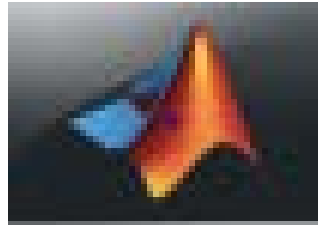


MATLAB

Fundamentals

2017

Parts 2 & 3: Matrices and Vectors



These notes are derived from several sources (see below) including MathWorks documentation which can be found at <https://uk.mathworks.com/help/matlab/>

B.C. Lowe, An Introduction to Matlab

David F. Griffiths, Dundee University, An Introduction to Matlab,
<http://www.maths.dundee.ac.uk/software/MatlabNotes.pdf>

David Houcque, Northwestern University, Introduction To Matlab For Engineering Students,
<https://www.mccormick.northwestern.edu/documents/students/undergraduate/introduction-to-matlab.pdf>

Other Matlab Resources:

<https://ocw.mit.edu/resources/res-18-002-introduction-to-matlab-spring-2008/lecture-notes/>

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2. Row and column vectors

A row vector is a matrix with one row. Think of it as a list of numbers in a row separated by spaces and with opening and closing brackets around the list.

A column vector is a column matrix and again. Think of it as a list of numbers written in a column.

See section 3 for details of working with matrices in matlab.

$a = [1 \ 2 \ 78 \ 4 \ -2]$ is a row vector and $b = \begin{bmatrix} 1 \\ 2 \\ 5 \\ 8 \end{bmatrix}$ is a column vector.

2.1 Entering vectors

Enter a and b as follows

```
>> clear all
>> a=[1 2 8 4 -2],b=[1;2;5;8]
a =
     1     2     8     4    -2
b =
     1
     2
     5
     8
```

Type a space between numbers in a row. Rows are separated by semi-colons. You can also press enter instead of a semi-colon.

2.2 Length of a vector

The number of entries is known as the length of the vector.

```
>> length(a), length(b)
ans =
     5
ans =
     4
```

2.3 Arithmetic operations

Row and column vectors can be multiplied by a scalar, and two or more vectors of the same length can be added or subtracted.

```
>> 2*a, 10*b
```

```
ans =  
      2      4     16      8     -4  
ans =  
     10  
     20  
     50  
     80
```

Note that each element of a has been multiplied by 2, and each element of b by 10.

```
>> c=[2 4 -1 -2 3];
```

```
>>a+c
```

```
ans =  
  
      3      6      7      2      1
```

```
>>3*a - 2*c
```

```
ans =  
-1     -2     26     16    -12
```

Elements can be sorted into ascending order:

```
>> sort(a)
```

```
ans =  
  
-2      1      2      4      8
```

```
>> sort(b)
```

```
ans =
```

```
1  
2  
5  
8
```

2.4 Colon notation

This is a short cut for producing row or column vectors whose elements increase uniformly. The square brackets are optional.

```
>> w=[3:7], s=[-5:1]
```

```
w =
```

```
      3      4      5      6      7
```

```
s =
```

```
     -5     -4     -3     -2     -1      0      1
```

By default the elements in s and w increment in steps of 1. We can have any increment we wish.

```
>>y=[-3:0.5:1]
```

```
y =
```

```
     -3.0000     -2.5000     -2.0000     -1.5000     -1.0000  
     -0.5000      0      0.5000      1.0000
```

```
>>x=[3.0:0.3:5]
```

```
x =
```

```
      3.0000      3.3000      3.6000      3.9000      4.2000  
4.5000      4.8000
```

Here the elements of x start at 3 and increase by 0.3 up to 4.8. Note 4.8 is the highest incremented value less than or equal to 5.

```
>>z=[3:-0.5:1]
```

```
z =
```

```
      3.0000      2.5000      2.0000      1.5000      1.0000
```

Here the elements of z decrease from 13 to 1 in steps of -0.5

The general form is $[a:b:c]$. The values start at a and increase by b until c is reached.

2.5 Transpose of a vector

Transposing a row vector gives a column vector with the same elements.

Similarly, transposing a column vector gives a row vector.

The transpose of a is called a' . Use the single quotes on the same key as the `@` symbol. In normal matrix notation a transpose operation is denoted by a superscript T

after the matrix variable name, eg A^T . Be aware that you must use the apostrophe for this operation in Matlab.

```
>> q=[1 2 3];r=[4;5;6];  
>> q,q',r,r'
```

```
q =  
    1    2    3  
ans =  
    1  
    2  
    3
```

```
r =  
    4  
    5  
    6
```

```
ans =  
    4    5    6
```

and we can transpose a transpose to return to the original matrix or vector.

```
>> r''
```

```
ans =
```

```
    4  
    5  
    6
```

2.6 Array product, division and power

If a and b are row (or column) vectors of the same length then the array product is $a.*b$. The elements of $a.*b$ are the products of the corresponding elements of a and b . This operation is sometimes referred to as the elementwise product or the Hadamard product.

```
>> a=[1 6 15 -7];b=[1 3 5 -7];  
>> a.*b
```

```
ans =  
    1    18    75    49
```

Similarly, the array division $a./b$ gives element by element division.

```
>> a./b
```

```
ans =  
    1    2    3    1
```

Array power raises each element to the power given.

```
>> a.^2,b.^3
```

```
ans =  
    1    36   225    49  
ans =
```

1 27 125 -343

3.0 Matrices

3.1 Entering matrices

Enter the matrix $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ as follows (and remember that A and a are different!)

```
» clear all
» A=[3 5;1 2]
A =
     3     5
     1     2
```

Type a space after each number. Rows are separated by semi-colons.

In the same way enter $B = \begin{bmatrix} 1 & 6 & 2 \\ 5 & 3 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix}$.

```
» B=[1 6 2;5 3 4],C=[2 -1 3;0 4 2]
B =
     1     6     2
     5     3     4
C =
     2    -1     3
     0     4     2
```

A has 2 rows and 2 columns and so its size is (2×2) . B and C are of size (2×3) .

This can be obtained using the *size* command:

```
» size(A),size(B)
ans =
     2     2
ans =
     2     3
```

If we need to, we can refer to a particular element of a matrix. For example, to obtain the element in the first row and second column of A enter

```
» A(1,2)
ans =
     5
```

To add the element in the second row, third column of B to this:

```
» A(1,2)+B(2,3)
ans =
```

3.2 Addition and Subtraction

Find $B + C$ and $B - C$:

» **B+C, B-C**

ans =

3	5	5
5	7	6

ans =

-1	7	-1
5	-1	2

Check that the answers are as you would expect.

Matrices can only be added or subtracted if they are of the same size, so

» **A+B**

??? Error using ==> +

Matrix dimensions must agree.

3.3 Multiplication of a matrix by a scalar

The elements of $2A$ are all twice the elements of A .

» **A, TWOTIMESA=2*A**

A =

3	5
1	2

TWOTIMESA =

6	10
2	4

We can $2B + 3C$ and $5B - 4C$ as follows:

» **2*B+3*C, 5*B-4*C**

ans =

8	9	13
10	18	14

ans =

-3	34	-2
25	-1	12

3.4 Multiplication of matrices

Matrices can be multiplied when the number of columns in the first matrix is the same as the number of rows in the second matrix, so the only products between A , B and C that we can find are AB and AC .

```
» A*B,A*C
```

```
ans =  
    28    33    26  
    11    12    10  
ans =  
     6    17    19  
     2     7     7
```

If you try any other combination an error message is given:

```
» B*C
```

```
??? Error using ==> *  
Inner matrix dimensions must agree.
```

Find $Q = FG$ and $R = GF$, where $F = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $G = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$.

```
» F=[1 3;2 4];G=[2 5;1 4];Q=F*G,R=G*F
```

```
Q =  
     5     17  
     8     26  
R =  
    12     26  
     9     19
```

This illustrates the fact that even when we can multiply two matrices in either order, the result may not be the same.

We can find powers of square matrices. To find Q^2 and R^3 :

```
>> FSQ=F^2,GCUB=G^3
```

```
FSQ =  
     7     15  
    10     22  
GCUB =  
    48    165  
    33    114
```

3.5 Transpose of a matrix

To transposes of A and C are usually called A^T and C^T , but A' and C' in MATLAB. Use the single quotes on the same key as @:

```
>> A',C'
ans =
     3     1
     5     2
ans =
     2     0
    -1     4
     3     2
```

3.6 Diagonal and Unit matrices

To input the **diagonal** matrix $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ we can proceed as in Section 4.1 or,

more conveniently:

```
>> H=diag([1,2,3])
H =
     1     0     0
     0     2     0
     0     0     3
```

To input the **unit** (identity) matrices $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ either use *diag*

or

```
>> I2=eye(2), I3=eye(3)
I2 =
     1     0
     0     1
I3 =
     1     0     0
     0     1     0
     0     0     1
```

Multiplying a matrix by a unit matrix leaves it unchanged.

» **A, A*I2, I2*A, B, I2*B, B*I3**

A =

```

      3      5
      1      2

```

ans =

```

      3      5
      1      2

```

ans =

```

      3      5
      1      2

```

B =

```

      1      6      2
      5      3      4

```

ans =

```

      1      6      2
      5      3      4

```

ans =

```

      1      6      2
      5      3      4

```

(What would happen if we tried to find $B \times I_2$ or $I_3 \times B$?)

3.7 The determinant of a square matrix

This is found using *det*. The determinants of $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 2 & 5 & 3 \end{bmatrix}$ are

» **P=[3 -4 2;2 -7 9;4 -2 -3];**

» **detA=det(A),detP=det(P)**

detA =

```

      1

```

detP =

```

     -3

```

You can only find the determinant of a square matrix:

» **det(B)**

??? Error using ==> det
Matrix must be square.

3.8 The inverse of a square matrix (A^{-1})

This is found using *inv*.

To find the inverses of A and P and verify that $A \times A^{-1} = I_2$ and $P \times P^{-1} = I_3$.

```
» IA=inv(A),A*IA,IP=inv(P),P*IP
```

```
IA =
```

```
    2.0000    -5.0000  
   -1.0000     3.0000
```

```
ans =
```

```
    1.0000         0  
   -0.0000     1.0000
```

```
IP =
```

```
  -13.0000     5.3333     7.3333  
  -14.0000     5.6667     7.6667  
   -8.0000     3.3333     4.3333
```

```
ans =
```

```
    1.0000     0.0000         0  
         0     1.0000     0.0000  
         0         0     1.0000
```

3.9 Solving systems of linear equations

The equations
$$\begin{aligned} 4x + y &= 13 \\ 6x + 2y &= 20 \end{aligned}$$
 can be written in matrix form as

$$\begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 20 \end{bmatrix}, \text{ or } AX = B.$$

The solution is $X = A^{-1}B$, so:

```
» A=[4 1;6 2]; %note that this overwrites the previous
```

```
A
```

```
» B=[13;20];
```

```
» solution=inv(A)*B
```

```
solution =
```

```
    3.0000  
    1.0000
```

so the solution is $x = 3, y = 1$.

$$x + 2y + z = 3$$

To find the solution of $2x + 5y + 2z = 5$ note that the matrix of coefficients is the

$$2x + 5y + 3z = 7$$

matrix P defined earlier, with inverse called IP , so

```
» solution=IP*[1;15;-9]
```

```
solution =
```

```
1.0000
```

```
2.0000
```

```
3.0000
```

so $x = 1, y = 2$ and $z = 3$.

MATLAB can solve these equations a second way, using backslash \:

```
» solution1=A\B,solution2=P\[1;15;-9]
```

```
solution1 =
```

```
3.0000
```

```
1.0000
```

```
solution2 =
```

```
1
```

```
2
```

```
3
```

The second method, using \, is more efficient and should always be used for large sets of equations.

Exercise 3

1. Enter $a = [6 \ -2 \ 1 \ 3]$, $b = [20 \ 4 \ -1 \ 7]$ and $c = \begin{bmatrix} 7 \\ 1 \\ 5 \end{bmatrix}$.

Find $3b$, $4c$ and $5a - 2b$.

Sort a and c into ascending order.

2. Enter $d = [4 \ 5 \ 6 \ 7 \ 8]$, $e = [2 \ 2.2 \ 2.4 \ \dots \ 4]$ and $f = [100 \ 95 \ 90 \ \dots \ 50]$
3. Find the vector whose elements are the
 - (i) products of those of a and b
 - (ii) quotients of those of a and b
 - (iii) quotients of those of b and a
 - (iv) the squares of those of d
 - (v) the cubes of those of e .
 - (vi) twice the cube minus the square of those of f .

4. Input the matrices $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 0 \\ -1 & 6 \\ 3 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 7 \\ 0 & 1 & 5 \end{bmatrix}$.

Find, if possible, (i) $A + B$, $2A - 3B$, $C - D$

(ii) AB , BA , AC , CB , DC

(iii) A^T , C^T

(iv) $\det A$, $\det C$ and $\det D$

(v) A^{-1} , B^{-1} , C^{-1} and D^{-1}

(vi) A^2 , B^3 and D^4 .

5. Find $AB - BA$. Is $AB = BA$?
6. Find $(A + B)^2$ and $A^2 + 2AB + B^2$. Are they equal?
Show that $(A + B)^2 = A^2 + AB + BA + B^2$.
7. Find $(AB)^2$ and A^2B^2 . Are they equal? Show that $(AB)^2 = ABAB$.
8. Show that $(AB)^T = B^T A^T$ and $(AB)^{-1} = B^{-1}A^{-1}$.
9. Show that (i) A satisfies the equation $A^2 - 4A + 3I = 0$
(ii) D satisfies the equation $A^3 - 6A^2 - 4A + 24I = 0$.
10. If $E = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $F = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ show that $E^{-1} = E^T$ and $F^{-1} = F^T$.
 E and F are called orthogonal matrices.
11. If $Q = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ show that $(QDQ^{-1})^4 = QD^4Q^{-1} = \begin{bmatrix} -44 & 90 \\ -30 & 61 \end{bmatrix}$.
Can you explain this result?
12. Solve the following sets of equations by matrix inversion.
- | | | |
|--|--|--|
| (i) $\begin{aligned} 2x + y &= 7 \\ x + 2y &= 2 \end{aligned}$ | (ii) $\begin{aligned} 2x + y + 3z &= 7 \\ -y + 7z &= 9 \\ y + 5z &= 3 \end{aligned}$ | (iii) $\begin{aligned} 3x + 2y - 4z + w &= 17 \\ 5x - 3y + 2z + 3w &= 2 \\ x + 4y + 5z + 3w &= 12 \\ 2x + 5y + 6z - 4w &= 9 \end{aligned}$ |
|--|--|--|