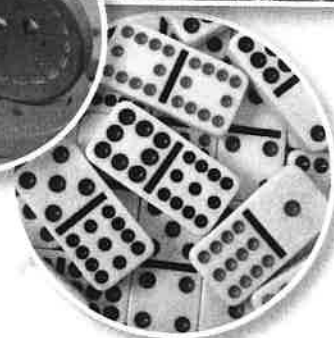


Improving Primary Mathematics

Teaching and Learning



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7 Learning difficulties in mathematics

Carol Rushworth-Little

Introduction

Dyscalculia is a problematic area for educators, not only because evidence is complex and contested, but also because it comes from scientific disciplines like neuro-psychology, with different theoretical perspectives and research methodologies. Some of these have implicit assumptions which conflict with current theories of mathematics learning.

(Gifford 2006: 36)

Many children have difficulties with some or many aspects of arithmetic, failing to make the expected progress in mathematics. A significant number of pupils have severe and specific difficulties which are sometimes described as dyscalculia. This chapter explores existing research and literature about dyscalculia and mathematical learning difficulties and raises awareness of the difficulties experienced by certain pupils that can impact upon their mathematical learning and development.

Chapter aims

This chapter discusses possible indicators and learning behaviours of pupils which, excluding language difficulties, are associated with a learning difficulty in mathematics. These are discussed through a case study which compares the observed learning behaviours and characteristics exhibited by pupils who have been identified as having a mathematical learning difficulty, to those characteristics identified by teachers. The aims of this chapter are to:

- Increase knowledge of current research around dyscalculia and mathematical learning difficulties

- Develop a wider understanding of factors that can affect mathematical learning
- Encourage teachers of mathematics to track back and delve deeper into the underlying issues that may be presenting stumbling blocks in learners' mathematical development.

The mathematics curriculum and children's learning

Within the *Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools* (Williams 2008) is a section on under-attainment in mathematics in primary schools in England identifying that, despite the introduction of the National Numeracy Strategy (NNS) in 1999, around 6 per cent of pupils each year fail to achieve a National Curriculum level 3 in mathematics by the time they leave the primary sector.

It is not a specifically English phenomenon; individual differences in arithmetic among pupils of the same age are also very great in most other countries (Dowker 2004).

The term 'mathematics' encompasses a wide range of interconnected skills and knowledge that, at primary school level, include number, using and applying, organizing and using data, and shape, space and measure.

While it is recognized that pupils can experience difficulties in learning in any (or all) of these mathematical strands, the term 'mathematical learning difficulty' is commonly used to describe those pupils who have significant difficulty with number.

In recent years, the term 'dyscalculia' has been introduced to describe the inability to acquire arithmetic skills and understand simple number concepts. There is ongoing debate surrounding the dyscalculia label, with dispute and lack of consensus between academics and researchers not only on cause and identification of dyscalculia but also on definition. While the discussion more comfortably resides with the academics, it is important for the educator, while focusing on supporting the struggling learner, and coping with parental pressure for diagnosis, to have an insight into current, research-informed perspectives.

Conceptual and theoretical perspectives: dyscalculia

Defining dyscalculia or mathematical learning difficulty

The term developmental dyscalculia was introduced by psychologist Ladislav Kosc in 1974. Subsequent studies have used different terms

to describe what would appear to be the same difficulties (Geary and Hoard 2001) including 'mathematical disability', 'arithmetic learning disability', 'number fact disorder' and 'psychological difficulties in mathematics'. There is ongoing debate as to what dyscalculia is and whether or when the term should be used. Some regard it as a brain-based disorder, involving abnormal function of those areas which are known to be involved in number and arithmetic. Others regard it in functional terms involving specific and severe mathematical difficulties without reference to a cause. In 2001 the DfES published a definition that involved identifying characteristics that describe the learning difficulty:

a condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.

(DfES 2001: 2)

This has become an accepted definition of dyscalculia within the UK although some consider that its manifestation is simply the lower end of the continuum of ability or achievement in mathematics (Dowker 2009).

Prevalence of dyscalculia

As there is no universally agreed definition of dyscalculia, there is a range of figures given for the prevalence of dyscalculia or specific mathematics learning difficulties. However, a large number of studies put the figure at about 6 per cent. Some quick arithmetic will identify that this is equivalent to 1 or 2 children in an average class of 30 pupils. Clearly this has major implications for every class teacher.

Reflection

Consider the fact that in an average class of 30 children it is likely that 2 pupils will have significant difficulty in understanding simple number concepts, will lack an intuitive grasp of numbers and will experience problems learning number facts and procedures.

What indicators might help you, as a teacher, to identify those pupils?

Identifying a dyscalculic pupil

The British Dyslexia Association (BDA) identifies behavioural commonalities or characteristics that can increase our understanding of the learning difficulties experienced by dyslexic pupils. It would be helpful if such a list of indicators existed for dyscalculia. Despite the lack of agreement surrounding this mathematical learning difficulty, Ronit Bird, a teacher of dyscalculic pupils, included the following list of indicators in her 'Dyscalculia Toolkit' published in 2007 (reproduced here with permission from Sage Publications).

Indicators for dyscalculia are:

- An inability to subitize (see without counting) even very small quantities
- An inability to estimate whether a numerical answer is reasonable
- Weaknesses in both long-term and short-term memory
- An inability to count backwards reliably
- A weakness in visual and spatial orientation
- Directional left/right confusion
- Slow processing speeds when engaged in mathematics activities
- Trouble with sequencing
- A tendency not to notice patterns
- A problem with all aspects of money
- A marked delay in learning to read a clock to tell the time
- An inability to manage time in their daily lives.

(Bird 2007)

A number of these characteristics could also describe a dyslexic pupil who has difficulty in acquiring mathematics skills and research suggests that there is an overlap or co-occurrence of dyscalculia with other learning difficulties, including dyslexia, dyspraxia and attention deficit hyperactivity disorder (ADHD) (Geary 2004).

It would be unwise to unquestioningly accept Bird's (2007) indicators of dyscalculia without exploring other perspectives. Consequently, the remainder of this chapter is divided into two sections; the first section looks at each of Bird's descriptors, identifies research that relates to it, and considers its impact on mathematical learning and development. The second section uses a case study to explore which characteristics were most commonly observed in pupils with a mathematical learning difficulty and compares these results with the indicators of dyscalculia that were most commonly identified by teachers.

What we know about indicators of dyscalculia or a mathematical learning difficulty

In this section, Bird's (2007) indicators of dyscalculia are examined in turn and discussed with reference to current research and theory.

An inability to subitize even very small quantities

Subitizing refers to the ability to look at a random cluster of objects or dots and know how many are there, without counting. Most adults can subitize between five and seven items.

A study that involved matching displays of dots found that dyscalculic children needed to count the number of dots whereas non-dyscalculic children could recognize the threeness of three without counting (Koonz and Berch 1996). It is believed that this fundamental capacity, recognizing small numerosities without counting (subitizing), could be tied to a child's understanding of number. Butterworth's (1999) work similarly shows that dyscalculic pupils lack the ability to subitize and he refers to this as part of 'the missing starter kit'.

Reflection



As teachers we need to be aware that, while the majority of pupils can 'see' that there are more dots on the right than there are on the left, some pupils will have to count how many dots are on each side of the diagram and then use their knowledge of number to identify the larger quantity. Consequently, the time required to complete what we consider to be a relatively simple task is greatly increased.

An inability to estimate whether a numerical answer is reasonable

Some pupils lack skills in estimation due to lack of experience in doing so in both the social and educational context. It is suggested that embedding 'real life' contexts into the teaching of mathematical concepts supports those pupils who do not make the intrinsic link between what happens in a mathematics lesson to the everyday arithmetic problems encountered in the world outside.

Weaknesses in both long-term and short-term memory

In identifying the impact of memory weakness upon mathematical learning, it is helpful to use a simplistic dual-store memory model (Atkinson and Shiffrin 1968).

In this model, long-term memory is recognized as a limitless storing of associations among items, while short-term memory is seen as a temporary storage system that holds items for around 20–30 seconds.

Weak long-term semantic memory

To be successful in basic mathematics pupils are required to remember a huge variety of facts and concepts. It is necessary to recall:

- written symbols, e.g. $+$, $-$, $\times$, $\div$, $=$ and the processes that they represent
- a large number of basic number facts such as $3 + 7 = 10$, $10 \times 6 = 60$, double 8 is 16.

Good long-term memory is also required for recall and application of procedural rules.

Geary (2004) found that many children with mathematics learning difficulties (MLD) had difficulties in retrieving basic arithmetic facts from long-term memory and this tended not to improve over time.

Reflection

Teachers frequently describe a pupil as having mathematical difficulties because 'he can't remember his times tables' or 'she doesn't even know the number bonds of 10'. When we test or assess a pupil in mathematics, does the test reflect their overall arithmetic ability or is it weighted in favour of the ability to quickly retrieve number facts from memory?

Weak short-term memory

Short-term memory can be regarded as the system for temporarily storing and managing information required to carry out complex cognitive tasks such as learning, reasoning and comprehension. Short-term memory can receive information from both visual and auditory input.

The most universally accepted model of *working memory* is that proposed by Baddeley (1986). Working memory is comprised of one supervising system, the central executive and two modality-specific storage systems, the phonological loop (auditory short-term memory) and the

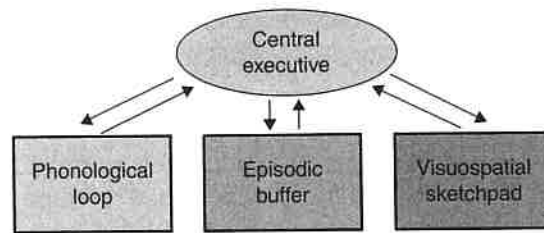


Figure 7.1 A simplified model of working memory

visuospatial sketchpad (visual short-term memory). The phonological loop is specialized for the storage and rehearsal of speech-based verbal information (Baddeley 1992), whereas the sketchpad is specialized for holding visual and spatial material (Logie 1986). See Figure 7.1.

It has been shown that working memory deficits in both the phonological and visuospatial domains contribute to mathematics difficulties (Wilson and Swanson 2001).

In a typical test of auditory memory span, an examiner reads a list of random digits aloud at the rate of one digit per second. At the end of a sequence, the person being tested is asked to recall the items in order. It is suggested that the number of items or digits that a child is able to hold in short-term memory aligns to their age in years up to the age of 7: an average 2-year-old will be able to retain two digits, an average 5-year-old five digits and so on. This appears to reach a ceiling at 7 years of age, with the average retention of digits for adults currently placed at seven.

The case study section of this chapter found that the majority of pupils identified as having a mathematical learning difficulty were unable to hold more than four digits in short-term auditory memory.

An element of *working memory* is the phonological loop where words are temporarily held while they are part of a process, such as adding numbers.

Reflection

Consider the difficulties experienced by pupils who are unable to hold more than four digits in short-term memory when they are participating in a fast paced oral and mental session that requires them to add two 2-digit numbers; such as 'What is the sum of 28 and 36?'

Working memory will be overloaded with the retention and recall of the digits and consequently the process of adding those numbers becomes impossible.

- How might the pupils respond?
- What can teachers do to support pupils with weak short-term memory skills?

Poor working memory in the visuospatial domain may result in an unreliable visuospatial sketchpad which will affect mathematical working not due to lack of understanding but because of mistakes when copying or transferring data.

Recent studies in Finland (Kytälä et al. 2010) identified that children with mathematical difficulties performed less well than age-related peers in all areas of working memory but *specifically* the visuospatial working memory.

An inability to count backwards reliably – sequencing

Although Bird (2007) suggests that an inability to count backwards reliably is an indicator of dyscalculia, this is not a widely acknowledged characteristic. Chinn and Ashcroft (2007) note that it is not uncommon to identify children who find counting backwards difficult; however, their findings link this difficulty to the dyslexic learner, which resonates with Butterworth and Yeo's (2004) findings of co-occurrence.

Directional left/right confusion

Directional confusion is a recognized characteristic of dyslexia and/or dyspraxia. The lack of directional consistency in mathematics can be the cause of major difficulty and confusion for a number of learners.

Consider the written sequence of the number 14, where the order, when the number is spoken, would seem to imply 4 followed by 10 but in fact means one ten followed by four units and is represented in that order, 14. Thus we write the number in the reverse *direction* to that in which it is said. (We say four ten but write ten four.) However, when the pupil moves beyond 'teen' numbers that rule changes; forty-two, representing four tens followed by two units, is represented as 42. (We say four tens and two and write it in the same order.) This time the direction is not reversed.

There is also a lack of consistency in the direction in which procedures are performed.

Consider the following two calculations and the confusion that could be experienced due to the directional inconsistency of written procedures. Let us explore two examples. First, the calculation $252 + 12$.

Procedure: align the numbers so that the first is *above* the second. Carry out the column addition procedure working *from the right to the left* and write the answer *underneath* the calculation.

$$\begin{array}{r} 252 \\ + 12 \\ \hline 264 \end{array}$$

Next, the calculation $252 \div 12$.

Procedure: align the numbers so that the first comes before the second on the same line. Carry out the division procedure working *from left to right* and write the answer *above* the calculation:

$$\begin{array}{r} 21 \\ 12 \overline{) 252} \end{array}$$

This lack of consistency in direction of basic procedures can be the source of considerable confusion for pupils with directional difficulties.

Slow processing speed when engaged in mathematics activities

Chinn's (1995) study compared the average time to stop work on 21 basic numeracy questions for a mainstream school population and a specialist dyslexia school population, and found that, on average, dyslexic learners took 50 per cent longer to stop attempting the task. The major focus of Chinn's (1995) work has been upon the mathematics difficulties experienced by dyslexic pupils; although Butterworth and Yeo (2004) propose that dyscalculia is best understood in terms of a specific numerical processing deficit, they allow that dyscalculic pupils may have problems with the speed at which they process numbers.

Trouble with sequencing

Sequencing difficulties have long been identified as characteristics of the dyslexic learner (Williams 2008).

Sharma (2003) believes that one of the manifestations of dyscalculia can be difficulty in following sequential directions and sequencing – including reading numbers out of sequence, substitutions, reversals, omissions and doing operations backwards.

Sequencing difficulties will affect a pupil's ability to recognize patterns in common number sequences such as odd and even numbers.

A tendency not to notice patterns

Pattern is at the heart of mathematical learning and understanding, with the ability to 'see' a pattern leading onto the more advanced concept of generalization. As many mathematical concepts are usually more easily understood if they can be associated with a visual image, impairment in the visuospatial sketchpad could result in the inability to notice visual sequences in number. Hannell (2005) suggests that pupils with dyscalculia may not use visual images effectively and may find reasoning that depends upon spatial reasoning difficult to understand. Similar, measurement, shape and space have a direct link to visuospatial abilities; weaknesses in this area could result in the inability to see patterns not only in number but also in shape.

Reflection

There are a number of reasons why a pupil may have difficulties with certain areas of mathematics. As teachers we see the 'result' of that difficulty (e.g. an inability to recognize pattern within the five times table) but when we are trying to unpick the 'cause' of the problem would we necessarily link it to an inability to 'see' patterns generally? How might we test that ability?

Difficulties experienced with understanding money and time

Bird's (2007) final three indicators can be regarded as presentational effects of dyscalculia rather than an underlying difficulty. Difficulties in handling and using money have been highlighted by a number of researchers who have worked with dyscalculic individuals. Sharma (1990) identifies difficulties found in handling a bank account, giving and receiving change and tipping. These findings are supported by Butterworth and Yeo (2004), who state that dyscalculics find it difficult to work with money and often find that shopping is a humiliating experience. They also comment on the difficulties presented by everyday planning, as those with dyscalculia struggle to learn to tell the time.

Emotional response to mathematics

One aspect of dyscalculia upon which all researchers seem to agree is the negative emotional impact that it has on those who live with the learning difficulty.

Butterworth and Yeo (2004) identify that most dyscalculic pupils do not enjoy number work, feel discouraged in mathematics lessons and as

a consequence many develop avoidance strategies. They believe that the lack of understanding of peers, parents and teachers can lead to mathematics anxiety, which further undermines their ability in making sense of numbers. Chinn and Ashcroft (2007) affirm this and suggest that it is the 'uniquely judgemental nature of mathematics, often exacerbated by those who design teaching programmes and those who actually teach mathematics that makes mathematics a topic that creates negative attributional style in so many students (and thus in adults)' (Chinn and Ashcroft 2007: 33).

Indicators of dyscalculia or a mathematical learning difficulty – report from a case study

This section looks at research conducted in 2009 and 2010. The aim of the study was to identify pupil indicators or learning behaviours associated with a specific mathematical learning difficulty or dyscalculia and to compare these observed characteristics with those identified by teachers. These characteristics were aligned to Bird's (2007) indicators of dyscalculia.

Context

The study focused upon 44 pupils identified as having specific learning difficulties linked to mathematics, and a group of 36 teachers and teaching assistants. Pupils with global learning difficulties were not included within the study. The group of 24 females and 20 males were aged between 7 and 16 years of age (31 primary school pupils and 13 secondary school pupils). I carried out the observed element of the research – assessing and recording mathematical understanding and difficulties through means of a diagnostic assessment test in a one-to-one situation. Throughout the assessment pupils were encouraged to explain their methods and reasoning both verbally and through written methods and jottings.

The group of teachers and teaching assistants (hitherto called 'teachers') was comprised of 4 teachers from the primary phase and 12 from the secondary phase of education. These teachers were asked to identify a typical profile of a pupil who they perceived as having a specific learning difficulty in mathematics and then to answer 11 questions with that pupil in mind. The questions were drawn from Bird's indicators of dyscalculia but with the omission of 'ability to manage time in daily life' as it was felt that teachers would not be able to comment with certainty upon this.

Results – an overview

The results of significant behavioural characteristics identified by observation of pupils are shown in Table 7.1 as are those regarded as significant

Table 7.1 Most common behavioural characteristics identified through observation

Behavioural characteristics	Observed	Identified by teaching staff
Pupil does not have a good short-term memory	84 per cent	31 per cent
Pupil is unable to identify number sequences	61 per cent	36 per cent
Pupil has slow processing speeds when engaged in maths activities	57 per cent	69 per cent
Pupil never notices pattern in number and shape	55 per cent	31 per cent
Pupil does not have the ability to make a reasonable estimate	52 per cent	61 per cent

by teachers. The purpose of this is to identify how closely the two sets of data align.

The highlighted characteristics are those that were identified as significant (greater than 50 per cent) both in observation of pupils and as recognized by teachers.

The significant behavioural characteristics identified by my observation of, and guided discussion with pupils, are identified in Table 7.1 in order of frequency of observed occurrence

The significant learning behaviours identified by teaching staff are listed in Table 7.2 in order of frequency.

Table 7.2 Most common behavioural characteristics identified by teaching staff

Behavioural characteristics	Observed	Identified by teaching staff
Pupil has slow processing speeds when engaged in maths activities	57 per cent	69 per cent
Pupil lacks the ability to estimate whether a numerical answer is reasonable	52 per cent	61 per cent
Pupil lacks the ability to count backwards reliably	30 per cent	53 per cent

Reflection

Take a moment to look again at the results in Table 7.1 and Table 7.2 and consider the correlation between characteristics that were identified as most common by teachers and the incidences of those observed. How does this relate to your perception and understanding of pupils with mathematical difficulties?

Results in greater detail

The results of the study highlighted seven significant features.

Weak short-term memory

Weak short-term memory skills was the most frequently observed characteristic of pupils but was identified as a problem by only 31 per cent of teaching staff. About 84 per cent of pupils, when tested on auditory recall of digits, achieved below age appropriate recall, yet only 31 per cent of the teachers perceived weak short-term memory as a characteristic of such pupils. In particular, there is a distinct lack of correlation between staff perception and pupil demonstration in this area for pupils of secondary school age; 12 out of 13 of these pupils had weak short-term memory but only 4 out of 12 of secondary phase teaching staff identified this as a characteristic.

Emma, aged 16, articulated the difficulty and frustration that she felt when asked to recall a five-digit sequence.

'When I see a maths problem written down that's not so bad but mental maths is a problem. When you gave me those numbers, just now, I could see them all in my head for a split second, then they all just sort of merged together and I couldn't remember any of them.'

It would appear that there is a lack of awareness of both the existence and impact of weak short-term memory upon arithmetic or mathematical competency in many teachers.

Difficulties in spotting patterns in number and shape

A significant number (55 per cent) of the pupils exhibited difficulties in spotting patterns in both number and shape. However, only 31 per cent of teachers identified this as an indicator of dyscalculia.

Joe, aged 8, has difficulty in spotting patterns in numbers. His teacher expressed concern that he could not see the pattern in the five times table.

'We've laid out number cards with multiples of 5 on them up to 50 and then got Joe to make each of the numbers using base 10 equipment so that he can see an image of the 10s and 5s that make up the pattern but he just doesn't seem to get it.'

I tested Joe to see if the inability to 'see' a pattern was linked specifically to number, or also to shape, by placing a repeated pattern of coloured pegs in a line and asking Joe to continue the pattern. He was able to continue a repeated pattern of red, yellow, red, yellow pegs but consistently failed to repeat patterns of three colours. The relatively small percentage of teachers identifying pattern spotting difficulties as a characteristic of dyscalculia could possibly be attributed to the limiting nature of the DfES (2001) definition of dyscalculia focusing specifically on number difficulties and not linking these to difficulties in spatial sense. Thus, a pupil who has difficulty in noticing patterns within shape could be displaying dyscalculic characteristics.

Reflection

Spotting pattern in both number and shape underpins the development and understanding of many mathematical concepts. Has the focus on identification of dyscalculia as being linked solely to number difficulties contributed to a lack of recognition of underlying visuospatial difficulties?

Inability to identify number sequences

61 per cent of pupils exhibited difficulty in identifying simple number sequences. Despite the fact that only 1 of the 12 secondary school teachers perceived that pupils had difficulty identifying number sequences, in actuality, 7 of the 13 secondary aged pupils could not identify odd and even numbers to 20.

Jenny, aged 13, said:

'I learned this in First School. If you pair your fingers then you get the even numbers – it doesn't really work after 10.'

Jenny can recite her two times table to 20 but makes no connection between this and pairing her fingers.

Chinn and Ashcroft (2007) claim that dyslexic pupils are unable to automatize many mathematical number sequences. Thus it would seem likely that if a pupil has a specific learning difficulty that impacts upon mathematical ability, be it dyslexia or dyscalculia, then an ability to recognize number sequences is certainly an identifiable characteristic.

Slow processing speeds when engaged in mathematics activities

Pupil response time when attempting ability-appropriate mathematical progress tests were noted. Results showed that 57 per cent of pupils required at least 50 per cent extra time to complete the assessment paper. Staff perception of slow processing speeds in mathematics was high: 75 per cent of primary level teachers and 58 per cent of secondary phase teachers (69 per cent overall) identified slow processing skills in mathematics as an indicator of a specific learning difficulty in this area. It is possible that the National Strategies, with their focus on pacey oral and mental sessions within the mathematics lesson, have heightened teacher awareness of speed of response. This lack of speed in number processing can present difficulties for teachers struggling to adhere to both the attainment and inclusion agendas within education. I once observed the 'oral and mental starter' session of a lower Key Stage 2 mathematics lesson. The focus of the observation was 'including pupils with a Special Educational Need (SEN) within the daily mathematics lesson'. The session had been well planned and the teacher had included questions differentiated by number size, to maximize the participation of the less able. I had the opportunity to focus upon Victoria's responses. Victoria had slow processing speeds in number; however, she had learned the method of calculation and was

keen to put up her hand to give the answer. Her solution to the problem was correct; unfortunately the answer that she gave was the correct answer to a problem set two minutes earlier. In this fast paced environment Victoria's success was perceived as failure.

Reflection

- Are you aware of any pupils who have slow processing speeds with number within your class?
- What strategies could you employ to ensure that they have enough time to answer a question correctly?

Inability to estimate

One of the most frequently commented upon characteristics of a pupil with mathematical difficulties was a lack of 'feel' for the size of number.

One primary school teacher described the difficulties of Charlotte, aged 8, as follows:

'It's as though Charlotte is just a random number generator, you know. I ask, 'What number is 1 more than 5?' and she says, '3? No 8. I don't know, 20?' It's as though she's just guessing or using some words that she's heard before.'

Alice, aged 10, also exhibited a lack of understanding of the size of numbers. She was asked to solve the mathematical problem $35 + 27$ and to explain her methods

Alice: You write down 35 and then put 27 underneath and then you just add them, so 5 add 7 (*counts on fingers*) is 12. Write it down. Then 3 + 2 is 5.

$$\begin{array}{r} 35 \\ 27+ \\ \hline 512 \end{array}$$

Tester: So you've got the answer five hundred and twelve. Do you need to check? Does it seem right that thirty-five add twenty-seven is five *hundred* and twelve?

Alice: Yes, that's right, that's how we do it.

There may be a number of reasons why Charlotte and Alice, in common with many pupils, respond with unreasonable answers. One of these may be an inability to estimate. The inability to provide a reasonable estimate was perceived as a characteristic of dyscalculia by 61 per cent of teaching staff in the study, whereas results of pupil observation identified 52 per cent of pupils overall having difficulty with estimation. An interesting result of this study was the strong gender bias in the results. There were more than twice as many female pupils unable to estimate than males with 71 per cent of female pupils unable to estimate compared with 35 per cent of male pupils. The results also indicated a perseverance of this difficulty for females across the age ranges, while this was not reflected in the boys' results.

Inability to count backwards reliably

Across the primary and secondary age range, 53 per cent of all teachers recognized that dyscalculic pupils had difficulty in counting backwards. This varied from 63 per cent in the primary phase of education to 33 per cent in the secondary school phase. These teacher perceptions aligned to observed pupil characteristics, where 64 per cent of pupils in the 7.1–8.0 age range were unable to count backwards reliably, whereas only 1 out of 13 pupils (8 per cent) had the same difficulty at secondary school level. This may suggest the possibility of developmental delay rather than a lifelong disability. However, it is also possible that older pupils have had the opportunity to experience a range of learning experiences that have helped in the development of this skill.

Negative emotional response to mathematics

A significant percentage of pupils had a negative emotional response to mathematics; a characteristic that was identified and recognized by teachers.

Discussion around mathematical difficulties with David, a 12-year-old, elicited the following:

- David:** One of the reasons that I find this stuff difficult is because of all the supply teachers last year. I got hot-seated a lot so I missed most of the stuff that got taught.
- Interviewer:** What's hot-seating all about? I don't think I've come across that.
- David:** It's like when you mess about and act up, you get hot-seated and you have to go and work outside the year head's office and they don't set you the work from class. They just give you easy stuff to shut you up – so I didn't learn anything last year.
- Interviewer:** Did you get hot-seated a lot the year before?
- David:** Not really. I just didn't understand a lot of what was going on.

David's avoidance strategy was to present behavioural difficulties that would have him removed from a situation where there was the potential for his peers to recognize his mathematical difficulties. He believed that recognition for being the 'bad boy' was preferable to being identified as 'stupid'.

Amy, aged 9, had her own avoidance strategies.

- Interviewer:** If you had to give a mark out of 10 for independent work in numeracy, that's working just on your own, what would you give that?
- Amy:** Probably 3.
- Interviewer:** What about working with a partner?
- Amy:** Could I choose the partner? (interviewer nods) Well if I could work with Jason he's really good so probably 10.
- Interviewer:** What about working in a group?
- Amy:** I think not so good, as they wouldn't always listen to what I said, so probably 5.

Amy is not alone in choosing to work with a partner who she identifies as more intelligent, believing that her lack of understanding will be masked. It is interesting to note that she has self-identified her role within

a group and believes that her ideas or mathematical suggestions would not be well received.

Reflection

Response to mathematics anxiety or a negative attitude towards mathematics can manifest itself in many ways, often with attempts to mask the emotion. David misbehaves so that he will be removed from the situation, Anna enjoys working with a mathematically stronger partner so that her difficulties are hidden. Can you identify other responses or strategies used by struggling pupils?

Summary

In conclusion, it is suggested that Bird's (2007) list of 'indicators of dyscalculia' should not be regarded as a 'tick list' for the identification, or labelling, of pupils, but more as a starting point in introducing teachers to factors that can present stumbling blocks in mathematical learning and development. Discussion of them in relation to current research helps teachers develop deeper understanding of mathematical difficulties.

The case study highlighted specific areas of shortfall in teacher understanding of difficulties experienced by dyscalculic pupils. Specifically there is a need for teachers to develop a greater understanding of the role that short-term and working memory plays in the ability to store and manipulate numbers mentally. They should also be encouraged to recognize the qualitative nature of dyscalculia that involves difficulty in spatial and visualization sense.

Despite the fact that this chapter has focused on establishing commonalities in the characteristics of a dyscalculic learner, it is important to recognize that no two children with arithmetic difficulties are the same. We need to find out the specific strengths and weaknesses of an individual pupil to enable us to provide appropriate support and to guide our teaching strategies.

The focus of this chapter has been to highlight the underlying difficulties that can affect mathematical progress and to prompt self-reflection on how pedagogy can be modified or improved to support pupils with mathematical difficulties. The Williams Report (2008) recognized that early intervention is a key feature in supporting the mathematical development of under-attaining pupils. It is also widely recognized that a

one-to-one remedial intervention, formulated in response to a diagnostic assessment of an individual's strengths and weaknesses, is the most effective way to support those with mathematical learning difficulties. Consequently, there has been no recommendation of any one intervention programme. However, those who wish to familiarize themselves with the effectiveness of intervention schemes may wish to read Dowker's (2009) comprehensive research on this matter.

Further reading

Bird, R. (2007) *Dyscalculia Toolkit* London: Sage.

This is a practitioner's guide to supporting pupils who have a specific learning difficulty in mathematics; it contains a number of teaching strategies and games.

Chinn, S. and Ashcroft, R. (2007) *Mathematics for Dyslexics Including Dyscalculia* (3rd edn). Chichester: Wiley.

A number of the indicators of dyscalculia explored in this chapter have reflected those mathematics difficulties experienced by dyslexic learners. This book provides a background understanding of why dyslexic pupils may have difficulties in mathematics and provides structured ideas for addressing the individual needs of the learner.

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