

Q1

$$\epsilon_x = 24 \times 10^{-6}$$

$$\epsilon_y = -18 \times 10^{-6}$$

$$\nu = 0.28$$

$$E = 170 \times 10^9 \text{ Pa}$$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{170 \times 10^9}{1-0.28^2} \times (24 \times 10^{-6} + (0.28 \times -18 \times 10^{-6})) = 3.5 \times 10^6 \text{ Pa} \quad (\text{tension})$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{170 \times 10^9}{1-0.28^2} \times (-18 \times 10^{-6} + (0.28 \times 24 \times 10^{-6})) = -2.08 \times 10^6 \text{ Pa} \quad (\text{compressive})$$

ii

$$\sigma_x = 488 \times 10^6 \text{ Pa}$$

$$\sigma_y = -44 \times 10^6 \text{ Pa}$$

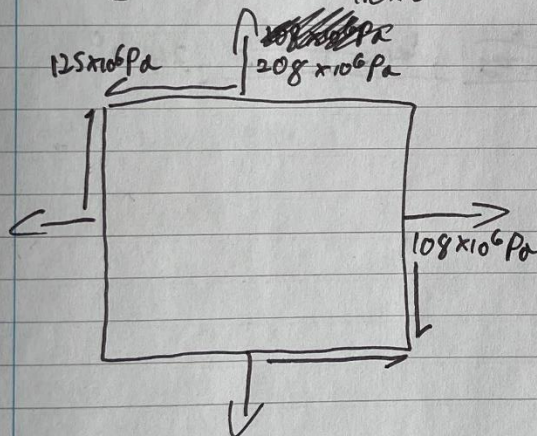
$$\nu = 0.28$$

$$E = 170 \times 10^9 \text{ Pa}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{170 \times 10^9} \times (488 \times 10^6 - (0.28 \times -44 \times 10^6)) = 5.9 \times 10^{-4}$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{170 \times 10^9} \times (-44 \times 10^6 - (0.28 \times 488 \times 10^6)) = -4.04 \times 10^{-4}$$

Q2



$$\sigma_1 = \frac{(\sigma_x + \sigma_y)}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = \frac{(108 \times 10^6 + 208 \times 10^6)}{2} + \frac{\sqrt{(108 \times 10^6 - 208 \times 10^6)^2 + (4 \times 125 \times 10^6)^2}}{2}$$

$$\sigma_1 = 292 \times 10^6 \text{ Pa}$$

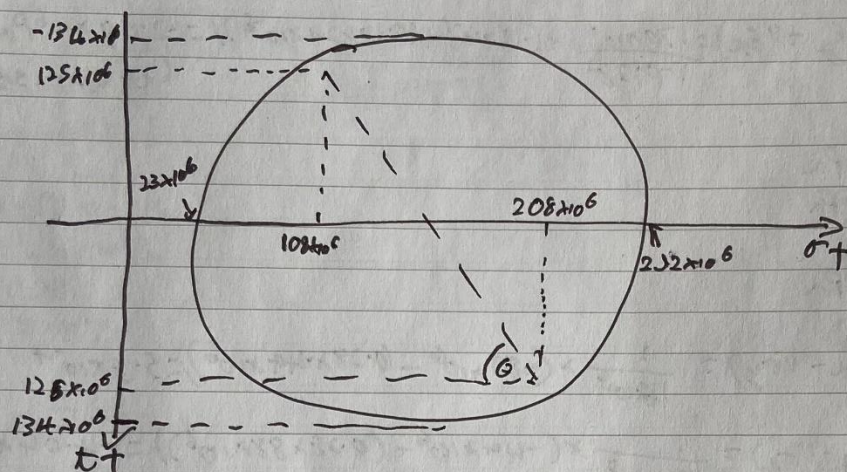
$$\sigma_2 = \frac{(\sigma_x + \sigma_y)}{2} - \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = \frac{(108 \times 10^6 + 208 \times 10^6)}{2} - \frac{\sqrt{(108 \times 10^6 - 208 \times 10^6)^2 + (4 \times 125 \times 10^6)^2}}{2}$$

$$\sigma_2 = 23.4 \times 10^6 \text{ Pa}$$

Q2ii) $\tau_{max} = \frac{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}{2} = \frac{((108 \times 10^6 - 208 \times 10^6)^2 + 4(125 \times 10^6)^2)}{2} = 134.6 \times 10^6 \text{ Pa}$

$\tau_{min} = \frac{-(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}{2} = \frac{-((108 \times 10^6 - 208 \times 10^6)^2 + 4(125 \times 10^6)^2)}{2} = -134.6 \times 10^6 \text{ Pa}$

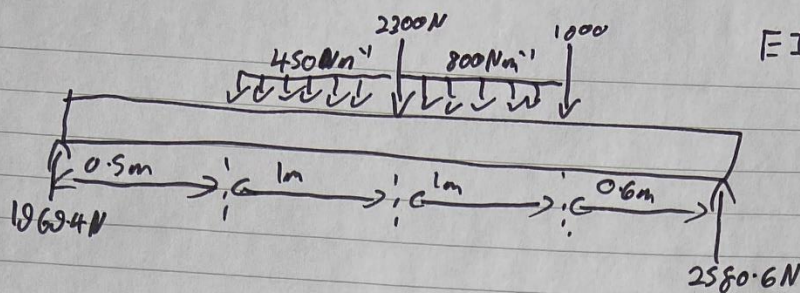
iii



$\tan 2\theta_p = \frac{2\tau}{\sigma_x - \sigma_y}$ $\theta_p = \frac{\tan^{-1} \frac{2\tau}{\sigma_x - \sigma_y}}{2} = \frac{\tan^{-1} \frac{125 \times 10^6}{108 \times 10^6 - 208 \times 10^6}}{2} = 34.1^\circ$

✓

(B3)



$$EI = 1.88 \times 10^6 \text{ Pa}$$

$$(i) \quad EI \frac{d^2 y}{dx^2} = 1969.4(x) - \frac{450(x-0.5)^2}{2} + \frac{450(x-1.5)^2}{2} - 2300(x-1.5) - \frac{800(x-1.5)^2}{2} + \frac{800(x-2.5)^2}{2} - 1000(x-2.5)$$

$$(ii) \quad EI \frac{dy}{dx} = \frac{1969.4(x)^2}{2} - \frac{450(x-0.5)^3}{6} + \frac{450(x-1.5)^3}{6} - \frac{2300(x-1.5)^2}{2} - \frac{800(x-1.5)^3}{6} + \frac{800(x-2.5)^3}{6} - \frac{1000(x-2.5)^2}{2} + A$$

$$(iii) \quad EI y = \frac{1969.4(x)^3}{6} - \frac{450(x-0.5)^4}{24} + \frac{450(x-1.5)^4}{24} - \frac{2300(x-1.5)^3}{6} - \frac{800(x-1.5)^4}{24} + \frac{800(x-2.5)^4}{24} - \frac{1000(x-2.5)^3}{6} + Ax + B$$

$$(iv) \quad \text{when } x=0 \quad y=0$$

$$EI y = 0 - 0 + 0 - 0 - 0 + 0 - 0 + 0 + A \cdot 0 + B$$

$$B = 0$$

$$\text{when } x=3.1 \quad y=0$$

$$0 = \frac{1969.4(3.1)^3}{6} - \frac{450(2.6)^4}{24} + \frac{450(1.6)^4}{24} - \frac{2300(1.6)^3}{6} - \frac{800(1.6)^4}{24} + \frac{800(0.6)^4}{24} - \frac{1000(0.6)^3}{6} + 3.1A$$

$$0 = 7224.18 + 3.1A$$

$$-3.1A = 7224.18$$

$$-A = \frac{7224.18}{3.1}$$

$$-A = 2330.38$$

$$A = -2330.38$$

(v)

$$\frac{EI y}{EI}$$

$$(\text{Centre} = 1.55)$$

$$EI y = \frac{1969.4(1.55)^3}{6} - \frac{450(1.05)^4}{24} + \frac{450(0.05)^4}{24} - \frac{2300(0.05)^3}{6} - 2330.38(1.55)$$

$$EI y = -2412.63$$

$$\theta = \frac{EI y}{EI} = \frac{-2412.63}{1.88 \times 10^6} = -1.28 \text{ mm}$$

Q4

$$D = 0.28 \text{ m}$$

$$R = 0.14 \text{ m}$$

$$L = 0.6 \text{ m}$$

$$n_L = 58\% = 0.58$$

$$n_C = 64\% = 0.64$$

$$P = 14.8 \text{ Bar} = 14.8 \times 10^5 \text{ Pa}$$

$$\sigma_{\max} = 150 \times 10^6 \text{ Pa}$$

$$(i) \sigma_c = \frac{PD}{2t n_L}$$

$$t = \frac{PD}{2\sigma_c n_L}$$

~~$$t = \frac{14.8 \times 10^5 \times 0.28}{2 \times 150 \times 10^6 \times 0.58}$$~~

$$t = \frac{14.8 \times 10^5 \times 0.28}{2 \times 150 \times 10^6 \times 0.58}$$

$$t = 2.38 \text{ mm}$$

$$(ii) \sigma_L = \frac{PD}{4t n_C}$$

$$t = \frac{PD}{4\sigma_L n_C}$$

$$t = \frac{14.8 \times 10^5 \times 0.28}{4 \times 150 \times 10^6 \times 0.64}$$

$$t = 1.08 \text{ mm}$$

(iii) minimum thickness is 2.38 mm

Q5

$$L = 1.38 \text{ m}$$

$$d_o = 19.05 \times 10^{-3} \text{ m} = 0.01905 \text{ m}$$

$$t = 1.2 \text{ mm}$$

$$d_i = 19.05 - 2(1.2) = 16.65 \text{ mm} = 0.01665$$

$$n = 0.25$$

$$E = 200 \times 10^9 \text{ Pa}$$

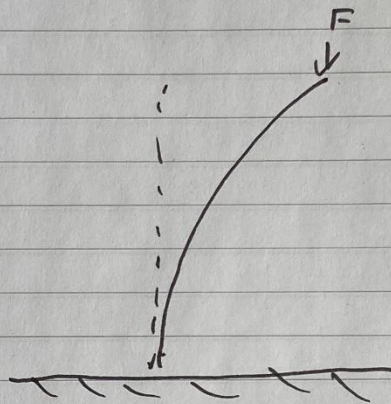
i

$$I = \frac{\pi}{64} (0.01905^4 - 0.01665^4) = 2.69 \times 10^{-9} \text{ m}^4 = 2692.24 \text{ mm}^4$$

$$A = \frac{\pi}{4} (0.01905^2 - 0.01665^2) = 6.73 \times 10^{-5} \text{ m}^2 = 67.55 \text{ mm}^2$$

ii

$$n = 0.25 \quad \text{because}$$



iii

$$F_{CR} = \frac{\pi^2 EI}{L^2} = \frac{0.25 \times \pi^2 \times 200 \times 10^9 \times 2.69 \times 10^{-9}}{1.38^2} = 697.05 \text{ N}$$

iv

$$K = \frac{I}{A} = \frac{2.69 \times 10^{-9}}{6.73 \times 10^{-5}} = 3.997 \times 10^{-5}$$

$$SR = \frac{L}{K} = \frac{1.38}{3.997 \times 10^{-5}} = 34555$$

Steel ring strut

$$K = \frac{I}{A} = \frac{2692.24}{67.55} = 39.86$$

$$SR = \frac{L}{K} = \frac{1380}{39.86} = 34.62$$

Steel $SR = 120$

$120 > 34.62$

so failure will occur at lower loads