

Question 1

Stress Analysis May 2022 Exam
Student Number:
A = 2 B = 0 C = 0 D = 3 E = 5 F = 8 G = 1

1. $\epsilon_x = 21 \times 10^{-6}$
 $\epsilon_y = -18 \times 10^{-6}$
 $\nu = 0.28$
 $E = 170 \times 10^9 \text{ Pa}$

a. $\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y)$

$$\sigma_x = \frac{170 \times 10^9}{1-0.28^2} \times (21 \times 10^{-6} + 0.28 \times (-18 \times 10^{-6}))$$

$$\sigma_x = \frac{170 \times 10^9}{1-0.28^2} \times (21 \times 10^{-6} - 5.04 \times 10^{-6})$$

$$\sigma_x = \frac{170 \times 10^9}{1-0.28^2} \times 15.96 \times 10^{-6}$$

$$\sigma_x = 4.603 \times 10^6 \text{ Pa} = 2.94 \times 10^6 \text{ Pa}$$

$$= 4.603 \text{ MPa} = 2.94 \text{ MPa (tensile)}$$

$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x)$

$$\sigma_y = \frac{170 \times 10^9}{1-0.28^2} \times (-18 \times 10^{-6} + 0.28 \times (21 \times 10^{-6}))$$

$$\sigma_y = \frac{170 \times 10^9}{1-0.28^2} \times (-18 \times 10^{-6} + 5.88 \times 10^{-6})$$

$$\sigma_y = \frac{170 \times 10^9}{1-0.28^2} \times (-12.12 \times 10^{-6})$$

$$\sigma_y = -2.24 \times 10^6 \text{ Pa}$$

$$= -2.24 \text{ MPa (compressive)}$$

b. $\sigma_x = 88 \times 10^6 \text{ Pa}$
 $\sigma_y = 41 \times 10^6 \text{ Pa}$
 $\nu = 0.28$
 $E = 170 \times 10^9 \text{ Pa}$

$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{170 \times 10^9} \times (88 \times 10^6 - 0.28 \times 41 \times 10^6)$

$$\epsilon_x = \frac{1}{170 \times 10^9} \times (88 \times 10^6 - 11.48 \times 10^6)$$

$$\epsilon_x = \frac{76.52 \times 10^6}{170 \times 10^9}$$

$$\epsilon_x = 4.50 \times 10^{-4}$$

$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{170 \times 10^9} \times (41 \times 10^6 - 0.28 \times 88 \times 10^6)$

$$\epsilon_y = \frac{1}{170 \times 10^9} \times (41 \times 10^6 - 24.64 \times 10^6)$$

$$\epsilon_y = \frac{16.36 \times 10^6}{170 \times 10^9}$$

$$\epsilon_y = 9.62 \times 10^{-5}$$

Question 2

$$2. \quad \sigma_x = 105 \times 10^6 \text{ Pa}$$

$$\sigma_y = 208 \times 10^6 \text{ Pa}$$

$$\tau_{xy} = 125 \times 10^6 \text{ Pa}$$

$$a. \quad \sigma_1 = \frac{(\sigma_x + \sigma_y)}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\sigma_1 = \frac{(105 \times 10^6 + 208 \times 10^6)}{2} + \frac{\sqrt{(105 \times 10^6 - 208 \times 10^6)^2 + 4 \times (125 \times 10^6)^2}}{2}$$

$$\sigma_1 = 291.69 \times 10^6 \text{ Pa}$$

$$= 291.61 \text{ MPa}$$

$$\sigma_2 = \frac{(\sigma_x + \sigma_y)}{2} - \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\sigma_2 = \frac{(105 \times 10^6 + 208 \times 10^6)}{2} - \frac{\sqrt{(105 \times 10^6 - 208 \times 10^6)^2 + 4 \times (125 \times 10^6)^2}}{2}$$

$$\sigma_2 = 21.31 \times 10^6 \text{ Pa}$$

$$= 21.31 \text{ MPa}$$

$$b. \quad \tau_{max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\tau_{max} = \frac{\sqrt{(105 \times 10^6 - 208 \times 10^6)^2 + 4 \times (125 \times 10^6)^2}}{2}$$

$$\tau_{max} = 135.2 \times 10^6 \text{ Pa}$$

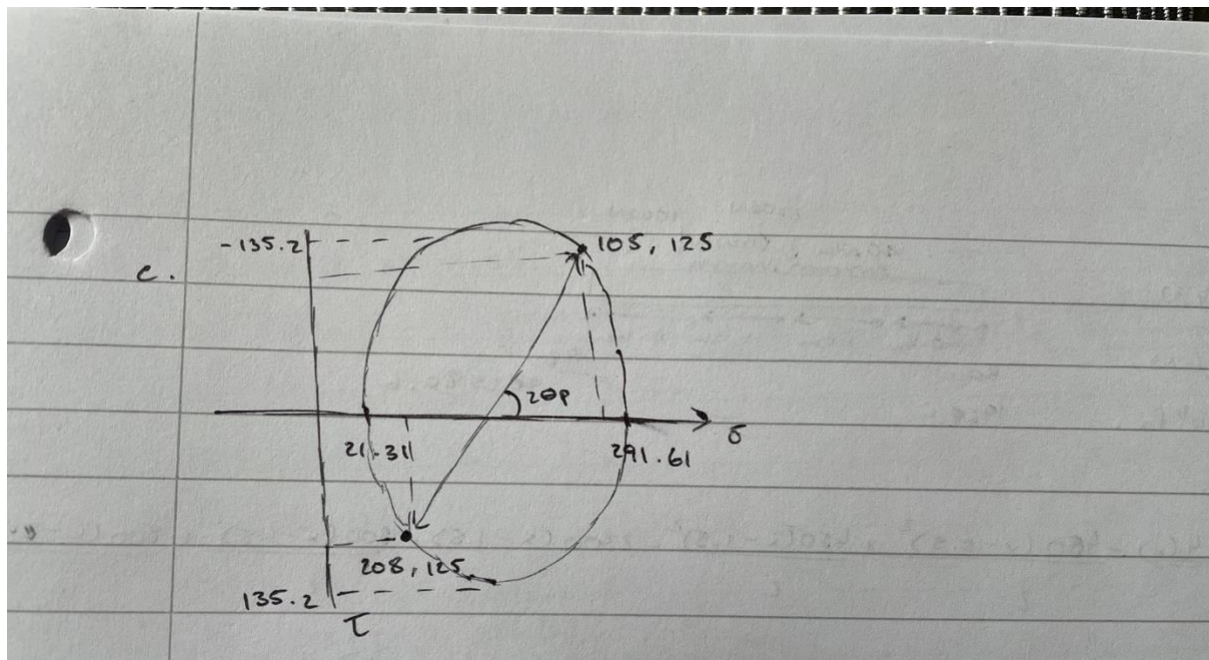
$$= 135.2 \text{ MPa}$$

$$\tau_{min} = -\frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\tau_{min} = -\frac{\sqrt{(105 \times 10^6 - 208 \times 10^6)^2 + 4 \times (125 \times 10^6)^2}}{2}$$

$$\tau_{min} = -135.2 \times 10^6 \text{ Pa}$$

$$= -135.2 \text{ MPa}$$



Question 3

3. $R_A = 1969.4 \text{ N}$
 $R_B = 2580.6 \text{ N}$
 $EI = 1.58 \times 10^6 \text{ Pa}$

a. $M = EI \frac{d^2 y}{dx^2}$

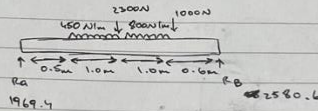
b. $y = \frac{FL^2}{16EI} + \frac{wl^3}{24EI}$

c. $y = \frac{FL^3}{48EI} + \frac{5wl^4}{384EI}$

d. $R_A = 1969.4 \text{ N}$

$R_B = 2580.6 \text{ N}$

$EI = 1.58 \times 10^6 \text{ Pa}$



$$EI \frac{d^4 y}{dx^4} = \frac{1969.4(x)}{2} - \frac{450(x-0.5)^2}{2} + \frac{450(x-1.5)^2}{2} - \frac{2300(x-1.5)}{2} - \frac{800(x-1.5)^2}{2} + \frac{800(x-2.5)^2}{2} - \frac{1000(x-2.5)}{2}$$

$$EI \frac{d^3 y}{dx^3} = \frac{1969.4(x)^2}{2} - \frac{450(x-0.5)^3}{6} + \frac{450(x-1.5)^3}{6} - \frac{2300(x-1.5)^2}{2} - \frac{800(x-1.5)^3}{6} + \frac{800(x-2.5)^3}{6} - \frac{1000(x-2.5)^2}{2} + A$$

$$EI y = \frac{1969.4(x)^3}{6} - \frac{450(x-0.5)^4}{24} + \frac{450(x-1.5)^4}{24} - \frac{2300(x-1.5)^3}{6} - \frac{800(x-1.5)^4}{24} + \frac{800(x-2.5)^4}{24} - \frac{1000(x-2.5)^3}{6} + Ax + B$$

$y = 0$ when $x = 0 \text{ m}$

$0 = 0 - 0 + 0 - 0 - 0 + 0 - 0 + B \quad B = 0$

$y = 0$ when $x = 3.1 \text{ m}$

$x = 0.5 + 1.0 + 1.0 + 0.6 = 3.1 \text{ m}$

$$0 = \frac{1969.4(3.1)^3}{6} - \frac{450(3.1-0.5)^4}{24} + \frac{450(3.1-1.5)^4}{24} - \frac{2300(3.1-1.5)^3}{6} - \frac{800(3.1-1.5)^4}{24} + \frac{800(3.1-2.5)^4}{24} - \frac{1000(3.1-2.5)^3}{6} + 2A$$

$$0 = \frac{1969.4(3.1)^3}{6} - \frac{450(2.6)^4}{24} + \frac{450(1.6)^4}{24} - \frac{2300(1.6)^3}{6} - \frac{800(1.6)^4}{24} + \frac{800(0.6)^4}{24} - \frac{1000(0.6)^3}{6} + 2A$$

$7474.32 - 250.13 = 7224.19$

$2A = 7224.19$

$A = \frac{7224.19}{2} = 3612.1$

3e

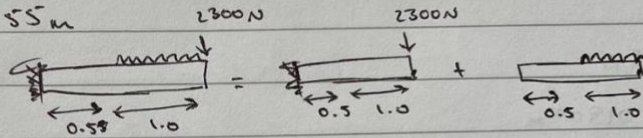
@ centre $x = 1.55 \text{ m}$

$$l = 1.55 \text{ m}$$

$$w = 450 \text{ N/m}$$

$$EI = 1.58 \times 10^6 \text{ Pa}$$

$$F = 2300 \text{ N}$$



$$y = \frac{FL^3}{48EI} + \frac{5wL^4}{384EI}$$

$$y = \frac{2300 \times 1.55^3}{48 \times 1.58 \times 10^6} + \frac{5 \times 450 \times 1^4}{384 \times 1.58 \times 10^6} = 1.17 \times 10^{-4}$$

Question 4

$$4. \quad d = 0.28 \text{ m}$$

$$L = 0.6 \text{ m}$$

$$n_L = 0.58$$

$$n_c = \cancel{0.61} 0.61$$

$$p = 14.5 \times 10^5 \text{ Pa}$$

$$\sigma = 150 \times 10^6 \text{ Pa}$$

$$t = ?$$

Circumferential

$$\sigma_c = \frac{pD}{2tn_L}$$

$$t = \frac{pD}{2\sigma_c n_L}$$

$$t = \frac{(14.5 \times 10^5) \times 0.28}{2 \times 150 \times 10^6 \times 0.58}$$

$$t = 2.33 \times 10^{-3} \text{ m}$$

4b

$$d = 0.28 \text{ m}$$

$$L = 0.6 \text{ m}$$

$$n_L = 0.58$$

$$n_c = 0.61$$

$$p = 14.5 \times 10^5 \text{ Pa}$$

$$\sigma = 150 \times 10^6 \text{ Pa}$$

Longitudinal

$$\sigma_L = \frac{pD}{4tn_c}$$

$$t = \frac{pD}{4\sigma_L n_c}$$

$$t = \frac{(14.5 \times 10^5) \times 0.28}{4 \times 150 \times 10^6 \times 0.61}$$

$$t = 1.11 \times 10^{-3} \text{ m}$$

c. $t_{min} = 2.33 \times 10^{-3} \text{ m}$

Question 5

E.

S. $L = 1.38 \text{ m}$

$$d_o = 0.01905 \text{ m}$$

$$t = 0.0012 \text{ m}$$

$$d_i = 0.01665 \text{ m}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$n = 0.25$$

a.
$$I = \frac{\pi}{64} (d_o^4 - d_i^4) = \frac{\pi}{64} \times (0.01905^4 - 0.01665^4)$$

$$I = 2.69 \times 10^{-9} \text{ m}^4$$

$$b. \quad n = 0.25$$

$$c. \quad F_{cr} = \frac{n\pi^2 EI}{L^2} = \frac{0.25 \times \pi^2 \times (200 \times 10^9) \times (2.69 \times 10^{-9})}{1.38^2}$$

$$F_{cr} = 697.05 \text{ N}$$

$$d. \quad SR = \frac{L}{K} \quad K = \sqrt{\frac{I}{A}}$$

$$A = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} \times (0.01905^2 - 0.01665^2) = 6.73 \times 10^{-5} \text{ m}^2$$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.69 \times 10^{-9}}{6.73 \times 10^{-5}}}$$

$$K = 0.00632$$

$$SR = \frac{L}{K} = \frac{1.38}{0.00632}$$

$$SR = 218.35$$