

Q1)

$$a)(i) \frac{3}{2x+5} = \frac{6}{x+2}$$

$$3(x+2) = 6(2x+5)$$

$$3x+6 = 12x+30$$

$$6-30 = 12x-3x$$

$$-24 = 9x$$

$$-2.67 = x$$

$$(ii) \frac{(x+1)}{10} - \frac{(7x+1)}{5} = \frac{x}{15}$$

(x30)

$$\frac{3}{30}(x+1) - \frac{6}{30}(7x+1) = \frac{2}{30}x$$

$$3x+3-42x-6 = 2x$$

$$-3 = 2x+42x-3x$$

$$-3 = 41x$$

$$-0.07 = x$$

$$(iii) 5^{x+2} = 3^x \times 7$$

$$\log 5^{x+2} = \log(3^x \times 7)$$

$$(x+2)\log 5 = \log 3^x + \log 7$$

$$(x+2)\log 5 = x \log 3 + \log 7$$

$$(x+2)0.70 = 0.48x + 0.85$$

$$0.70x + 1.40 = 0.48x + 0.85$$

$$0.22x = -0.55$$

$$x = \frac{-0.55}{0.22}$$

$$x = -2.5$$

$$(iv) \log(x+5) + \log(x+3) = 2 \log(3)$$

$$\log(x+5)(x+3) = \log(3x-2)^2$$

$$x^2 + 8x + 15 = 9x^2 - 12x + 4$$

$$0 = 8x^2 - 20x - 11$$

using Quadratic Formula

$$\text{gives } x = 2.96$$

$$x = -0.46$$

$$b) \quad SL = \frac{u+Q}{7u+T} \quad [u]$$

$$SL(7u+T) = u+Q$$

$$3SLu + SLT = u+Q$$

$$3SLu - u = Q - SLT$$

$$u(3SL - 1) = Q - SLT$$

$$u = \frac{Q - SLT}{3SL - 1}$$

$$L=1$$

$$Q=4$$

$$T=2$$

$$u = \frac{4 - 10}{35 - 1} = \frac{-6}{34} = \underline{\underline{-0.2 \text{ to 1 dp}}}$$

$$c)(i) \quad 2x^2 + 11x + 5 = 0$$

$$(2x+1)(x+5) = 0$$

$$x = -\frac{1}{2} \quad x = -5$$

$$(ii) \quad x^2 - 5 = 4x$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$x = 5 \quad x = -1$$

$$(iii) \quad x^2 = -5x + 14$$

$$x^2 + 5x - 14 = 0$$

$$(x+7)(x-2) = 0$$

$$x = -7 \quad x = 2$$

d) (i) $3z^2 + 13z = -4$

$$3z^2 + 13z + 4 = 0$$

$$(3z+1)(z+4)$$

(ii) $x^2 - y^2$

$$(x+y)(x-y) \quad [\text{diff. between 2 squares}]$$

(iii) $uy - ur + ty - tr$

$$u(y-r) + t(y-r)$$

$$(y-r)(u+t)$$

e) $x=3 \quad x=-1$

$$x-3=0 \quad x+1=0$$

$$(x-3)(x+1)=0$$

$$x^2 - 2x - 3 = 0$$

$$[\text{also } y = x^2 - 2x - 3 \text{ accepted}]$$

Q2)

a) $12x^2 + 2x - 5 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(12)(-5)}}{2(12)}$$

gives $x = -0.73$ & $x = 0.57$

b) $y = x^2 + 3x + 1$

$$\frac{dy}{dx} = 2x + 3 \leftarrow \text{Sub numbers into this!}$$

(i) $x = 1 \rightarrow \frac{dy}{dx} = 5$

(ii) $x = 0 \rightarrow \frac{dy}{dx} = 3$

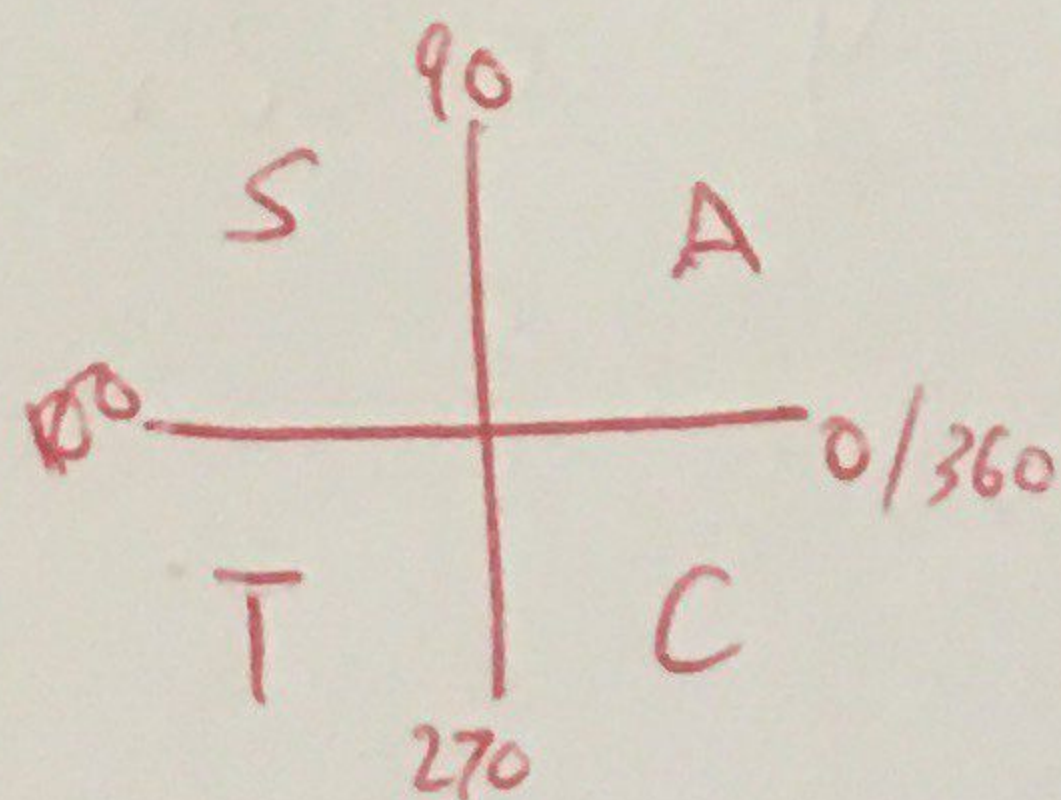
(iii) $x = -1 \rightarrow \frac{dy}{dx} = 1$

c) (i) $\sin x = 0.4$

$$x = \sin^{-1}(0.4)$$

$$x = 23.58^\circ$$

$$x = 156.42^\circ$$



(iii) $4 \tan x = 3$

$$\tan x = 3/4$$

$$x = 36.87^\circ$$

$$x = 216.87^\circ$$

(ii) $5 \cos x = 2$

$$\cos x = 2/5$$

$$x = 66.42^\circ$$

$$x = 293.58^\circ$$

* Remember you can Sub all of your answers into the questions to check that you are correct!

d) $(x-2)(x-3)$

$$(\tan x - 2)(\tan x - 3) = 0$$

$$\tan x = 2$$

$$x = 63.43$$

$$x = 243.43$$

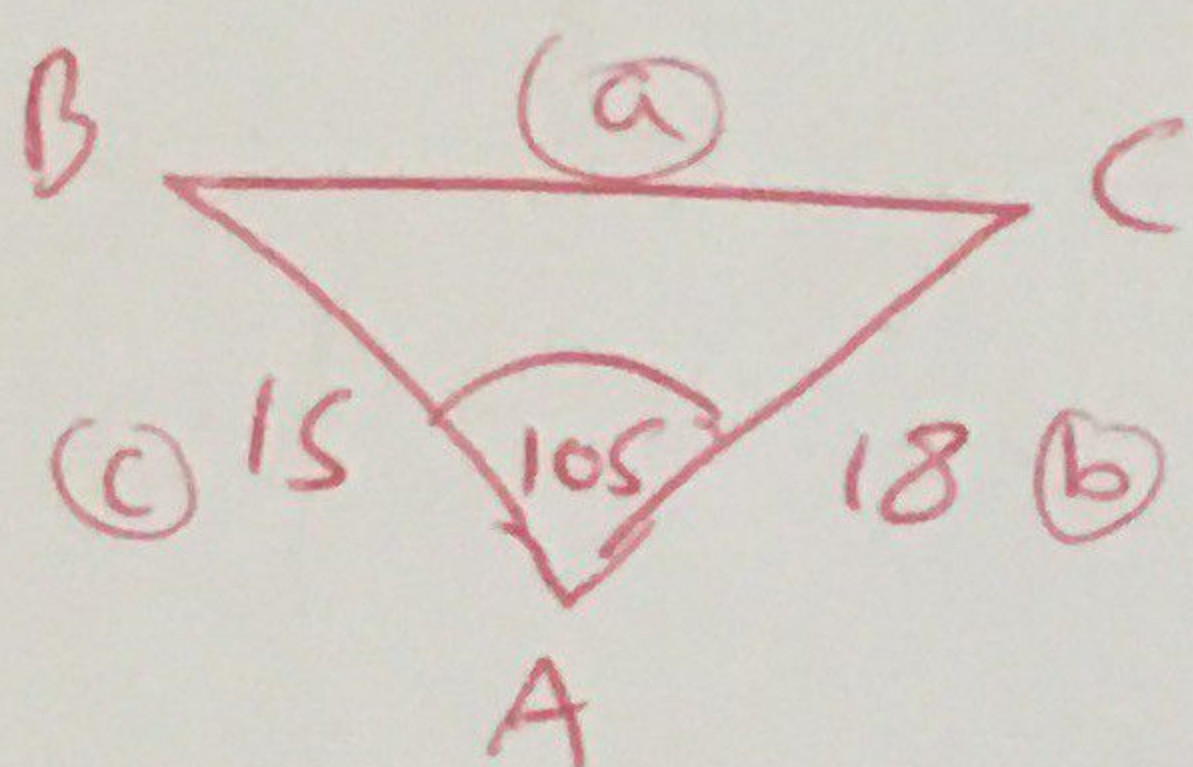
$$\tan x = 3$$

$$x = 71.57$$

$$x = 251.57$$

e) $= 1$

f) Cosine Rule



$$(BC)^2 = (18)^2 + (15)^2 - 2(18)(15)\cos 105$$

$$(BC)^2 = 324 + 225 + 139.76$$

$$BC = \sqrt{688.76}$$

$$BC = 26.24m$$

g) $y = \cos^{-1}\left(\frac{s}{z-x}\right)$

$$\cos y = \frac{s}{z-x}$$

$$z-x = \frac{s}{\cos y}$$

$$z = \frac{s}{\cos y} + x$$

Q3)

a) (i) $y = 5x^3 + 2x^3 - x + 1$

$$\frac{dy}{dx} = 15x^2 + 6x^2 - 1 \quad \text{i.e.} \rightarrow \underline{\underline{21x^2 - 1}}$$

(ii) $y = (5x + 1)^2$

$$y = 25x^2 + 10x + 1$$

$$\frac{dy}{dx} = 50x + 10$$

(iii) $y = x^{1/2} + 5x^{-1}$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - 5x^{-2}$$

(iv) $y = \sin 2x + 4\cos 3x + \tan x \leftarrow \text{all from formula book!}$

$$\frac{dy}{dx} = 2\cos 2x - 12\sin 3x + \sec^2 x$$

(v) $y = \frac{\sin x}{e^{4x}} \leftarrow \text{Quotient Rule}$

$$\frac{dy}{dx} = \frac{(e^{4x})(\cos x) - (\sin x)(4e^{4x})}{(e^{4x})^2}$$

(vi) $y = (3x^5 - 4x + 1)^{1/2} \leftarrow \text{Chain Rule}$

$$\frac{dy}{dx} = \frac{1}{2}(3x^5 - 4x + 1)^{-1/2} (15x^4 - 4)$$

(vii) $y = 5x^3 \sin x \leftarrow \text{Product Rule.}$

$$\frac{dy}{dx} = (\sin x)(15x^2) + (5x^3)(\cos x)$$

$$b) \quad y = 2x^3 + 1$$

$$\frac{dy}{dx} = 6x^2$$

$$\frac{d^2y}{dx^2} = 12x$$

$$\left. \begin{array}{l} \frac{dy}{dx} = 6x^2 \\ \frac{d^2y}{dx^2} = 12x \end{array} \right\} \begin{array}{l} 2(12x) + 6x^2 - 20 - 6x = 6x^2 + 18x - 20 \\ 24x + 6x^2 - 20 - 6x = 6x^2 + 18x - 20 \\ 6x^2 + 18x - 20 = 6x^2 + 18x - 20 \end{array}$$

hence Proved

$$c) \quad y = 2x^4 + 3x^2$$

$$\frac{dy}{dx} = 8x^3 + 6x$$

$$\frac{d^2y}{dx^2} = 24x^2 + 6$$

$$\frac{d^3y}{dx^3} = 48x$$

~~$$\frac{d^4y}{dx^4} = 48$$~~

$$d) \quad z = x^5y^2 + x^3 - 2y^2 + 5$$

$$\frac{\partial z}{\partial x} = 5x^4y^2 + 3x^2$$

$$\frac{\partial^2 z}{\partial x^2} = 20x^3y^2 + 6x$$

$$\frac{\partial^2 z}{\partial y \partial x} = 10x^4y$$

$$\frac{\partial z}{\partial y} = 2x^5y - 4y$$

$$\frac{\partial^2 z}{\partial y^2} = 2x^5 - 4$$

$$\frac{\partial^2 z}{\partial x \partial y} = 10x^4y$$

$$e) \quad y = 2x + 2 \quad (1)$$

$$y = x^2 + 8x + 7 \quad (2)$$

$$(2) - (1)$$

$$0 = x^2 + 6x + 5$$

$$0 = (x+5)(x+1)$$

$$x = -5$$

$$x = -1$$

← Sub in (1)

$$y = -8$$

$$y = 0$$