

Name: Michael Anderson P151178

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Date: 11/5/20

Time: 1pm

Module: M6X00420 - Stress Analysis + FEA

1) Left Blank

2) (a) Prove $\gamma = \frac{Q}{AG}$

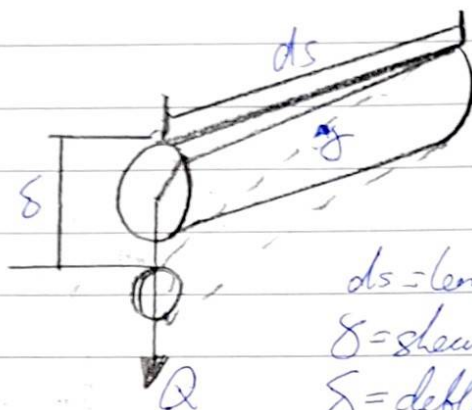
$$\text{As } G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\gamma} \quad \text{--- (1)}$$

$$\text{so } G = \frac{\tau}{\gamma} \quad \text{--- (2)}$$

$$\text{As } \tau = \frac{Q}{\text{CSA}} = \frac{Q}{A} \quad \text{--- (3)}$$

$$\text{Sub (3) into (1)} \rightarrow G = \frac{Q}{A\gamma}$$

$$\text{Make } \gamma \text{ subject} \rightarrow \gamma = \frac{Q}{AG}$$



ds = length of bar

γ = shear strain

δ = deflection

Q = Shear load

ii) Prove $U = \frac{Q^2 L}{2AG}$

$$\text{From graph } U = \frac{1}{2} \gamma Q$$

$$\text{As } \gamma = \frac{Q}{AG}$$

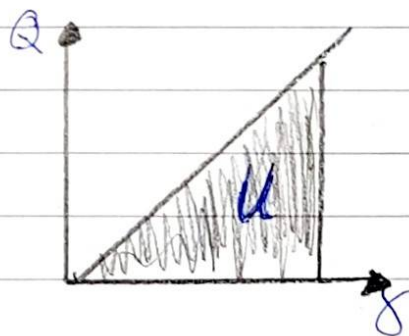
$$\text{Sub } \gamma \rightarrow U = \frac{1}{2} \cdot \frac{Q}{AG} \cdot Q$$

$$\text{Condense} \rightarrow U = \frac{Q^2}{2AG}$$

$$\text{Integrate for strain energy} \rightarrow U = \int_0^L \frac{Q^2}{2AG} ds \text{ * with respect to length of bar}$$

hence,

$$U = \frac{Q^2 L}{2AG}$$



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② i) Hollow Box

$$CSA = (30 \times 20 \text{ mm}) \times 10^{-6} \text{ m}^2$$

$$t = 2.5 \text{ mm} = \times 10^{-3} \text{ m}$$

$$L = 2.4 \text{ m}$$

$$\sigma = 480 \text{ MPa} = \times 10^6 \text{ Pa}$$

$$E = 190 \text{ GPa} = \times 10^9 \text{ Pa}$$

$$U = \frac{P^2 L}{2AE} \quad \text{As } P^2 = \sigma^2 A^2$$

$$U = \frac{\sigma^2 A^2 L}{2AE}$$

$$U = \frac{\sigma^2 AL}{2E} = \frac{(480 \times 10^6)^2 \times (2.25 \times 10^{-4}) \times (2.4)}{2 \times (190 \times 10^9)}$$

$$A = [(30 \times 20) \times 10^{-6}] - (2.5 \times 1.5) \times 10^{-6}$$

$$A = 2.25 \times 10^{-4} \text{ m}^2$$

$$U = \underline{\underline{327.4 \text{ J}}}$$

ii) Prove $h = \frac{\sigma^2 V}{2EW}$ - Extension negligible

$$\text{As } PE = mgh$$

If all PE becomes U

$$\text{And } U = \frac{P^2 L}{2AE}$$

$$\text{If } P^2 = \sigma^2 A^2 \rightarrow U = \frac{\sigma^2 A^2 L}{2AE}$$

$$\text{So } U = \frac{\sigma^2 AL}{2E}$$

$$\text{If } V = AL \rightarrow U = \frac{\sigma^2 V}{2E}$$

$$\text{So } mgh = \frac{\sigma^2 V}{2E}$$

$$\text{make 'h' subject} = h = \frac{\sigma^2 V}{2Emg}$$

as W = mg then

$$h = \frac{\sigma^2 V}{2EW}$$

iii) $M = 20 \text{ kg}$

$$h = 450 \text{ mm} = 10^{-3} \text{ m}$$

$\sigma = ?$

$$\sigma = \sqrt{\frac{2hEW}{V}} = \sqrt{\frac{2 \times (450 \times 10^{-3}) \times (190 \times 10^9) \times (20 \times 9.81)}{(2.25 \times 10^{-4}) \times (2.4)}}$$

$$\sigma = 249 \times 10^6 \text{ Pa}$$

$$\sigma = \underline{\underline{249.26 \text{ MPa}}}$$

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(20) iv) Circular Pipe

$$d = 20 \text{ mm} = \underline{\underline{20 \times 10^{-3} \text{ m}}}$$

$$L = 2.4 \text{ m}$$

$$\sigma_{\text{max}} = 249.26 \text{ MPa} = \underline{\underline{249.26 \times 10^6 \text{ Pa}}}$$

$$\epsilon = ?$$

$$M = 20 \text{ Kg}$$

$$h = 450 \text{ mm} = \underline{\underline{450 \times 10^{-3} \text{ m}}}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (20 \times 10^{-3})^2}{4} = \underline{\underline{3.14 \times 10^{-4} \text{ m}^2}}$$

$$\text{Outer } A = \underline{\underline{3.14 \times 10^{-4} \text{ m}^2}}$$

$$\underline{\underline{E = 190 \times 10^9 \text{ Pa}}}$$

$$h = \frac{\sigma^2 V}{2 E W}$$

$$W = Mg \quad V = AL$$

$$2 E W h = \cancel{\sigma^2 V} \quad \sigma^2 V$$

$$\text{so } \frac{2 E m g h}{\sigma^2} = AL$$

$$\frac{2 E m g h}{L} = A$$

$$\text{As } A_{\text{circular}} = \frac{\pi}{4} (R_o^2 - R_i^2) \\ = \frac{\pi}{4} (R_o^2 - R_i^2)$$

$$\frac{2 E m g h}{L} = \frac{\pi}{4} (R_o^2 - R_i^2)$$

$$\frac{4 \times 2 E m g h}{\pi L} = R_o^2 - R_i^2$$

$$\frac{8 E m g h}{\pi L} - R_o^2 = -R_i^2$$

$$\text{so } \frac{8 \times (190 \times 10^9) \times (20 \times 9.81) \times (450 \times 10^{-3})}{\pi \times (2.4)} - \left(\frac{20 \times 10^{-3}}{2} \right)^2 =$$

$$A = \frac{\pi}{4} (R_o^2 - R_i^2)$$

$$A = \sqrt{\frac{4 R_o^2}{\pi}} = \underline{\underline{0.01128 \text{ m}^2}}$$

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③ Hollow cylinder - Torsion

$$\text{outer } d = 75 \text{ mm} = \times 10^{-3} \text{ m}$$

$$t = 5 \text{ mm} = \times 10^{-3} \text{ m}$$

$$\sigma_{\text{max}} / \sigma_{\text{yield}} = 325 \text{ MPa} = \times 10^6 \text{ Pa}$$

$$D = 0.3$$

$$n = 6$$

$$\text{Hoop stress} = \frac{Pd}{2t} = \sigma_1$$

$$\text{Longitudinal stress} = \frac{Pd}{4t} = \sigma_2$$

$$\text{Radial Stress} = 0 = \sigma_3$$

$$\text{① Tresca} = \sigma_1 - \sigma_3 = \sigma_y \rightarrow \frac{Pd}{2t} - 0 = \frac{\sigma_y}{n}$$

$$\frac{Pd}{2t} / 0 = \frac{\sigma_y}{n}$$

$$\text{As } P = \frac{F}{A}$$

$$P \times A = F$$

$$\frac{Pd}{2t} = \frac{\sigma_y}{n} \rightarrow P = \frac{2\sigma_y}{nd}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (75 \times 10^{-3})^2}{4} - \frac{\pi (65 \times 10^{-3})^2}{4}$$

$$A = \underline{\underline{1.09955 \times 10^{-3} \text{ m}^2}}$$

$$F = P \times A = 7.22 \times 10^6 \times 1.0996 \times 10^{-3}$$

$$F = \underline{\underline{7.938 \times 10^3 \text{ N}}}$$

$$F \times R = T \rightarrow 7.938 \times 10^3 \times \left(\frac{75 \times 10^{-3}}{2} \right)$$

$$T = \underline{\underline{297.71 \text{ N/m}}}$$

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$$\textcircled{3} \textcircled{6} \text{ Von Mises} = \sqrt{(61 - 62)^2 + (62 - 63)^2 + (63 - 61)^2} = \frac{26y^2}{n^2}$$

$$\left[\left(\frac{p_d}{2t} - \frac{p_d}{4t} \right)^2 + \left(\frac{p_d}{4t} - 0 \right)^2 + \left(0 - \frac{p_d}{2t} \right)^2 \right] = \frac{3y^2}{n^2}$$

as $\sigma_3 = 0$

$$\sigma_1 = \frac{p_d}{2t}$$

$$\sigma_2 = \frac{p_d}{4t}$$

$$\left[\left(\frac{p_d}{2t} - \frac{p_d}{4t} \right) \left(\frac{p_d}{2t} - \frac{p_d}{4t} \right) + \left(\frac{p_d}{4t} - 0 \right) \left(\frac{p_d}{4t} - 0 \right) + \left(0 - \frac{p_d}{2t} \right) \left(0 - \frac{p_d}{2t} \right) \right] = \frac{26y^2}{n^2}$$

$$\left[\left(\frac{p_d^2}{4t^2} \right) - \left(\frac{p_d^2}{8t^2} \right) - \left(\frac{p_d^2}{8t^2} \right) + \left(\frac{p_d^2}{16t^2} \right) + \left(\frac{p_d^2}{16t^2} \right) + \left(\frac{p_d^2}{4t^2} \right) \right] = \frac{26y^2}{6^2}$$

$$\left[\frac{p_d^2}{t^2} \times \left(\frac{1}{4} - \frac{1}{8} - \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{4} \right) \right] = \frac{26y^2}{6^2}$$

$$\frac{3}{8} \times \frac{p_d^2}{t^2} = \frac{26y^2}{6^2}$$

$$\frac{3 p_d^2}{8 t^2} = \frac{26y^2}{6^2}$$

$$P^2 = \frac{8 \times 26y^2 t^2}{3 d^2 \times 6^2}$$

$$P = \sqrt{\frac{8 \times 2 \times 6y^2 \times t^2}{3 d^2}} = \sqrt{\frac{16 \times (325 \times 10^6)^2 \times (5 \times 10^{-3})^2}{3 \times (75 \times 10^{-3})^2 \times 6^2}}$$

$$P = 8.339 \times 10^6 \text{ Pa} \quad \text{if } A = 1.09955 \times 10^{-3} \text{ m}^2$$

$$F = PA = 8.339 \times 10^6 \times 1.09955 \times 10^{-3}$$

$$F = 9169.7 \text{ N}$$

$$T = F \times R = 9169.70 \times \left(\frac{75 \times 10^{-3}}{2} \right)$$

$$T = 343.86 \text{ Nm}$$

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④ $L = 8m$

Pinned-Pinned

$$P_e = \frac{\pi^2 EI}{L^2}$$

$$E = 200 GPa = 2 \times 10^{11} Pa$$

$$A = 7500$$

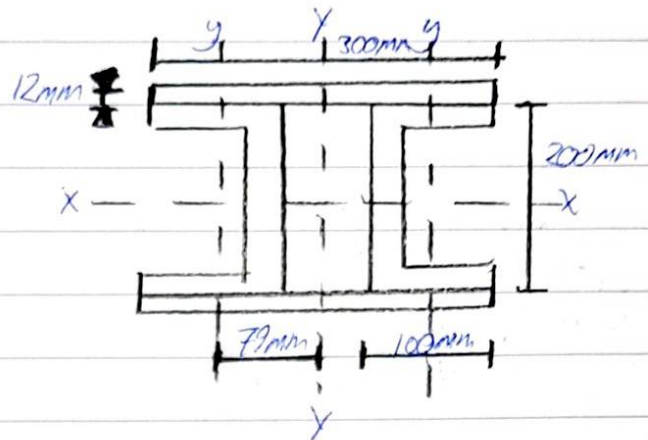
$$\sigma = 320 MN/m^2 = 3.2 \times 10^8 Pa$$

1 section

$$I_{xx} = 26.2 \times 10^6 mm^4$$

$$I_{yy} = 1.57 \times 10^6 mm^4$$

$$A = 4400 mm^2$$



⑤ I_{xx} vs I_{yy}

$$I_{xx} = 2(I_{xx \text{ section}}) + 2(I_{xx \text{ plate}})$$

$$= 2(26.2 \times 10^6) + 2(I + Ah^2)$$

$$= 2(26.2 \times 10^6) + 2\left(\frac{130.5}{12} + (300 \times 10^2) \times (100+6)^2\right)$$

$$= (52.4 \times 10^6) + 2\left(\frac{300 \times 12^3}{12} + (3600) \times (106)^2\right)$$

$$= (52.4 \times 10^6) + 2(40.49 \times 10^6)$$

$$= (52.4 \times 10^6) + (80.99 \times 10^6)$$

$$I_{xx} = 133.39 \times 10^6 mm^4$$

$$I_{xx} = 133.39 \times 10^{-6} m^4$$

$$I_{yy} = 2(I_{yy \text{ section}}) + 2(I_{yy \text{ plate}})$$

$$= 2(1.57 \times 10^6) + 2\left(\frac{130.5}{12}\right)$$

$$= 2(1.57 \times 10^6) + (4400) \times (7)^2 + 2\left(\frac{12 \times 300^3}{12}\right)$$

$$= 2(29.03 \times 10^6) + 2(27 \times 10^6)$$

$$= (58.06 \times 10^6) + (54 \times 10^6)$$

$$I_{yy} = 112.06 \times 10^6 mm^4$$

$$I_{yy} = 112.06 \times 10^{-8} m^4$$

$I_{xx} > I_{yy}$ hence will buckle about I_{yy}

⑥ $K = \sqrt{\frac{I}{A}} \rightarrow \sqrt{\frac{I_{yy}}{A}} \rightarrow \sqrt{\frac{112.06 \times 10^6}{16000 \times 10^6}} = 0.08369m$

$$A = 2(4400) + 2(300 \times 12) = 16000$$

$$A = 8800 + 3600 = 12400 mm^2 \times 10^{-6} m^2$$

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⑦ Slenderness Ratio $= \frac{L}{K} = \frac{8}{0.084} = 95.59$

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$$\textcircled{4} \textcircled{d} \text{ Euler} = \frac{\pi^2 EI}{L^2} \rightarrow \frac{\pi^2 \times (200 \times 10^9) \times (112.06 \times 10^8)}{(8)^2} = \underline{3.46 \times 10^6 \text{ N}}$$
$$= \underline{3.46 \text{ MN}}$$

$$\textcircled{e} \text{ Rankine} \rightarrow \frac{\sigma_y A}{1 + a \left(\frac{L}{K} \right)^2} \rightarrow \frac{(320 \times 10^6) \times (16 \times 10^3)}{1 + \left(\frac{1}{7500} \right) \times (9559)^2} = \underline{2.308 \times 10^6 \text{ N}}$$
$$= \underline{2.31 \text{ MN}}$$

\textcircled{f} New Plate dimensions

$$A = (224 \times 6 \text{ mm}) \times 10^{-6} \text{ m}^2$$

$$I_{xx} = 2(26.2 \times 10^6) + 2 \left(\frac{224 \times 6^3}{12} + (224 \times 6) \times 106^2 \right)$$

$$I_{xx} = (52.4 \times 10^6) + 2(15.11 \times 10^6)$$

$$I_{xx} = (52.4 \times 10^6) + (30.21 \times 10^6)$$

$$I_{xx} = \underline{82.6 \times 10^6 \text{ mm}^4} \text{ or } \underline{82.6 \times 10^{-6} \text{ m}^4}$$

$$\text{Euler} = \frac{\pi^2 EI}{L^2}$$

$$\text{Rankine} = \frac{\sigma_y A}{1 + a \left(\frac{L}{K} \right)^2}$$

$$I_{yy} = 2 \left((1.57 \times 10^6) + (400) \times (79)^2 \right) + 2 \left(\frac{6 \times 224^3}{12} \right)$$

$$I_{yy} = 2(29.03 \times 10^6) + 2(5619 \times 10^6)$$

$$I_{xx} > I_{yy} \text{ so } I_{yy} = \text{buckle } I_{yy} = (58.06 \times 10^6) + (11.24 \times 10^6)$$

$$I_{yy} = \underline{69.3 \times 10^6 \text{ mm}^4} \text{ or } \underline{69.3 \times 10^{-6} \text{ m}^4}$$

$$K = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{69.3 \times 10^6}{11488 \times 10^6}} = \underline{0.0776 \text{ m}}$$

$$A = (2 \times 400) + (2 \times 224 \times 6) = \underline{11488 \times 10^{-6} \text{ m}^2}$$

$$\text{Euler} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times (200 \times 10^9) \times (69.3 \times 10^6)}{8^2} = \underline{2.137 \times 10^6 \text{ N}}$$

$$= \underline{2.137 \text{ MN}}$$