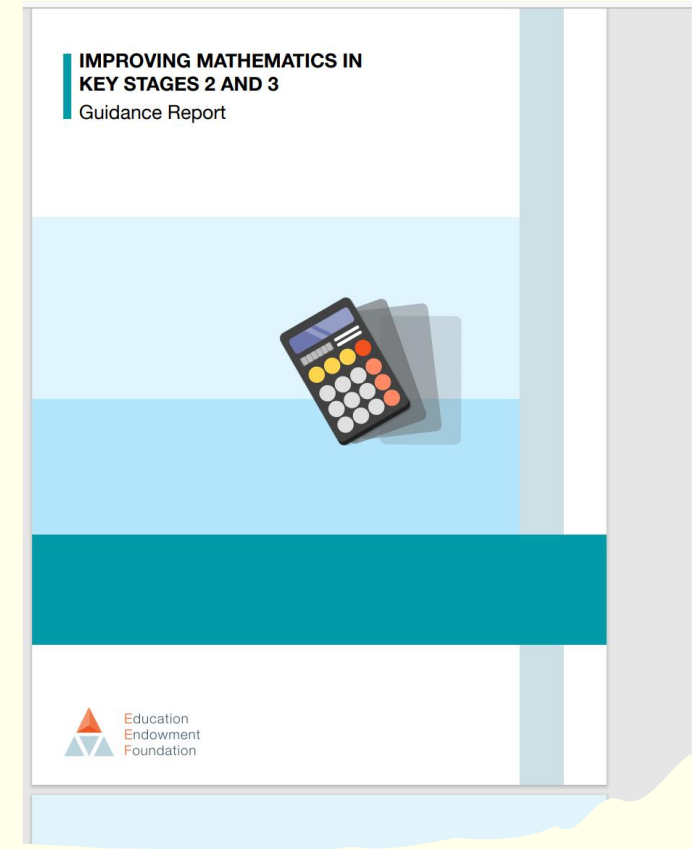


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200
1822 - 2022

Trawsnewid Addysg, Trawsnewid Bywydau
Transforming Education, Transforming Lives



Research:
Education Endowment Foundation (EEF) (2022)
'Improving Mathematics in Key Stages 2 and 3'
Guidance Report suggests some key actions that can
help pupils to improve their skills in problem-solving.

[EEF-Improving-Mathematics-in-Key-Stages-2-and-3-2022-Update.pdf](#)

Recommendation 1	Use assessment to build upon pupils' existing knowledge and understanding	8
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Box D: What is a problem-solving strategy?

A problem-solving strategy is a general approach to solving a problem. The same general strategy can be applied to solving a variety of different problems. For example, a useful problem-solving strategy is to identify a simpler but related problem. Discussing the solution to the simpler problem can give insight into how the original, harder problem may be tackled and the underlying mathematical structure.

A strategy is different from an algorithm, which is a well-established sequence of predetermined steps that are executed in a particular order to carry out a commonly-required procedure.

Evidence summary

- The review identified three relevant meta-analyses concerned with the teaching of problem-solving. These provide some evidence for the use of problem-solving, although the effects were varied.
- A systematic review by the US What Works Clearinghouse provides evidence for specific teaching approaches. The evidence strength is judged to be strongest in support of the use of visual representations and worked examples, and encouraging pupils to monitor and reflect on the problem-solving process.

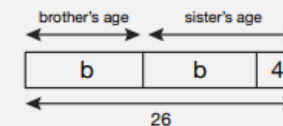
Box E: Using the bar model to compare strategies

A class was working on this problem:

*'A sister is 4 years older than her brother.
The total of their ages is 26.
How old are they?'*

One pupil used the bar model to solve the problem:

So the brother is 11 years old and the sister is 15 years old.



$$b + b + 4 = 26$$

$$\text{so } b + 4 = 22$$

Another pupil said, 'Couldn't we just halve 26 and then add and take away 4? So their ages are $13 + 4$ and $13 - 4$, which is 17 and 9.'

Another pupil said, 'No, you don't add and take away 4, you add and take away 2'. The teacher drew this revised bar model diagram:

'How does this help us to see whether it's 4 or 2 that you add and subtract?'



A pupil said, 'There are two lots of b in the 26, there is only one 4, so it was correct to add and subtract 2, not 4.'

Strategaethau Effeithiol ar gyfer Datrys Problemau

- Dewis tasgau datrys problemau y mae disgyblion yn eu gwneud ar eu cyfer ddim yn cael atebion parod
- Eu dysgu i ddefnyddio a chymharu gwahanol dulliau
- Dangos iddynt sut i holi a defnyddio eu gwybodaeth bresennol i ddatrys problemau
- Defnyddio enghreifftiau wedi'u gweithio i'w galluogi i ddadansoddi y defnydd o wahanol strategaethau
- Gofyn i ddisgyblion fonitro, myfyrio ar, a chyfathrebu eu datrys problemau.

Effective Strategies for Problem Solving

1. Select problem solving tasks for which pupils do not have ready-made solutions
2. Teach them to use and compare different approaches
3. Show them how to interrogate and use their existing knowledge to solve problems
4. Use worked examples to enable them to analyse the use of different strategies
5. Require pupils to monitor, reflect on, and communicate their problem solving.

1. Dewiswch dasgau datrys problemau gwirioneddol nad oes gan ddisgyblion ddulliau wedi'u hymarfer yn dda i'w datrys. Weithiau cymerir datrys problemau i olygu cwestiynau arferol wedi'u gosod mewn cyd-destun, neu 'broblemau geiriau', sydd wedi'u cynllunio i ddangos y defnydd o ddull penodol.

Ond os yw'n ofynnol i fyfyrwyr gyflawni gweithdrefn neu algorithm penodol i gyrraedd yr ateb, nid yw'n ddatrys problemau mewn gwirionedd; yn hytrach, dim ond ymarfer y weithdrefn ydyw.

1. Select genuine problem-solving tasks that pupils do not have well-rehearsed, readymade methods to solve. Sometimes problem-solving is taken to mean routine questions set in context, or 'word problems', designed to illustrate the use of a specific method.

But if students are only required to carry out a given procedure or algorithm to arrive at the solution, it is not really problem solving; rather, it is just practising the procedure.

2. Addysgu disgyblion i ddefnyddio a chymharu gwahanol ddulliau. Yn aml mae sawl ffordd o fynd i'r afael â phroblem.

Gellir dysgu llawer trwy archwilio gwahanol atebion i'r un broblem a chwilio am debygrwydd mewn dulliau datrys gwahanol broblemau.

Dysgwch ddisgyblion i holi a defnyddio eu dealltwriaeth o fathemateg presennol i ddatrys problemau.

Dylid annog disgyblion i chwilio am eu gwybodaeth am broblemau tebyg y maent wedi dod ar eu traws am strategaethau a oedd yn llwyddiannus, ac am ffeithiau a chysyniadau a allai fod yn berthnasol.

2. Teach pupils to use and compare different approaches. There are often multiple ways to approach a problem.

Much can be learned by examining different solutions to the same problem and looking for similarities in solution approaches to different problems. Teach pupils to interrogate and use their existing mathematical knowledge to solve problems.

Pupils should be encouraged to search their knowledge of similar problems they have encountered for strategies that were successful, and for facts and concepts that might be relevant.

3. Annog disgyblion i ddefnyddio cynrychioliadau gweledol.

Helpu myfyrwyr i ddefnyddio diagramau a chynrychioliadau priodol sy'n rhoi mewnwleddiad i strwythur problem ac i'w fformiwla mathemategol.

3. Encourage pupils to use visual representations.

Help students to make use of appropriate diagrams and representations that provide insight into the structure of a problem and into its mathematical formulation.

$$\begin{aligned} 4x + 5(-4) &= 16 \\ 4x - 20 &= 16 \quad (+20) \quad (9, -4) \\ 4x &= 36 \quad (\div 4) \\ x &= 9 \end{aligned}$$

4. Defnyddiwch enghreifftiau wedi'u gweithio i alluogi disgyblion i ddadansoddi'r defnydd o wahanol strategaethau. Mae enghreifftiau wedi'u gweithio, neu 'broblemau wedi'u datrys', yn cyflwyno'r broblem ac ateb cywir gyda'i gilydd, maent yn dileu'r angen i gyflawni'r gweithdrefnau sydd eu hangen i gyrraedd yr ateb ac yn galluogi disgyblion i ganolbwyntio ar y rhesymu a'r strategaethau dan sylw.

Gall enghreifftiau wedi'u gweithio fod yn gyflawn, anghyflawn neu anghywir, yn fwriadol yn cynnwys gwallau a chamsyniadau cyffredin i ddysgwyr eu datgelu. Mae dadansoddi a thrafod enghreifftiau yn helpu myfyrwyr i ddatblygu dealltwriaeth ddyfnach o'r prosesau rhesymegol a ddefnyddir i ddatrys problemau.

4. Use worked examples to enable pupils to analyse the use of different strategies. Worked examples, or 'solved problems', present the problem and a correct solution together, they remove the need to carry out the procedures required to reach the solution and enable pupils to focus on the reasoning and strategies involved.

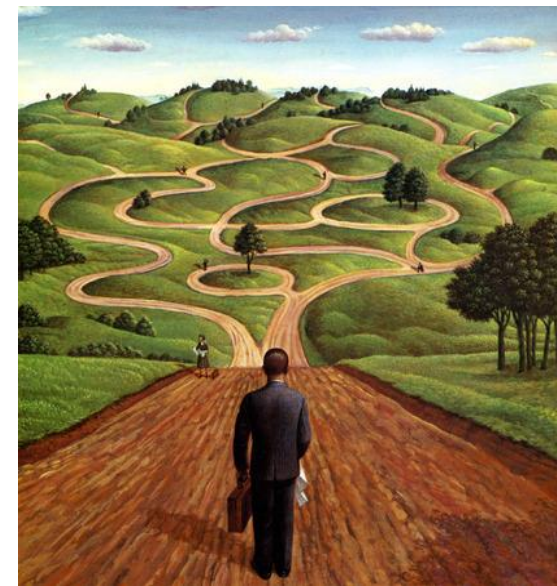
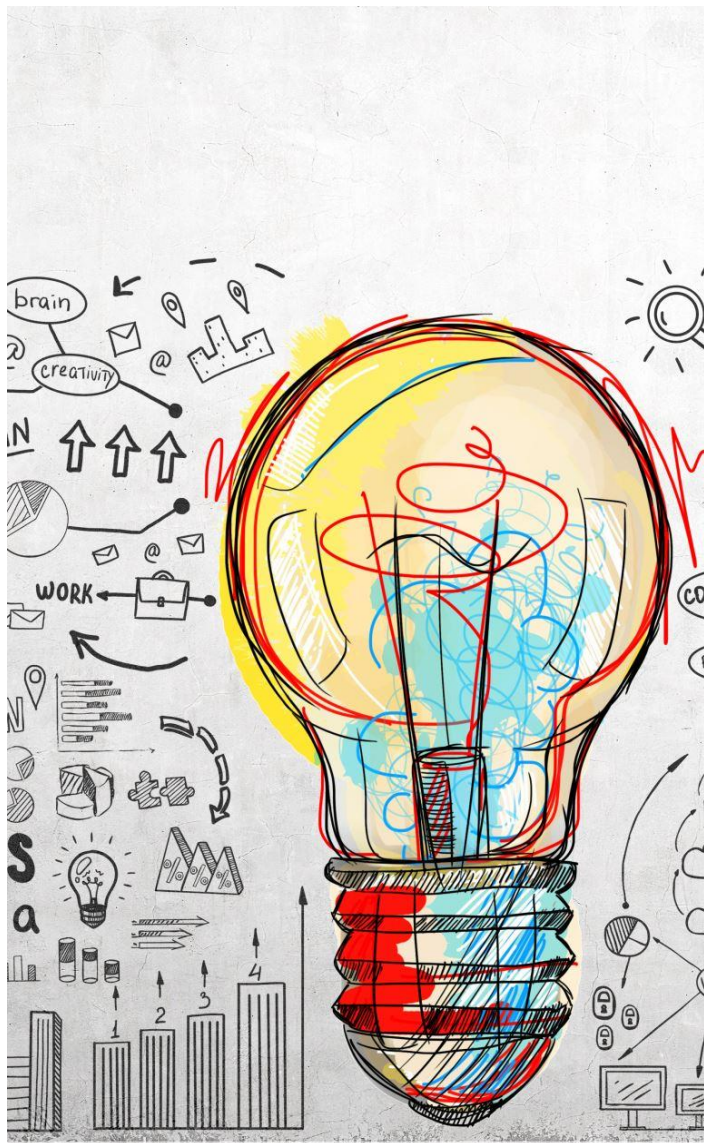
Worked examples may be complete, incomplete, or incorrect, deliberately containing common errors and misconceptions for learners to uncover. Analysing and discussing worked examples helps students develop a deeper understanding of the logical processes used to solve problems.

5. Gofyn i ddisgyblion fonitro, myfyrio ar, a chyfathrebu eu rhesymu a'u dewis o strategaeth. Wrth weithio ar broblem, annog disgyblion i ofyn cwestiynau fel, 'Beth ydw i'n ceisio gweithio allan?', 'Sut ydw i'n mynd amdano?', 'A yw'r dull rydw i'n ei gymryd yn gweithio?', a 'Pa ddulliau eraill y gallwn i roi cynnig arnynt?'

Pan fydd y broblem wedi'i chwblhau, anogwch ddisgyblion i ofyn cwestiynau fel, 'Beth weithiodd yn dda wrth ddatrys y broblem hon?', 'Beth nad oedd yn gweithio'n dda?', 'Pa broblemau eraill y gellid eu datrys drwy ddull tebyg?', 'Pa broblemau tebyg i'r un hon rydw i wedi'u datrys yn y gorffennol?' Dylai disgyblion gyfathrebu eu meddwl ar lafar ac yn ysgrifenedig - gan ddefnyddio cynrychioliadau, ymadroddion a hafaliadau - i athrawon a disgyblion eraill.

5. Require pupils to monitor, reflect on, and communicate their reasoning and choice of strategy. While working on a problem, encourage pupils to ask questions like, 'What am I trying to work out?', 'How am I going about it?', 'Is the approach that I'm taking working?', and 'What other approaches could I try?'

When the problem is completed, encourage pupils to ask questions like, 'What worked well when solving this problem?', 'What didn't work well?', 'What other problems could be solved by a similar approach?', 'What similar problems to this one have I solved in the past?' Pupils should communicate their thinking verbally and in writing—using representations, expressions, and equations—to both teachers and other pupils.



Ble ydw i'n dechrau?

Mae modelau datrys problemau yn ddull y gellir ei ddefnyddio i helpu dysgwyr i sefydlu ffordd o fynd i'r afael â phroblem.

Where do I start?

Models of problem solving are a method that can be used to help learners establish a way of tackling a problem.

Model Datrys Problemau

George Polya

(1887 – 1985)



Yn ei lyfr *How to Solve it* (1945) nododd Polya **bedair egwyddor sylfaenol** datrys problemau.

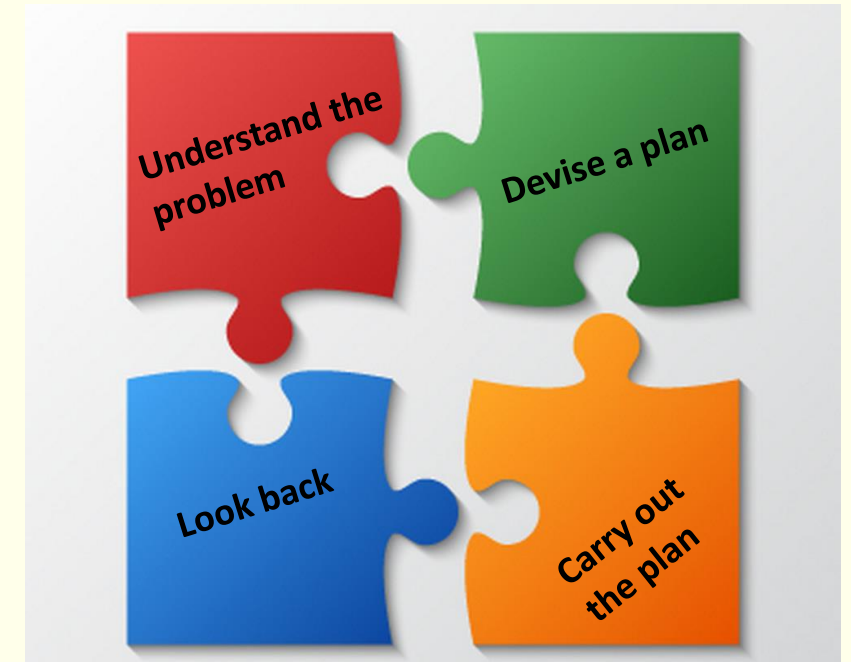


G. PÓLYA, 1887–1985

Problem Solving Model

George Polya

(1887 – 1985)



[polya.pdf \(berkeley.edu\)](http://polya.pdf(berkeley.edu))

In his book *How to Solve it* (1945) Polya identified **four basic principles** of problem solving.

Egwyddor Gyntaf Polya: **Deall y broblem**

Gall hyn ymddangos yn rhy amlwg fel egwyddor gyntaf, ond mae dysgwyr yn aml yn cael eu rhwystro yn eu hymdrechion i ddatrys problemau dim ond oherwydd nad ydynt yn ei ddeall yn llawn, neu hyd yn oed yn rhannol. Bydd llawer o ddysgwyr yn cymhwyso algorithm neu dechneg a ddysgwyd yn ddiweddar heb dreulio amser i ddeall yn llawn beth y gofynnir i'w wneud mewn gwirionedd

Dysgodd Polya athrawon i ofyn cwestiynau i ddysgwyr fel:

- Ydych chi'n deall yr holl eiriau a ddefnyddir yn y broblem?
- Beth y gofynnir i chi ddod o hyd iddo neu ei ddangos?
- Allwch chi ailddatgan y broblem yn eich geiriau eich hun?
- Allwch chi feddwl am lun neu ddiagram a allai eich helpu i ddeall y broblem?
- A oes digon o wybodaeth i'ch galluogi i ddod o hyd i ateb?

Polya's First Principle: **Understand the problem**

This may seem too obvious as a first principle, yet learners are often hindered in their efforts to solve problems simply because they do not understand it fully, or even in part. Many learners will apply a recently learnt algorithm or technique without spending time to fully understand what is actually being asked to do

Polya taught teachers to ask learners questions such as:

- Do you understand all the words used in the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

Ail Egwyddor Polya: Dyfeisio cynllun

Mae Polya yn sôn bod yna lawer o ffyrdd rhesymol o ddatrys problemau. Mae'r sgil i ddewis strategaeth briodol yn cael ei ddysgu orau trwy ddatrys llawer o broblemau. Mae rhestr rhannol o strategaethau yn cynnwys:

- Chwilio am batrwm
- Dyfalu a gwirio
- Treial a gwella
- Creu rhestr drefnus
- Creu tabl
- Arlunio llun
- Arlunio diagram/diagram graddfa wedi'i labelu
- Defnyddio cymesuredd
- Dileu posibiliadau
- Ystyried achosion arbennig
- Datrys problem symlach
- Defnyddio fformiwla
- Ffurfio a datrys hafaliad

Polya's Second Principle: Devise a plan

Polya mentions that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems. A partial list of strategies includes:

- Look for a pattern
- Guess and check
- Trial and improvement
- Make an orderly list
- Create a table
- Draw a picture
- Draw a labelled diagram/scale diagram
- Use symmetry
- Eliminate possibilities
- Consider special cases
- Solve a simpler problem
- Use a formula
- Form and solve an equation

Trydydd Egwyddor Polya: Cyflawni'r cynllun

Mae'r cam hwn fel arfer yn haws na dyfeisio'r cynllun. Yn gyffredinol, mae angen gofal ac amynedd ar ddatryswyr problemau, o ystyried bod ganddynt y sgiliau angenrheidiol. Gellir annog dysgwyr i barhau â'r cynllun y maent wedi'i ddewis, fodd bynnag, os yw'n parhau i beidio â gweithio, ei daflu a dewis un arall.

Pedwerydd Egwyddor Polya: Edrych yn ôl

Mae Polya yn sôn y gellir ennill llawer trwy ofyn i ddysgwyr gymryd amser i fyfyrrio ac edrych yn ôl ar yr hyn maen nhw wedi'i wneud, beth oedd yn gweithio, a beth ddim. Bydd gwneud hyn yn eu galluogi i ragweld pa strategaeth i'w defnyddio i ddatrys problemau yn y dyfodol.

Polya's Third Principle: Carry out the plan

This step is usually easier than devising the plan. In general, problem solvers need care and patience, given that they *have the necessary skills*. Learners can be encouraged to persist with the plan that they have chosen, however, if it continues not to work discard it and choose another.

Polya's Fourth Principle: Look back

Polya mentions that much can be gained by asking learners to take time to reflect and look back at what they have done, what worked, and what didn't. Doing this will enable them to predict what strategy to use to solve future problems.

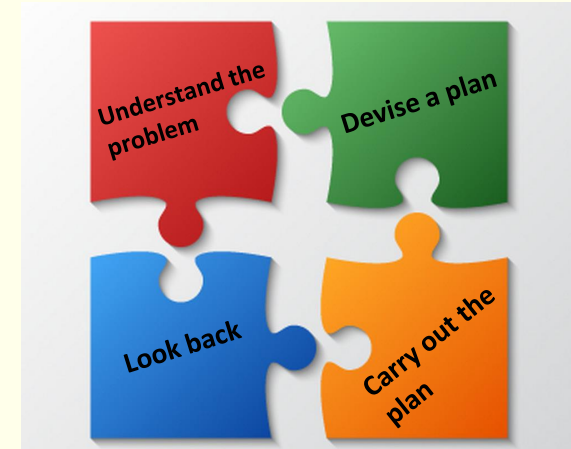
Beth yw'r cysylltiad rhwng Model Datrys Problemau Ploya a'r Cwricwlwm i Gymru?

- gosod safonau uchel eu hunain a cheisio a mwynhau her
- yn adeiladu corff o wybodaeth ac yn meddu ar y sgiliau i gysylltu a chymhwyso'r wybodaeth honno mewn gwahanol gyd-destunau
- yn **cwestiynu ac yn mwynhau datrys problemau**
- yn gallu cyfathrebu'n effeithiol mewn gwahanol ffurfiau a lleoliadau, gan ddefnyddio Cymraeg a Saesneg
- yn gallu esbonio'r syniadau a'r cysyniadau maen nhw'n dysgu amdanynt
- gall ddefnyddio rhif yn effeithiol mewn gwahanol gyd-destunau
- deall sut i ddehongli data a chymhwyso cysyniadau mathemategol
- defnyddio technolegau digidol yn greadigol i gyfathrebu, dod o hyd i wybodaeth a dadansoddi
- Ymgymryd ag ymchwil a **gwerthuso'n feirniadol** yr hyn maen nhw'n ei ddarganfod ac yn barod i ddysgu trwy gydol eu bywydau



What is the link between Ploya's Problem Solving Model and the Curriculum for Wales?

- set themselves high standards and seek and enjoy challenge
- are building up a body of knowledge and have the skills to connect and apply that knowledge in different contexts
- are **questioning and enjoy solving problems**
- can communicate effectively in different forms and settings, using both Welsh and English
- **can explain the ideas and concepts they are learning about**
- **can use number effectively in different contexts**
- **understand how to interpret data and apply mathematical concepts**
- use digital technologies creatively to communicate, find and analyse information
- undertake research **and evaluate critically what they find**
and **are ready to learn throughout their lives**





Eich tro chi!

It's your turn!



Cynlluniwch set o gwestiynau datrys problemau i naill ai **gam cynnydd 1, 2 neu 3**.
Sicrhewch bod y cwestiynau'n **addas** i'r Cam Cynnydd rydych chi wedi ei ddewis.
Defnyddiwch y **fframweithiau** (Moodle)

Plan a set of problem-solving questions for either **progression stage 1, 2 or 3**.
Make sure the questions are **suitable** for the Progress Step you have chosen.
Use the **frameworks** (Moodle)

