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Lesson study as a fundamental situation for the knowledge of teaching

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Abstract

Purpose – The purpose of this paper is to elaborate on the commonality between the Theory of Didactical Situations (TDS) and lesson study to propose a model of lesson study using both the predominant graphical representation of lesson study by Lewis and the model of the didactical situation at the heart of TDS by Brousseau.

Design/methodology/approach – Starting by describing and adapting the predominant graphical representation of lesson study by Lewis and the model of the didactical situation at the heart of TDS by Brousseau, the paper integrates the two representations to highlight the commonalities between the students' learning situation and the teachers'. Based on this integrated graphical representation, the key phases of lesson study are then conceptualised by the mean of the dialectic between didactical and adidactical situation.

Findings – The reflection about the use of the TDS graphical representation embedded in the lesson study diagram helps the reflection on the use of TDS itself to analyse lesson study. This theoretical analysis describes the process of teacher learning in lesson study and the link between their learning and the student's. It also shows that lesson study is a good candidate for the fundamental situation of the knowledge for teaching.

Originality/value – The graphical conceptualisation of lesson study as a learning situation for teachers offers new insight about how teachers learn in lesson study.

Keywords Mathematics, Lesson study, Didactique, Theory of didactical situations, Fundamental situation

Paper type Conceptual paper

1. Introduction

The call for work on theorisation of lesson study have been numerous since the years 2010 and journals have made special issues about attempts towards this theorisation, both in subject matter education, especially mathematics (for e.g. Hart *et al.*, 2011; Huang and Shimizu, 2016; Quaresma *et al.*, 2018) and in more specific lesson study journal (e.g. Runesson, 2015). I have been a participant of this call and of these attempts (Clivaz, 2015a, b, 2017; Clivaz and Takahashi, 2018; Ni Shuilleabhain and Clivaz, 2017). In particular, Ni Shuilleabhain and Clivaz built a “cake-model” to analyse the use and the potential development of mathematical knowledge for teaching during lesson study process (Ni Shuilleabhain and Clivaz, 2017). These analyses, and specifically this cake-model uses the Theory of Didactical[1] Situations (TDS) by Brousseau (1997) to analyse lesson study. Using part of this French model to analyse lesson study led me to wonder about the particular commonality between lesson study and TDS (Clivaz, 2015a, 2018). This paper elaborates further on this proximity to propose a model of lesson study using both the predominant graphical representation of lesson study by Lewis (Lewis and Hurd, 2011) and the model of the didactical situation at the heart of TDS (Brousseau, 1997).

The first part of this paper, will reflect on the role of the research lesson in Lewis' schema and propose a variation of the schema. Brousseau's conception of teaching and learning in a didactical situation will then be presented. These two schemas will guide the question if lesson study is a learning situation for the teachers and how to model this situation. The second part will elaborate on the graphical combination of the two models, based on the



concept of paradidactical situation by Winsløw *et al.* (2018). The conclusion part will discuss on the hypothesis of lesson study being a fundamental situation for the knowledge of teaching.

2. Lesson study and didactical situations: two graphical representations for two learning situations

Lesson study and didactical situation certainly could not be reduced to graphical representation. Nevertheless, discussions on graphical representation could aid in understanding the ideas of the processes involved. Moreover, the reflection about the use of the TDS graphical representation embedded in the lesson study diagram can help the reflection on the use of TDS itself to analyse lesson study.

2.1 One lesson study schema

Lesson study is a simple idea. If you want to improve instruction, what could be more obvious than collaborating with fellow teachers to plan instruction and examine its impact on students? Although the idea may be simple, lesson study is a complex process. (Lewis and Hurd, 2011, p. 3)

One of the fascinating features of lesson study is its simplicity. The simplicity of the structure encapsulating the complexity of the process is shown in the most common schema of lesson study by Lewis (see e.g. Lewis, 2016, p. 572; Lewis and Hurd, 2011, p. 2; Lewis *et al.*, 2006, p. 4) in its more or less detailed forms. Outside Japan, including in Lausanne Laboratory Lesson Study, this graphical representation is often used to present the lesson study process, to teachers or to researchers (Figure 1).

Several researchers have proposed slightly different graphical representation. These representations emphasise on, for example, the alternative to revise and re-teach or not (Murata, 2011, p. 2), the importance of educational values guiding the whole process (Fujii, 2014, p. 78; 2017, p. 99), or the importance of *kyouzai kenkyuu* and of knowledgeable other (Takahashi and McDougal, 2016, p. 521).

In lesson study, the research lesson is the inquiry part of the process in many ways. For instance, preparation of research lesson guides the study of the curriculum and the

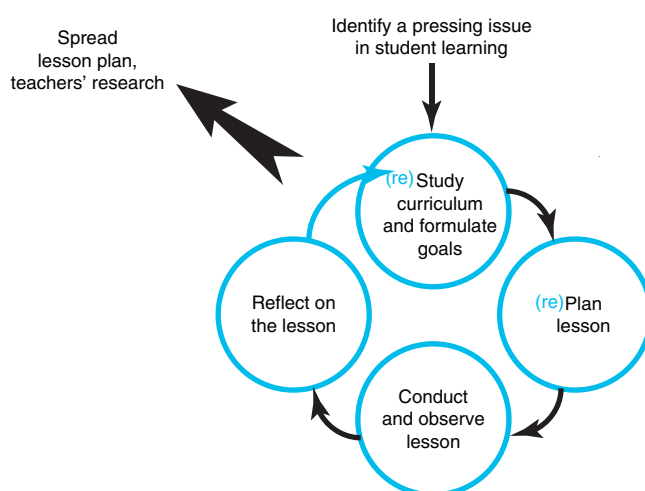


Figure 1.
Lesson study cycle

Sources: Adapted from Lewis *et al.* (2006, p. 4; Clivaz, 2016, p. 98)

planning of lesson. Instruction and observation of research lesson, on the other hand, are the source of post lesson discussions and reflections. They also are one of the main parts of the diffusion of the teachers' research. Therefore, it is no exaggeration to say that, research lesson is at the core of the lesson study process. Nevertheless, in all the above graphical representations, including Figure 1, this central part is not shown. In some representations, (Winslow *et al.*, 2018, p. 132), the research lesson even is almost positioned outside the lesson study process. To better illustrate the position of research lesson while maintaining the simplicity of Figure 1 (and even simplifying the text), I propose that research lesson to be positioned in the centre of the graphical model (Figure 2). This will also allow a better representation of the interactions of every phase of the process with the research lesson.

2.2 The learning situation in TDS

In TDS, Brousseau defines a very particular type of learning situations, called "adidactical situations". They are defined as learning situations in which the teacher has successfully hidden her will and intervention as a determinant of what the student has to do; they are those which function without the intervention of the teacher at the level of knowledge. (Brousseau, 1997, p. 236). In modern didactique, then, teaching is the devolution (the attribution of responsibility) to the student, of an adidactical, appropriate situation and learning is the student's adaptation to this situation (Brousseau, 1997, p. 31).

In a previous paper (Clivaz, 2015a), I have compared TDS and lesson study and given a number of TDS key concepts. Here, I will describe more precisely on the didactical situation at the core of the TDS in relation with its graphical representation (Figure 3).

In order to model the learning of a knowledge, Brousseau's idea is to put the student in a position where he needs this knowledge and could re-invent it. In this situation, the student interacts directly with the problem, his[2] previous knowledge, the manipulatives (if any). All these elements constitute the environment of the student, what Brousseau calls the milieu, "what the students exercise their actions on and what gives them objective responses" (Brousseau and Warfield, 2014, p. 166). The teacher is not part of the milieu since any intention to teach or to learn is not part of the adidactical situation. The situation where the student interacts with the milieu is called adidactique. But, in fact, the teacher has created the milieu, and she is interacting with the adidactical situation. The situation

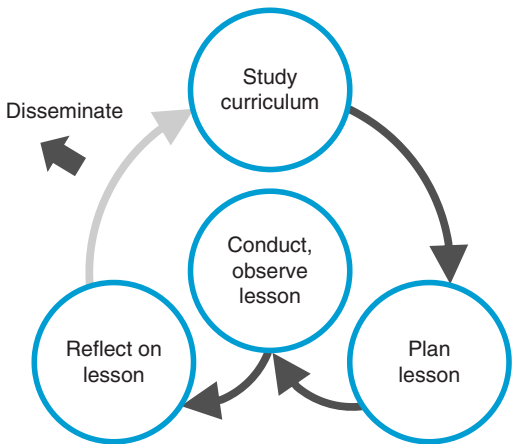
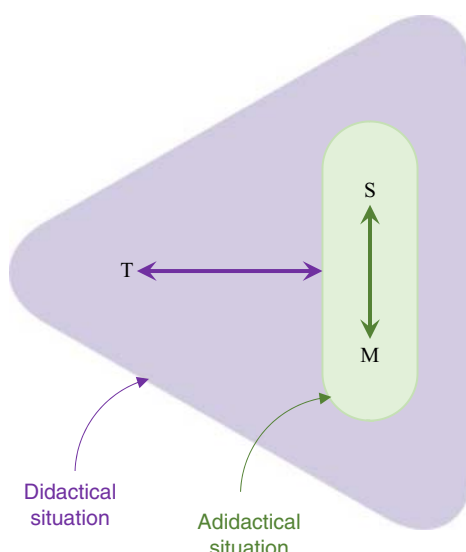


Figure 2.
Lesson study cycle



Notes: T is the teacher, S is the student and M is the milieu

Source: Based on Brousseau (1997, p. 56)

Figure 3.
The milieu of
the teacher

encompassing the adidactical situation, the teacher and the relations between these elements constitute the didactical situation. Brousseau distinguishes four phases of the didactical situation: action, formulation, validation and institutionalisation.

Students directly act (situation of action) on the milieu, articulate the ideas that have been developing implicitly (situation of formulation) and need to convince themselves or each other of the validity of the ideas they had developed (situation of validation). The final situation, institutionalisation, marks the return of the teacher in the didactical situation (Clivaz, 2015a, p. 250).

The didactical situation in TDS, both by its aims and structure, and by its phases share many common points with Japanese structured problem-solving lessons (Clivaz, 2015a). In the following, I will mostly consider a mathematics lesson study about a problem-solving research lesson. One reason to do so is that this type of research lesson is prevalent in mathematics lesson study, both in Japan (Fujii, 2017) and abroad. The other reason is that mathematics lesson study is quite prototypical of other lesson studies and I will advocate in the conclusion of this paper that the proposed graphical model of lesson study is valid for other subjects.

3. A graphical representation of lesson study as a didactical situation for didactical knowledge

3.1 Lesson Study as a learning situation for the teachers

The lesson is a situation built for the student to learn, which is to say it is a didactical situation for the student, but it is not for the teacher. There is no doubt that teachers learn things when teaching, but a lesson is not designed for them to learn. Moreover, the work of teaching is almost too complex and too stressful for the teacher to learn during the lesson. If we take into consideration the fact that the teacher is alone in the classroom and the number of lessons a teacher is teaching every week, the conditions are not favouring the learning for the teacher. Teacher training, initial and in-service, know that and they either happen independently of the classroom or they put different setting for the lesson: external

observer, extra time to plan or to analyse the lesson, focus on particular and problematic teaching and learning points. It is also the case for lesson study which combines all these characteristics. In this sense, the lesson study process is a didactical situation for the teacher (Clivaz, 2015b) and the lesson might be considered as a milieu for the teacher.

The TDS has not only been used for designing lessons in a lesson study process, but also to analyse lesson study. Miyakawa and Winsl w (2009) are pioneers in this direction and I followed their impetus (e.g. Clivaz, 2016, 2018; N  Shuilleabhain and Clivaz, 2017). Recently Winsl w *et al.* (2018, p. 130) have proposed a graphical representation of the teacher's milieu (Figure 4).

Figure 4 shows that the didactical situation, the lesson, becomes a milieu for the teachers' learning. In the following sections, I will explicate this representation to distinguish the phases of the lesson study process and to specify what are didactical and adidactical interaction. I will also clarify in which aspects the teacher is the same or not the same in the two occurrences in Figure 4 while describing in a finer grain lesson study as a professional didactical situation.

3.2 Lesson study as a didactical situation for didactique

The call for research on the teaching and learning of teaching and learning, more specifically for a didactique of *didactique des math matique* has been made by many researchers, implicitly or explicitly (Portugais, 1995). More generally, research on the teachers have been developed in the international community of mathematics education since 1980 (Da Ponte and Chapman, 2006) and also in the French *didactique des math matiques* since 1990 (Clivaz, 2014, pp. 41-44). Particularly notable in this line of research are the research about "didactical engineering for development and training" (Perrin-Glorian, 2011) or "cooperative engineering" (Sensevy *et al.*, 2013). This paper is also heading in the same direction where the hypothesis of lesson study being a didactical situation for teacher learning (Clivaz, 2015b) and graphical representation by Winsl w *et al.* (2018) is an occasion for me to propose a graphical model and further modelling of lesson study as a didactical situation for teachers.

When researchers propose graphical representation to represent the classroom situation and the teacher training situation, they often represent both situations with similar structure. That is the case of Portugais (1995, p. 282), of Wake *et al.* (2016, p. 248) or of the graphical representation by Winsl w, Bahn and Rasmussen in Figure 4. The question is then how to link the two systems of classroom situation and the teacher training situation. Wake *et al.* (2016, p. 248) propose a boundary space or object. Portugais (1995, p. 282) and Figure 4 propose to see the teacher, in her two roles, as a common point. The model proposed in this paper will take into account the different actions teachers have on the research lesson at different phases of lesson study.

When studying curriculum and planning the lesson, the teachers try to solve a professional problem. They have identified a pressing issue and they want to solve that problem. In order to do so, they study all possible resources and they design an experiment to test and if possible validate their solution to the problem. During these occasions, they are not trying to acquire new knowledge, even if they are aware they might acquire some. In this

Figure 4.
The learning situation
of lesson studies: the
interplay between the
teachers (T) and their
milieu (M), the
didactical situation



sense, these two situations can be considered as *adidactique* for them. More precisely both the curriculum studying phase and the lesson planning phase are influencing the research lesson. Obviously, the study of the curriculum is also influencing the planning of the lesson. These two situations are represented in Figure 5 where the research lesson is represented by the representation of the student-milieu-teacher didactical situation. In these two *adidactical* situation for the teachers, the milieu, what the teachers exercise their actions on, is the research lesson. It is a situation of action. So far, the milieu is not giving any response. When writing the research lesson plan, or what Takahashi and McDougal (2016) call a research lesson proposal, it is a situation of formulation.

The research lesson is an *adidactical* situation for the teachers in itself. The milieu of that situation for the teachers is the didactical situation of the students. The instructor teacher, as well as the students, the students' milieu and the lesson plan, are part of this milieu for the teachers who observe the lesson. These observer teachers are outside this milieu. For them, when they write their observations, it is a situation of formulation. In fact, the position of an observing teacher is quite complex since the lesson she is observing could have been taught by herself. In fact, she is observing her avatar (Cameron, 2009) teaching the lesson (Clivaz, 2016). She is outside the milieu for the teachers and her avatar is inside it. In this sense, the milieu is giving a retroaction.

The research lesson, both as it was and as it was observed, is the milieu of the post-lesson discussion. This time the milieu is giving objective responses. It is an *adidactical* situation of validation. During the discussion, the lesson planning and the curriculum studying phase might also influence the discussion.

During all the phases analysed above, the teachers' goal is not to acquire professional knowledge. As are students during a problem-solving lesson, they might be aware that the real goal of the problem-solving process is to acquire knowledge, but their action is not determined by that goal. It is only motivated by their interaction with the milieu for the teachers, the research lesson. And, if students produce knowledge that is not new (their teacher already knows this knowledge), teachers in lesson study produce new knowledge,

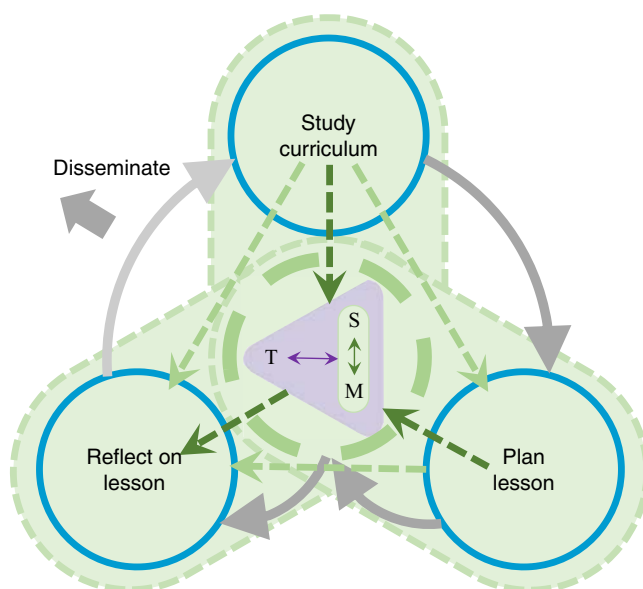


Figure 5.
Lesson study as an
adidactical situation

not only about teaching, but “knowledge in teaching” (Watson, 2008, p. 1). These situations are didactical for the teachers.

But, the whole lesson study process is happening in a formalized and quite strict setting. The succession of the phases, often secured by a facilitator, is the didactical situation encompassing the adidactical situation. This adidactical situation is constituted by the reunion of the four adidactical situations described above (Figure 6) with the milieu of the didactical situation being the research lesson.

Some parts of the lesson study process have a didactical aspect. The first aspect is the entry point of the process. The decision to participate in a lesson study and the discussions about the choice of the topic are didactical. Teachers participate in lesson study because they want to improve their professional knowledge. They choose a pressing issue because they want to learn about it. Another didactical part is the final comment by a knowledgeable other (Watanabe and Wang-Iverson, 2005) at the end of the post-lesson discussion. By “(1) bringing new knowledge from research and the curriculum; (2) showing the connection between the theory and the practice; and (3) helping others learn how to reflect on teaching and learning” (Takahashi, 2014, p. 13), the knowledgeable other makes the occasion of a decontextualisation and a depersonalisation of the knowledge acquired during the process. This happens in the same manner as, in the didactical situation for the students.

It is the Situation in which the teacher takes the ideas the class has developed, reviews them, shapes them up, [...] provides them with labels [and] provides the bridge between the class’s production and the concepts and terms accepted by the world at large and in particular the standard curriculum. (Warfield, 2014, p. 66)

This is a situation of institutionalisation. A third didactical moment is the discussion and the decision to study further the curriculum, devise a refined lesson and teach it in another class. In the debate between re-teaching (Yoshida, 2011) or not re-teaching (Fujii, 2014), our research in the Lausanne lesson study groups shows that the “refine or not refine” question was not about designing a better lesson, but it is about “will we learn more, as professionals,

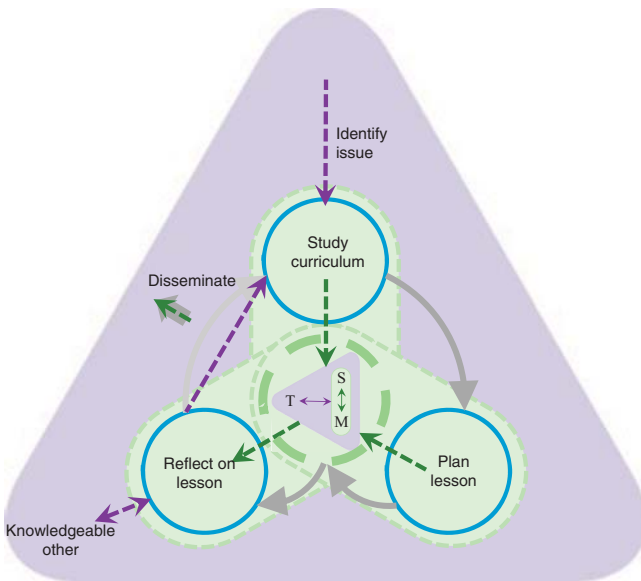


Figure 6.
Lesson study as a
didactical situation

in digging more this topic and redesigning a lesson to analyse what the changes will make for difference in student's learning". This kind of discussion is clearly about the professional learning and not just about solving the professional problem.

Last but not least, the dissemination of the process and the writing of a sharable lesson report, lesson plan or paper in a professional journal is adidactical. The teachers want to share the process, the result of the process, the knowledge they have acquired and produced. They do not do that to learn more. This phase is close to the summing up, the *matome* moment in a Japanese lesson. It is a situation of formulation for the teachers.

3.3 Didactical contract and didactical engineering

This paper exemplified the case of mathematics and problem-solving lesson, but I will advocate that similar process can be analysed for other subject matters.

In fact, even if the research lesson does not have a situation with a part of *adidacticity* in its centre, the lesson study process still has the essential adidactical features present. This is true for mathematics lesson as well as for other subjects. Nevertheless, the fact that the teachers can feel, and sometimes express that they are somehow in the same adidactical situation probably is important. As described by Clivaz (2016), in Lausanne groups, teachers have repeatedly made the comparison between their situation in lesson study process and the student's during a problem-solving lesson: the goal seems to be to prepare a lesson (for the teachers) or to solve the problem (for the students) but the real goal is to learn or produce professional knowledge (for the teacher) or mathematical knowledge (for the students). This didactical contract, this "set of (specific) behaviours of the teacher which are expected by the student and the set of behaviours of the student which are expected by the teacher" (Brousseau, Brousseau and Warfield, 2014, p. 204) is parallel in the case of a lesson study about a teaching situation with a part of adidacticity. For French *didactique des mathématiques*, analysing classroom situation in terms of didactical contract has been extremely fruitful to analyse the learning or the not learning of the students in a classroom situation. Many examples are given by Brousseau, Sarrazy and Novotná (2014). Analysing lesson study process in the term of didactical contract could bring interesting results as well in unfolding the links between lesson study structure, including the situations described above, and teachers' learning. The notion of didactical contract, in conjunction with the graphical description of lesson study as a didactical situation for teachers might also help to clarify the specificity of lesson study compared to other design who might seem similar such as cooperative engineering (Sensevy *et al.*, 2013) or other design-based research (Cobb *et al.*, 2003). Closer to French *didactique*, Miyakawa and Winsløw (2009) have made a detailed comparison of lesson study with didactical engineering methodology (Artigue, 2014). It would probably be interesting to contrast the didactical engineering especially designed for development and training (Perrin-Glorian, 2011) with lesson study. The comparison about the origin of the process, about the adidactical part of the mutual action on/from the research lesson on the process, about "the theoretical control of the construction and of the realisation of the lessons" (Perrin-Glorian, 2011, p. 70) and about the production of resources should be the subject of future research.

4. Conclusion: a fundamental situation of knowledge for teaching

This paper proposes a new graphical model of the lesson study process and of teacher learning in this process. However, it does not discuss on the type of knowledge at stake in the lesson study process. Several researches have shown that they are related to what can be referred to content knowledge and pedagogical content knowledge in the sense of Shulman (1986, 1987) or as *didactique* in a French speaking culture. But this paper is pushing forward a stronger hypothesis about lesson study in the framework of TDS.

This hypothesis is that lesson study is a good candidate for the fundamental situation of the knowledge for teaching.

In TDS, the fundamental hypothesis is that “every mathematical concept is the solution of at least one specific system of mathematical conditions, which itself can be interpreted by at least one situation” (Brousseau and Warfield, 2014, p. 165). In this sense, a mathematical knowledge can be modelled in a fundamental situation, or a family of situations, who will preserve and even give back the sense of this knowledge (Brousseau, 2003). The hypothesis here, is that lesson study can model the different knowledge for and in teaching and preserve and even give back the sense of this knowledge (Clivaz, 2015b). The didactical situations of action, of formulation and of validation as well as of institutionalisation, and moreover the interactions of these situations around the milieu of the research lesson described in this paper, are important elements in order to give the sense of professional knowledge.

The study of the use and development of the mathematical knowledge for teaching (Ball *et al.*, 2008) with the tools of the TDS have exemplified the case of mathematics (Clivaz, 2016; Ni Shuilleabhain and Clivaz, 2017). Categories of mathematical knowledge for teaching are a refinement for mathematics of some of Shulman’s (1987) categories of teacher knowledge: content knowledge, curriculum knowledge, and pedagogical content knowledge. Further inquiry should be made to review research in mathematics and in other subjects about the way lesson study interacts with these categories of knowledge, as well as with the more general categories: general pedagogical knowledge, with special reference to classroom management; knowledge of learners and their characteristics; knowledge of educational contexts; knowledge of educational ends, purposes, and values. This inquiry, as well as new fine-grained analysis of the use and development of knowledge for teaching in the lesson study didactic situation might help to better understand how teachers learn in lesson study, but also to better understand the knowledge for and in teaching itself.

Notes

1. Didactic has a pejorative sense of “overly inclined to lecture others”. According to Warfield, V. 2014. *Invitation to didactique*, New York, Springer. it is the reason why it has been difficult to promote its signification “as a noun denoting a field that studies questions raised by teaching and learning (of mathematical knowledge) in the milieu of school” *ibid.* Like Warfield, I shall use didactical as an adjective and the French noun *didactique* to avoid the pejorative connotation.
2. In this paper, as Brousseau (1997) did, the student takes the masculine form and the teacher takes the feminine form.

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