

Engineering Report:
State-Space Representation and Transfer Function
Analysis

Yacob Alkarim

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1 Introduction

This report will conduct a PID analysis, applying and analysing system representations, time domain analysis, and frequency domain analysis.

2 Task 1: PID control

2.1 Task 1.A

Proportional-integral-derivative (PID) is a control system that is often utilized in engineering practice because of its simplicity, stability, and ease of tuning (Chen, R. (2024)). Most of PID controllers have been developed on the state-space model with this assumption that all state variables are measurable or non-input-output model for a linear system (Rooholahi, B. et.al (2015)). Often used in fields like manufacturing industries and chemical industry.

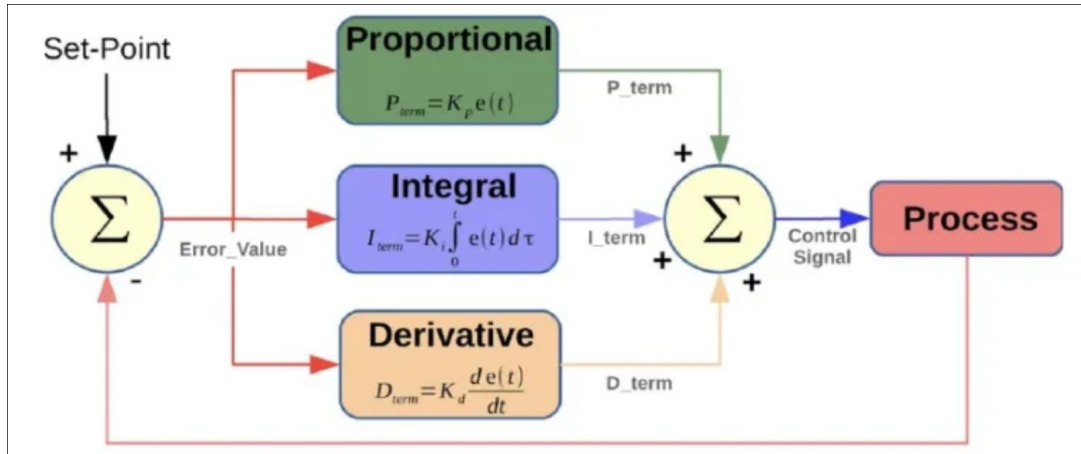


Figure 1: PID control process

Figure (1) shows how a PID controller, the proportional control, integral control and derivative control with each part having their respected uses.

Proportional control is used for correction, for example, a small error results in small corrective action, while a large error triggers a stronger response.

$$u(t) = K_p \cdot e(t) \quad (1.1)$$

Integral control reduces the time to converge to the set point and eliminates some of the steady state error.

$$u(t) = k_i \cdot \int_0^t e(t) dt \quad (1.2)$$

Derivative control improves tuning time and system stability.

$$u(t) = k_d \cdot \frac{de(t)}{dt} \quad (1.3)$$

In conclusion a PID controller is a generic closed loop control feedback mechanism widely used in industrial control systems. The PID attempts to correct the error between a measured process variable and a desired set point by calculating and then outputting a corrective action that can adjust the process accordingly (Hasan, S., et (2014)).

$$u(t) = K_p \cdot e(t) + K_i \cdot \int_0^t e(t)dt + K_d \cdot \frac{de(t)}{dt} \quad (1.4)$$

2.2 Task 1.B A Review of Ziegler-Nichols tuning Methods

What Ziegler-Nicholas tuning methods allow us to do, is find the K_p , K_i , and K_d . Looking at the first method to the Ziegler-Nicholar tuning methods. It by using a unit step function graph.

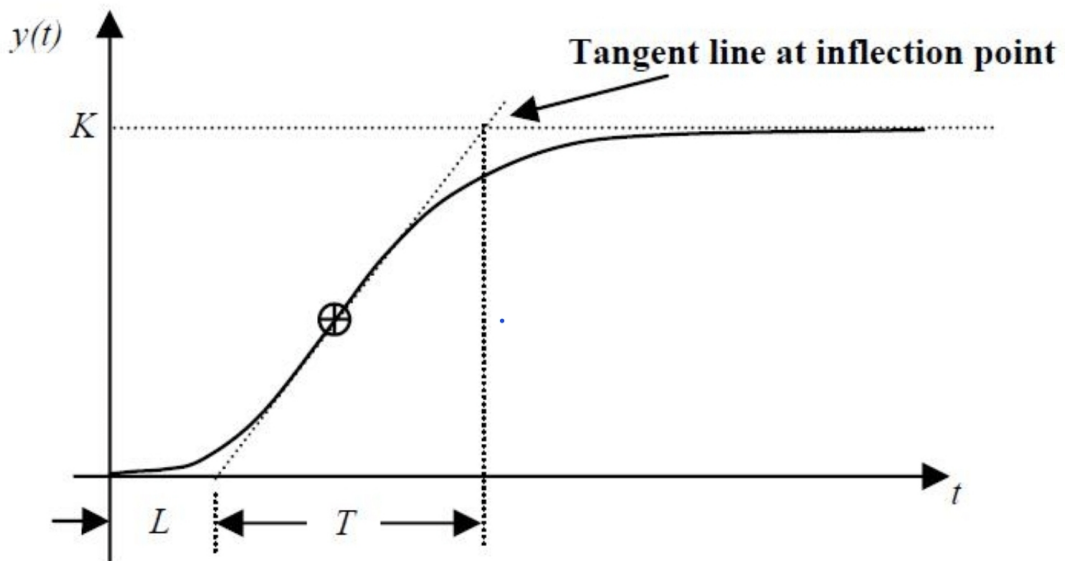


Figure 2: Ziegler-Nichols unit step function.

When looking at this unit step function we need to find the tangent line. We find this tangent line by finding the inflection point. The inflection point is when the unit step function is starting to move horizontally to the x(axis). The tangent line is found by finding the derivative of that function. Then allowing us to find the L, and T variables. In figure (2), we can see that when the tangent line intersect the x(axis) this then being the L function. To find T you would need to go from the top of the tangent line. Then from the top trace down to the x(axis), from there you're T variable will be from the L variable to the trace point one the x(axis).

$$u(t) = \begin{cases} 0 & t < 0, \\ 1 & t \geq 0, \end{cases} \quad (1.5)$$

However we still need to solve for K_p , K_i , and K_d . This is done by substituting the L, and T variables found into a table seen in figure (3). Might need to go more in-detail on the table. Like why is there a 0.9 a 2 soo on.

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9 \frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2 \frac{T}{L}$	$2L$	$0.5L$

Figure 3: Ziegler-Nichols Open loop table.

The second Ziegler Nichols method is similar however different. In this method you are task to solve for K_{crit} , K critical. This is when the system starts to become marginally unstable. This done by increasing the K_p to K_{crit} by only increasing the proportional control very slowly. In figure (4) you can see the variable P_{crit} P critical this is just $K_p = K_{crit}$.

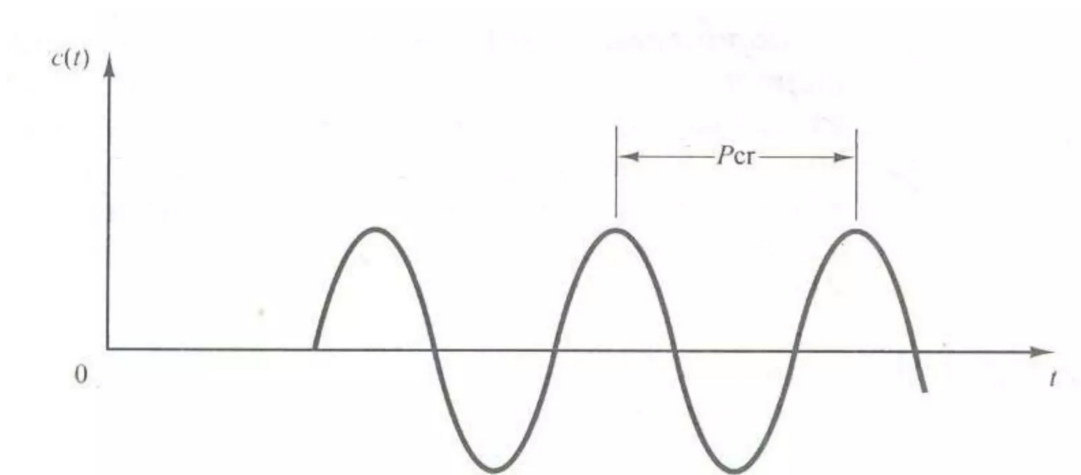


Figure 4: PID at point of instability

With figure (5) you apply your variables and then able to find your corresponding results.

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Figure 5: Ziegler Nichols P critical Table

Looking into the main limitations and the potential drawback of both of Ziegler Nichols' techniques. One glaring issue is the manual tuning, looking at figure (4) we can see the "unstable oscillation". The issue that arises with this is too much tuning will cause a excess response and overshoot (Das, Susmita., 2012). Resulting in manual tuning not being applied in practice as it laborious, time consuming, particularly is a process that has large time constants (Hang, C.C.,et al. 1991). Furthermore drawbacks with the Ziegler-Nichols rules too little process information is used and the closed loop systems that are obtained lack robustness (Åström and Murray, 2008).

2.3 Task 1.C

In task 1.C we are told to find to determine the PID control parameters for a plant with a open loop transfer function using the Zeigler-Nichols oscillation method.

$$\frac{s + 2}{(s + 3)(s + 3)}$$

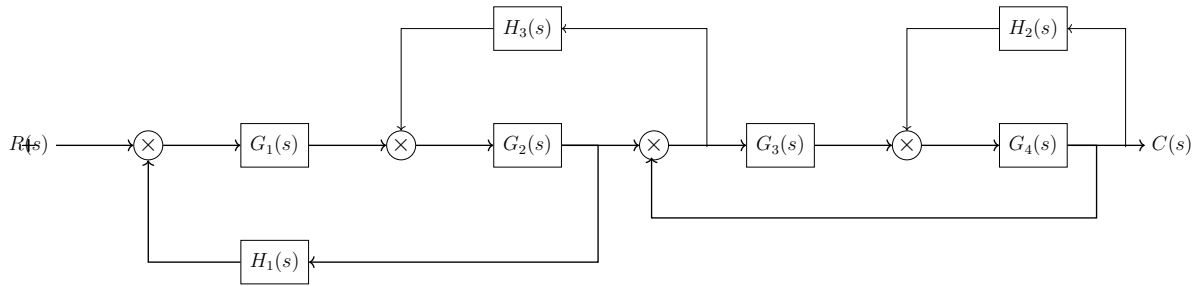
The first step in solving this is by find the inverse Laplace transform. As this would allow for us to convert the input variable (s) into a time domain variable (t).

$$\mathcal{L}^{-1} \left\{ \frac{s + 2}{(s + 3)^2} \right\} = e^{-3t} - te^{-3t} = (1 - t)e^{-3t}$$

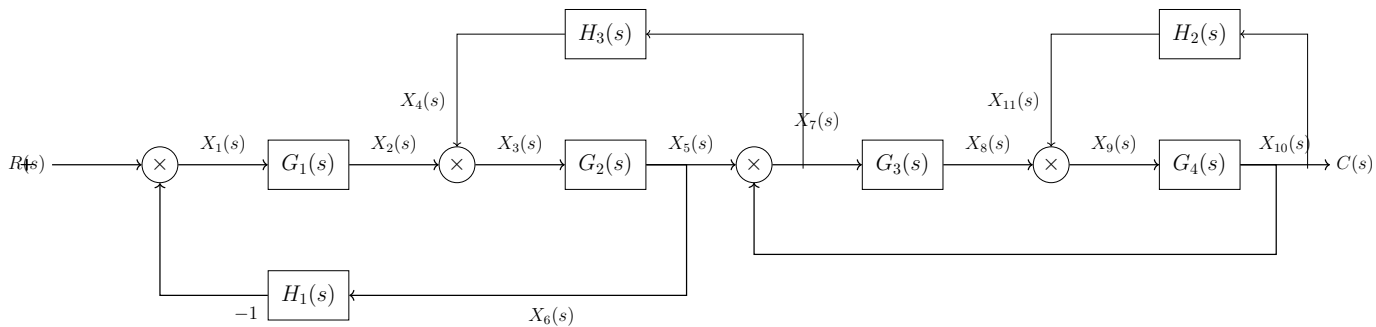
3 Task 2: System Representation.

3.1 Problem description

With-in this task I'm to create a signal flow diagram that aligns with the given system block diagram and then use Mason's Gain formula to derive the overall transfer function of the system.



The first step we do is converter this block diagram into a signal flow diagram. We do this by making the line going into the blocks the new summing blocks and convert the blocks in to the signal.



From here convert the blocks into branch which represent the signal of the block.

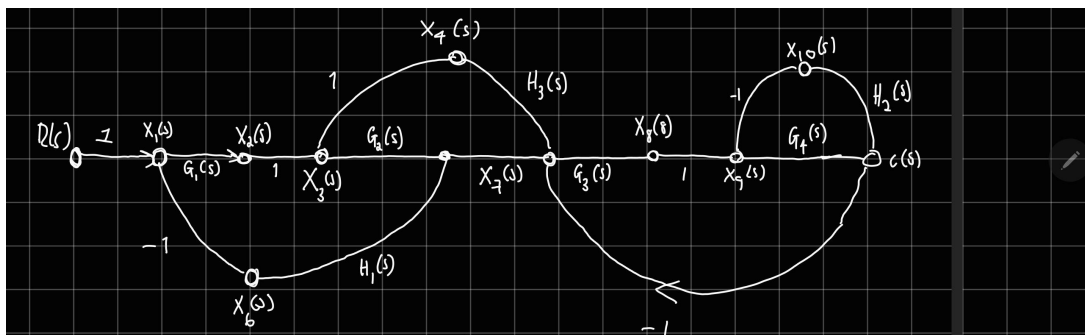


Figure 6: Conversion of block digram

After converting the block diagram into a signal flow diagram we simplify. We can do this by multiplying the connected signals by 1 or -1 one. However they mustn't be

already connected to something prior. For example we have at the start of the signal flow diagram we have a $G_1(s)$ connected to a $x_3(s)$ which then has a 1 on the right of it. We can simplify this to just get $G_1(s)$ as a signal by itself.

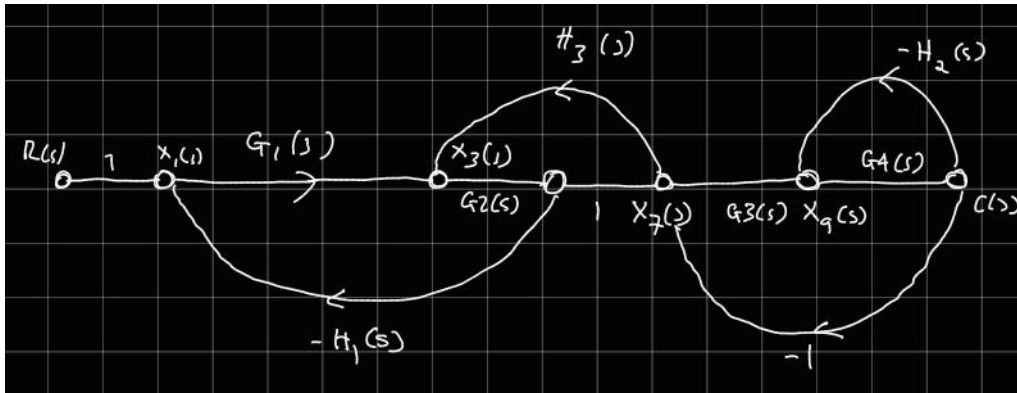


Figure 7: Signal flow Simplified

We then apply this simplified signal flow diagram into the mason gain formula.

$$T(s) = \frac{1}{\Delta} \sum_{k=1}^p T_k \Delta_k$$

P = The amount of forward paths. These are paths that do not flow back into the signal. In the signal flow diagram we only have 1 that doesn't flow back into the signal. You can see this in figure 7.

T_K = total gain with-in the system.

$$T_k = G_1(s) \cdot G_2(s) \cdot G_3(s) \cdot G_4(s)$$

This is the total gain within that 1 path forward.

In this section we look into the loops that we have and see what the signal is doing.

If you look back to figure 7 you can see that we have 4 loops.

$$\text{Loop 1} = G_1(s) \cdot G_2(s) \cdot (-H_1(s))$$

$$\text{Loop 2} = G_2(s) \cdot 1 \cdot H_3(s)$$

$$\text{Loop 3} = 1 \cdot G_3(s) \cdot G_4(s)$$

$$\text{Loop 4} = G(s) \cdot H_2(s)$$

There are no non-touching loops since all loops share at least one common node.

Therefore, Since there are no non-touching loops:

$$\Delta = 1 - (-G_1G_2H_1 + G_2H_3 + G_3G_4 - G_4H_2)$$

Simplifying:

$$\Delta = 1 + G_1G_2H_1 - G_2H_3 - G_3G_4 + G_4H_2$$

Since all loops touch the forward path:

$$\Delta_1 = 1$$

Using Mason's Gain Formula:

$$\frac{C(s)}{R(s)} = \frac{\sum P_k \Delta_k}{\Delta}$$

Since there is only one forward path:

$$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1}{\Delta}$$

Substituting:

$$P_1 = G_1(s)G_2(s)G_3(s)G_4(s)$$

and

$$\Delta_1 = 1$$

Therefore,

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)G_3(s)G_4(s)}{1 + G_1(s)G_2(s)H_1(s) - G_2(s)H_3(s) - G_3(s)G_4(s) + G_4(s)H_2(s)}$$

Hence, the overall transfer function is:

$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)G_3(s)G_4(s)}{1 + G_1(s)G_2(s)H_1(s) - G_2(s)H_3(s) - G_3(s)G_4(s) + G_4(s)H_2(s)}$
--

4 Task 3: System Representation

4.1 Task 3.A

State space modeling is a mathematical representation of a physical system by means of input and outputs, state variables, and differential equations (Mumtaz, Farhan, et al (2021)). Furthermore, a state space model is a linear representation of a dynamic system in either continuous or discrete form.

$$\dot{x} = Ax + Bu \quad (\text{Continuous})$$

$$y = Cx + Du \quad (\text{Discrete})$$

The formulas above are how you solve the transfer function. The continuous equation is also often referred to as the state equation, and the discrete equation is often referred to as the output equation.

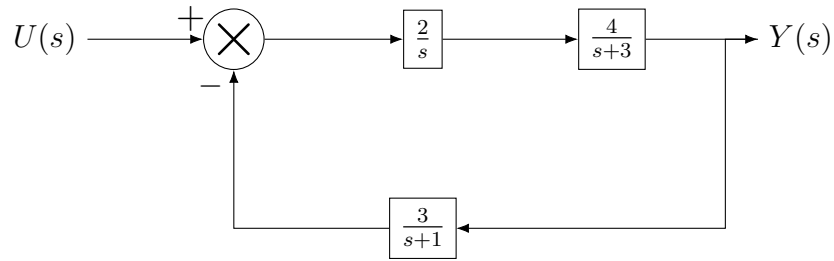
The matrix A which is a state of transmission matrix is a matrix that connects the state at time to the state at t-1. It captures the dynamics of the system. While the matrix B at time, controls input matrix accounts for any external control or effect on the system. The matrix C describes how the states are combine to get the outputs (Vekariya P.R, et al. (2024)). And the matrix D means that there is a direct connection between the input and the output without the intervention of the state $\dot{x}$. This results in D in most cases be 0 (Tokarzewki, Jerzy. al,(2001))

How are state space models able to represent dynamic systems is by looking at each of the state variables individually and seeing how they relate to each other. When $\dot{x}$ changes you look into A and B individually and look into how they effect the system. You can do this further for every state when you have you can see how the system is effected when taken to the second derivative $\ddot{x}$, $\ddot{\ddot{x}}$ and so on.

4.2 Task 3: Block Diagram to State-Space Model Conversion

4.3 3.B Problem Description

Figure 1 shows a block diagram of the feedback control system. The objective is to derive the closed-loop transfer function, convert it to a differential equation, and then express the system in state-space form.



4.4 Closed-Loop Transfer Function

4.4.1 Forward Path

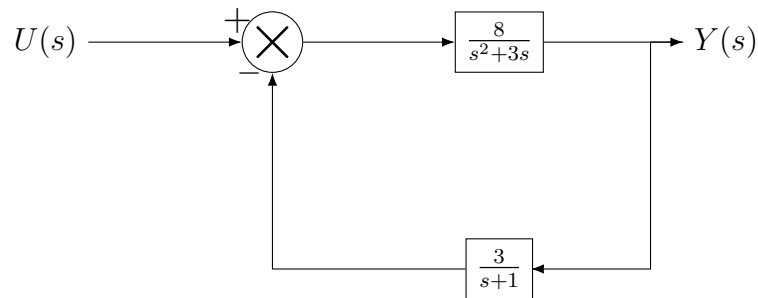
The forward transfer function $G(s)$ is the product of the two series blocks:

$$G(s) = \frac{2}{s} \cdot \frac{4}{s+3} = \frac{8}{s^2+3s}$$

4.4.2 Feedback Path

The feedback transfer function is:

$$H(s) = \frac{3}{s+1}$$



4.4.3 Closed-Loop Transfer Function

The closed-loop transfer function is:

$$R(s) = \frac{G(s)}{1 + G(s)H(s)}$$

Substituting:

$$R(s) = \frac{\frac{8}{s^2+3s}}{1 + \left(\frac{8}{s^2+3s}\right)\left(\frac{3}{s+1}\right)}$$

Simplifying:

$$R(s) = \frac{\frac{8}{s^2 + 3s}}{\frac{(s^2 + 3s)(s + 1) + 24}{(s^2 + 3s)(s + 1)}} = \frac{8(s + 1)}{(s^2 + 3s)(s + 1) + 24}$$

Expanding the denominator:

$$(s^2 + 3s)(s + 1) = s^3 + 4s^2 + 3s$$

Therefore:

$$R(s) = \frac{8(s + 1)}{s^3 + 4s^2 + 3s + 24}$$

4.5 Differential Equation Form

Since $G(s) = Y(s)/U(s)$:

$$(s^3 + 4s^2 + 3s + 24)Y(s) = 8(s + 1)U(s)$$

Expanding:

$$s^3Y(s) + 4s^2Y(s) + 3sY(s) + 24Y(s) = 8sU(s) + 8U(s)$$

Converting to the time domain (where s represents differentiation):

$$y^{(3)} + 4\ddot{y} + 3\dot{y} + 24y = 8\dot{u} + 8u$$

Here, $y^{(3)}$ denotes the third derivative, $\ddot{y}$ the second derivative, and $\dot{y}$ the first derivative.

4.6 State-Space Representation

4.6.1 State Variable Selection

Using the controllable canonical form, the state variables are chosen such that:

$$x_1 = x_1, \quad x_2 = \dot{x}_1, \quad x_3 = \dot{x}_2$$

4.6.2 State Equations

The state equations become:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -24x_1 - 3x_2 - 4x_3 + u$$

4.6.3 Output Equation

From the numerator $8s + 8$, the output equation is:

$$y = 8x_1 + 8x_2$$

4.6.4 Matrix Form

The system is expressed as $\dot{x} = Ax + Bu$ and $y = Cx + Du$:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -3 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 8 & 8 & 0 \end{bmatrix} x$$

4.7 Result for Task B

The final state-space representation is:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -3 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 8 & 8 & 0 \end{bmatrix}, \quad D = 0$$

4.8 Task 3.C Problem Description

A linear time-invariant system is described by the following state-space representation:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 2 \\ 1 & 3 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_B u$$
$$y = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_C \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad D = 0$$

The objective is to determine the equivalent transfer function $G(s) = Y(s)/U(s)$.

4.9 Methodology

The transfer function is derived using the standard conversion formula:

$$G(s) = C(sI - A)^{-1}B \quad (1)$$

4.10 Calculations

4.10.1 Step 1: Compute $sI - A$

The identity matrix for this second-order system is:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (2)$$

Therefore,

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} s+2 & -2 \\ -1 & s-3 \end{bmatrix}$$

4.10.2 Step 2: Compute $(sI - A)^{-1}$

For a 2×2 matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse is $\frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Solving for the ad-bc:

$$ad - bc = (s+2)(s-3) - (-2)(-1) = s^2 - s - 8$$

The inverse is:

$$(sI - A)^{-1} = \frac{1}{s^2 - s - 8} \begin{bmatrix} s-3 & 2 \\ 1 & s+2 \end{bmatrix}$$

4.10.3 Step 3: Compute $G(s)$

Substituting into the formula:

$$G(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \frac{1}{s^2 - s - 8} \begin{bmatrix} s - 3 & 2 \\ 1 & s + 2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

First, multiply the matrix by B :

$$\begin{bmatrix} s - 3 & 2 \\ 1 & s + 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ s + 2 \end{bmatrix}$$

Then, multiply by C :

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ s + 2 \end{bmatrix} = s + 2$$

Thus,

$$G(s) = \frac{s + 2}{s^2 - s - 8}$$

4.11 Results of task C

The transfer function equivalent to the given state-space model is:

$$G(s) = \frac{s + 2}{s^2 - s - 8}$$

5 Task 4: Time Domain Analysis

5.1 Root locus analysis.

Introduced in 1948 by Walter R. Evans, the root locus analysis technique helps represent any physical system using the transfer function. The idea of the root locus method is to use the poles and zeros of open-loop transfer functions to plot the root loci of the characteristic equations of closed loop transfer functions based on some rules, so that stability as well as other performance indexes of the closed-loop systems can be analyzed through the root loci (TIE. Lin al.(2020). The core function of the root locus is the S-plane, this is when you have a Real on the x-axis and the Imaginary on the y-axis. Each location on the S-plane corresponds to a waveform in the time domain represented by e^{st} . With this equation we are able to see that if S is a real value is -1 we will have exponential decay seen in (figure 6). The opposite is when you have a real of 1 you will have exponential growth. The further away you are from the 0 the faster the decay or growth is. The imaginary corresponds to sinusoidal or oscillation motion

applying the same idea to the real, the further you are from 0 the faster the frequency is. Seen in (figure 6) we have a combination of both you get a sinusoidal growth or decay.

While both Poles and zero are needed to characterize the response of a forced input. Only the poles are needed to dictate a unforced response to a set of initial conditions. This then ties directly to the stability of a system.

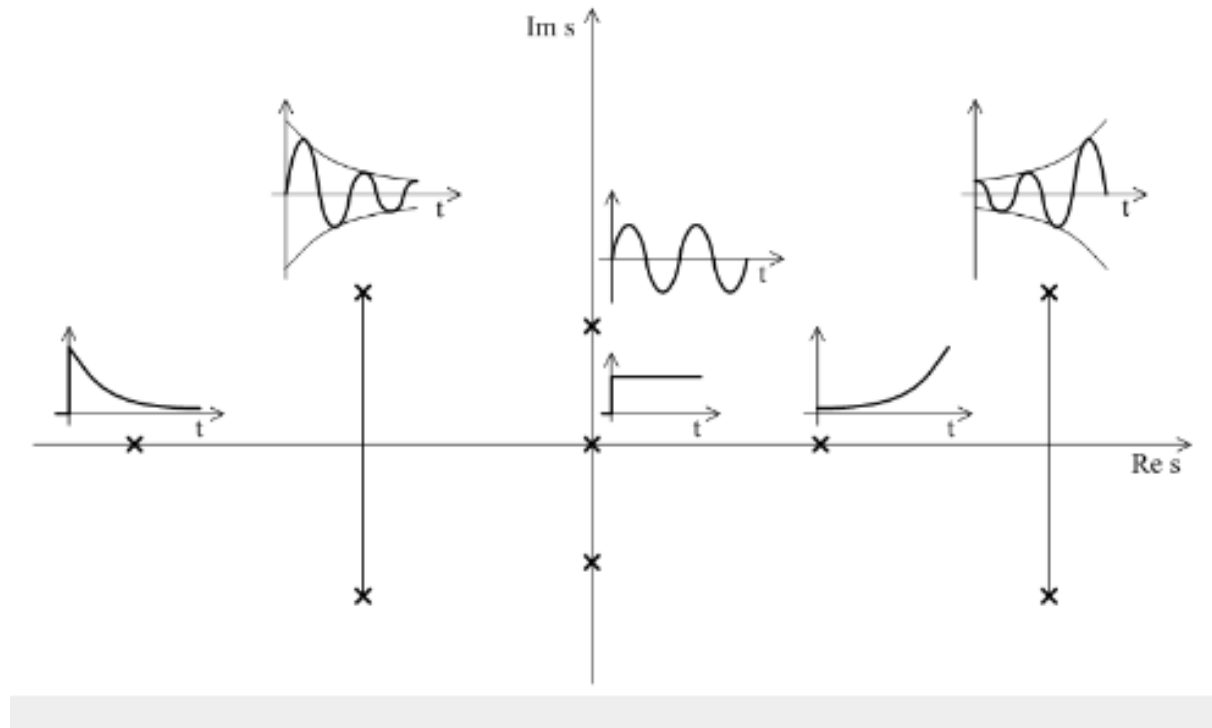


Figure 8: Poles of Transfer functions.

Consider the characteristics equation. This is just a simple closed loop function.

$$1 + KG(s) = 0$$

Stability is dictated by the characteristic equation. When all its roots lie in the left half of the s-plane, the closed-loop system is stable. The root locus method enables analysis of how these roots shift as the gain K changes (TIE. Lin al.(2020)).

Further more root locus is able to give insight to system stability is by using a damping ratio this is when you try to get 0 to 1. There are 4 different types of damping there is undamped, under-damped, critically damped, and over damped. The best out of the 4 is critically damped as it is able to get from 0 to 1 as efficient as possible seen in (figure 7). There is no over shoot as seen in the under-damped. Overdamped is when the system takes twice as long to get to 1. (feel like i need to explain more).

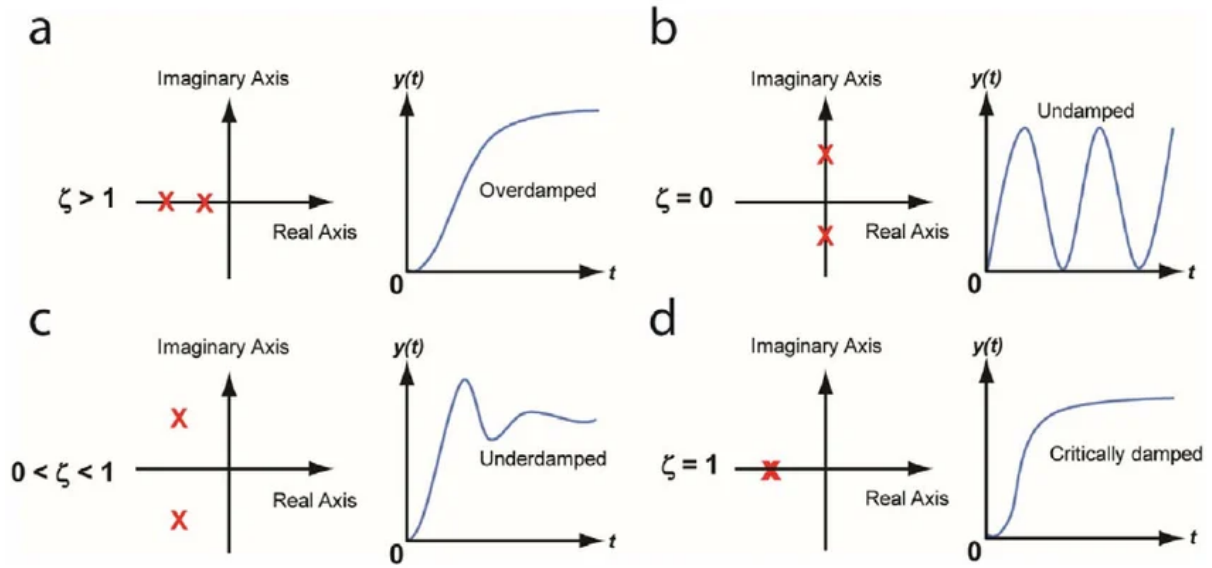
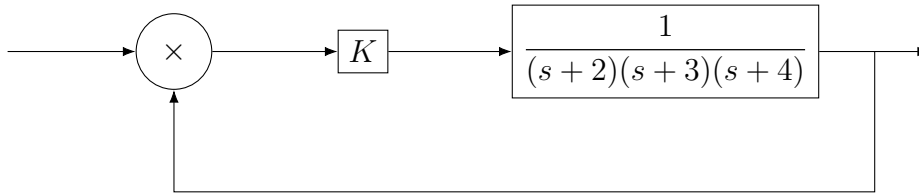


Figure 9: Damping Corresponding to the S-plane

The s-plane can be used for robustness as well as transient design, and it can be help to diagnose the faulty elements in an unstable close loop system (Cavicchi, Thomas J. at(1996)).

5.2 Determining value of K from a root locus diagram.



The first thing we do is convert the image into a transfer function. Must note that the line is the $H=1$.

$$G = \frac{G}{1 + GH}$$

$$H = 1$$

Step 1: Transfer function

$$TF = \frac{\frac{K}{(s+2)(s+3)(s+4)}}{\left(1 + \frac{K}{(s+2)(s+3)(s+4)} \times 1\right)}$$

$$TF = \frac{K}{(s+2)(s+3)(s+4) + K}$$

Step 2: We characterize the function. What we do is we take the denominator of the equation and equal it too zero.

$$(s + 2)(s + 3)(s + 4) + K = 0$$

Step 3: Trying to determine start and end point.

When $K = 0$

$$(s + 2)(s + 3)(s + 4) = 0$$

Poles: $s = -2, s = -3, s = -4$

The pole will be the starting point and will go along the x-axis.

When $K = \infty$

When characterize this we get no zero.

Now we want to determine what on the Real Axis is root locus. The rule for this is when there is a number of poles that is even resulting it not being on root locus. However is the number of pole is odd it would result in there being root locus. In this section we are trying to create something on the s-plane. Using these rules are able to do that.

Step 4: Determining Root locus

Section 1: $S < -4$

Poles to the right $\rightarrow -4, -3, -2 = 3$

poles 3 is odd, so this section is on root locus.

Section 2: $-4 < S < -3$

Poles to the right $\rightarrow -3, -2 = 2$

poles 2 is even, so this section is not on root locus.

Section 3: $-3 < S < -2$

Poles to the right $\rightarrow -2 = 1$

pole 1 is odd, so this section is on root locus.

Section 4: $S < -2$

Poles to the right $\rightarrow 0$

pole 0 is even, so this section is on root locus.

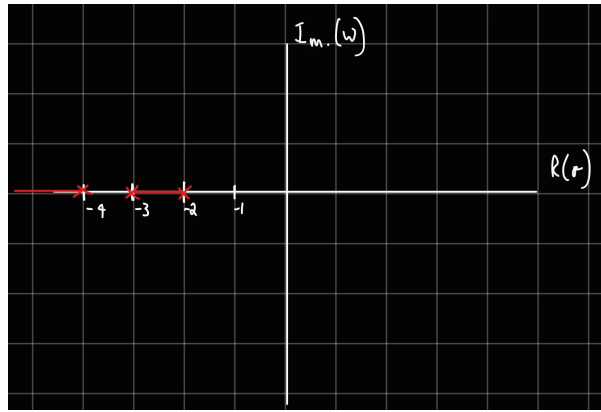


Figure 10: S-plane plots

Step 5: finding the breaking point.

$$(s + 2)(s + 3)(s + 4) + K = 0$$

Rearrange for u :

$$K = -(s + 2)(s + 3)(s + 4)$$

Expand:

$$K = -(s^3 + 9s^2 + 26s + 24)$$

Differentiate:

$$\frac{du}{ds} = -(3s^2 + 18s + 26)$$

Make equal to zero:

$$3s^2 + 18s + 26 = 0$$

$$S = \frac{-18 \pm \sqrt{18^2 - 4(3)(26)}}{2(3)}$$

$$S = \frac{-18 \pm \sqrt{324 - 312}}{6}$$

$$S = \frac{-18 \pm \sqrt{12}}{6}$$

$$S = \frac{-18 \pm 3.46}{6}$$

$$S = -2.423 \quad \text{or} \quad S = -3.576$$

Has to lay in the root locus so -3 and -2 to be on root locus.

$$S = -2.423$$

Step 6: Angle of Breakaway Direction

$$\theta = \frac{(2n + 1)180^\circ}{n - m}$$

n = number of holes

m = number of zeros

q = integer counter starting at 0.

$$q = 0, 1, 2$$

$$\theta = \frac{(2 \times (0) + 1)(180^\circ)}{3} = 60^\circ$$

$$\theta = \frac{(2 \times (1) + 1)(180^\circ)}{3} = 180^\circ$$

$$\theta = \frac{(2 \times (2) + 1)(180^\circ)}{3} = 300^\circ$$

Step 7: Value of K

$$(s + 2)(s + 3)(s + 4) + K = 0$$

$$s = j\omega$$

$$s^3 + 9s^2 + 26s + (24 + K) = 0$$

$$(-j\omega)^3 - 9(j\omega)^2 + 26(j\omega) + (24 + K) = 0$$

$$(-9\omega^2 + 24 + K) + (-j\omega^3 + 26j\omega) = 0$$

$$(-9\omega^2 + 24 + K) + (-j\omega^2 + 26j\omega) = 0$$

$$(-9\omega + 24 + K) + j(\omega^3 + 26\omega) = 0$$

$$-9\omega^2 + 24 + \omega = 0$$

$$K = 9\omega^2 - 24$$

$$-\omega^3 + 26\omega = 0$$

$$\omega(-\omega^2 + 26) = 0$$

$$\omega^2 = 26$$

$$K = 9\omega^2 - 24$$

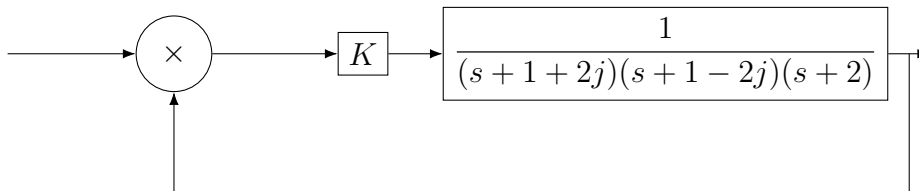
$$K = 9(26) - 24$$

$$K = 234 - 24$$

$$K = 210$$

This then results in anything $\omega > 210$ the system will become unstable.

5.3 Task 4 question 2



Step 1: Solve for the transfer function.

Transfer function:

$$\frac{K}{(s+1+2j)(s+1-2j)(s+2)} \times 1$$

Transfer function:

$$\frac{K}{(s+1+2j)(s+1-2j)(s+2)+k}$$

Step 2: characterize the equation

When solving for this equation we have plots going into the imaginary axis.

$$(s+1+2j)(s+1-2j)(s+2)+k=0$$

Step 3: Determine the start and end point

$$(s+1+3j)(s+1+2j)(s+2)+K=0$$

When $K = 0$

$$(s+1+3j)(s+1+2j)(s+2)=0$$

Poles: $s = -1 - 2j$, $s = -1 + 2j$, $s = -2$

When $K = \infty$

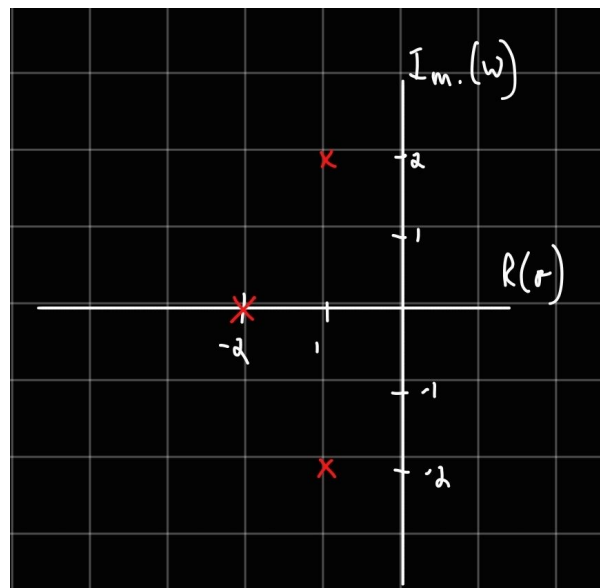


Figure 11: S-plane plots

Step 4: Determining the real-axis

Section 1: $S < -2$: No poles to the right (0). Zero is even meaning that section one is not on root locus.

Section 2: $-2 < S < -1$: Only one pole -2 to the right (1) this mean that this segment is on root locus. So ∞ to -2 is on locus.

Step 5: Finding the breaking point.

$$K = -(s+1+2j)(s+1-2j)(s+2)$$

$$= (s^2 + 2s + 5)(s + 2)$$

$$= s^3 + 2s^2 + 5s + 2s^2 + 4s + 10$$

$$= s^3 + 4s^2 + 9s + 10$$

For breakaway point:

$$\frac{dK}{ds} = 0$$

$$3s^2 + 8s + 9 = 0$$

Using quadratic formula:

$$s = \frac{-8 \pm \sqrt{8^2 - 4(3)(9)}}{6}$$

$$= \frac{-8 \pm \sqrt{64 - 108}}{6}$$

$$= \frac{-8 \pm j\sqrt{44}}{6}$$

$$s = -1.33 \pm 1.10j$$

Step 6: Angel of break away.

$$\theta = \frac{(2q + 1)180^\circ}{3}$$

6 Task 5: Frequency domain analysis

Bode diagrams are frequently used to graphically describe a system's frequency response. A Bode diagram is a plot that shows how a system's magnitude and phase response change with frequency. Bode diagrams make it simpler to study and build circuits or systems by giving a clear and succinct picture of a system's frequency response characteristics (Samreen Fizaat(2022)). The diagram constitutes two parts or panels. One is the magnitude diagram and the other is the phase diagram. The magnitude diagram is 20 times the logarithm of the magnitude of the transfer function of the system. The horizontal axis is the frequency in the logarithmic scale (Yazdan Bavafa-Toosi at (2019)). Bode's development of the terms gain margin and phase margin, either of which can be

clearly understood on a Bode plot, gave engineers the ability to examine the stability of the system (Samreen Fizaat(2022)). A Nichols chart illustrates a linear control system's frequency response by graphing open-loop magnitude (gain) on the vertical axis against phase angle on the horizontal axis within a single plot. This configuration supports stability analysis and design by tracing the system's open-loop frequency characteristics. The resulting curve is overlaid onto a pre-drawn grid of closed-loop magnitude (M) and phase (N) contours allowing viewing of the closed loop behavior from open loop data (Richard C. Dorf at (2018)). When combining both the magnitude and the phase into a cartesian plot it will allow control engineers to directly visualize, analyze, and design feedback control systems (Ellis, George at (2012)).

Looking into application of the Bode diagram and Nichols chart in control system analysis and design. The Nichols chart is often used in application that require tuning and optimizing loop performance. As stated earlier adjusting the gain and phase margins to meet stability requirements. In the paper Modern Control engineering it states The designer can graphically determine the phase margin, gain margin, resonant peak magnitude, resonant frequency, and bandwidth of the closed loop system from the plot of the open-loop locus (Ogata, Katsuhiko at(2010)). The Bode diagram while working with both the gain and phase margins it combines both of them together to show what the system look like.

PID control is a fundamental control algorithm, which is widely used in automatic control technology, including temperature control, speed control of motors, level control in tanks, robotics, and many other automated systems (Wei, Yumou at (2023)). PID controllers are widely used in control systems to regulate and stabilize processes. Proper tuning of PID parameters is crucial to achieve optimal control performance (Wei, Yumou at (2023)).

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