

Math Exam

$$u = x^2 \quad v = 2x+1$$

$$du = 2x \quad dv = 2$$

$$1. a) y = \frac{x^2}{2x+1}$$

$$u \frac{dv}{dx} - v \frac{du}{dx}$$

$$\frac{(2x+1)(2x) - (x^2)(2)}{(2x+1)^2}$$

$$\frac{4x^2 + 2x - 2x^2}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{2x^2 + 2x}{(2x+1)^2}$$

$$c) y = \sqrt{x^2+1}$$

$$y = (x^2+1)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} (x^2+1)^{-\frac{1}{2}} \times 2x$$

$$\frac{dy}{dx} = x(x^2+1)^{-\frac{1}{2}}$$

$$i.i) y = 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x$$

$$\frac{dy}{dx} = \frac{9}{2}x^{\frac{1}{2}} - 6x^{-3} - 6$$

$$= \frac{9}{2}x^{\frac{1}{2}} - \frac{6}{x^3} - 6$$

$$ii) y = \sec^3(4x)$$

$$\frac{dy}{dx} = 3 \sec^2(4x) \times \sec(4x) \tan(4x) \times 4$$

$$= 3 \sec^3(4x) \tan(4x) \times 4$$

$$= 12 \sec^3(4x) \tan(4x)$$

$$v) y = (x + \sin 2x)$$

$$\frac{dy}{dx} = 2 \cos 2x + 1$$

$$vi) \frac{1-10x}{(2x-1)^2}$$

$$u = 1-10x$$

$$v = (2x-1)^2$$

$$du = -10 \quad dv = 2(2x-1) \times 2$$

$$= 4(2x-1)$$

$$\frac{u dv - v du}{(2x-1)^4}$$

$$\frac{dy}{dx} = \frac{-10}{(2x-1)^2} - \frac{4(1-10x)}{(2x-1)^3}$$

$$b) y = \frac{1}{2x} + \cos 2x$$

$$\frac{dy}{dx} = -x^{-2} + -2 \sin 2x$$

$$\frac{d^2y}{dx^2} = 2x^{-3} - 4 \cos 2x$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 8 \sin 2x$$

$$\frac{d^4y}{dx^4} = 24x^{-5} + 16 \cos 2x$$

$$\frac{d^5y}{dx^5} = -120x^{-6} - 32 \sin 2x$$

$$\frac{d}{dx} (\cos x - \cot x)$$

$$= -\sin x - (-\csc^2 x)$$

$$= -\sin x + \csc^2 x$$

$$\frac{dy}{dx} = \frac{-10}{(2x-1)^2} - \frac{4(1-10x)}{(2x-1)^3}$$

$$b) y = \frac{1}{2x} + \cos 2x$$

$$\frac{dy}{dx} = -x^{-2} + -2 \sin 2x$$

$$\frac{d^2y}{dx^2} = 2x^{-3} - 4 \cos 2x$$

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$$\frac{d^4y}{dx^4} = 24x^{-5} + 16 \cos 2x$$

$$\frac{d^5y}{dx^5} = -120x^{-6} - 32 \sin 2x$$

$$\frac{d^6y}{dx^6} = 720x^{-7} - 64 \cos 2x$$

$$\frac{d^7y}{dx^7} = -5040x^{-8} + 128 \sin 2x$$

$$\frac{d^8y}{dx^8} = 40320x^{-9} - 256 \cos 2x$$

$$c) \text{ Let } y = \ln(\operatorname{cosec} x - \cot x)$$

$$\frac{dy}{dx} = \frac{1}{(\operatorname{cosec} x - \cot x)} \times (-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x)$$

$$= \frac{-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x}{\operatorname{cosec} x - \cot x}$$

$$2. a) i) \int \frac{2x}{(4+3x^2)} dx \quad u = 3x^2 + 4$$

$$\frac{du}{dx} = 6x$$

$$dx = \frac{du}{6x}$$

$$\frac{dx}{6x} = \frac{du}{6x}$$

$$\int \frac{2x}{u} \frac{du}{6x}$$

$$\frac{1}{3} \int \frac{1}{u} du$$

$$\frac{1}{3} (\ln u)$$

$$= \frac{\ln u}{3} + C \quad \ln(3x^2 + 4) + C$$

$$(i) \int_0^4 e^{\frac{x}{2}} dx$$

$$u = \frac{x}{2}$$

$$\frac{du}{dx} = \frac{1}{2}$$

$$dx = 2du$$

$$2 \int_0^4 e^u du$$

$$2e [2e^u]_0^4$$

$$[2e^{\frac{x}{2}}]_0^4$$

$$(2e^{\frac{4}{2}}) - (2e^{\frac{0}{2}})$$

$$14.78 - 2$$

$$\underline{12.78}$$

$$(ii) \int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$$

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$$b) \int \frac{4}{x(1+4\ln x)^2} dx$$

$$u = 1 + 4\ln x$$

$$\frac{du}{dx} = \frac{4}{x}$$

$$y = \int \frac{du}{u}$$

$$\int \frac{1}{u^2} du$$

$$\int u^{-2} du$$

$$= \frac{u^{-1}}{-1}$$

$$= -\frac{1}{u}$$

$$= -\frac{1}{1+4\ln x} + C$$

$$c) \int_2^{1.5} \frac{3e^{2x}}{x^2} dx$$

$$\int 3e^{2x} (e^x - 1)^{\frac{1}{2}} dx$$

$$u = e^x - 1$$

$$\frac{du}{dx} = e^x$$

$$dx = \frac{du}{e^x}$$

$$\int 3e^{2x} (e^x - 1)^{\frac{1}{2}} \frac{du}{e^x}$$

$$\int 3e^{2x} (e^x - 1)^{\frac{1}{2}} \frac{du}{e^x}$$

$$= \int 3e^{2x} (e^x - 1)^{\frac{1}{2}} \frac{du}{e^x}$$

$$c) \text{ Let } y = \ln(\operatorname{cosec} x - \cot x)$$

$$\frac{dy}{dx} = \frac{1}{(\operatorname{cosec} x - \cot x)} \times (-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x)$$

$$= \frac{-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x}{\operatorname{cosec} x - \cot x}$$

$$2. a) i) \int \frac{2x}{(4+3x^2)} dx \quad u = 3x^2 + 4$$

$$\frac{du}{dx} = 6x$$

$$dx = \frac{du}{6x}$$

$$\frac{dx}{6x} = \frac{du}{6x}$$

$$\int \frac{2x}{u} \frac{du}{6x}$$

$$\frac{1}{3} \int \frac{1}{u} du$$

$$\frac{1}{3} (\ln u)$$

$$= \frac{\ln u}{3} + C \quad \ln(3x^2 + 4) + C$$

$$c) \int \ln^5 \frac{3e^{2x}}{\sqrt{e^x - 1}} dx$$

$$u = e^x - 1$$

$$\frac{du}{dx} = e^x$$

$$3 \int \frac{e^{2x}}{\sqrt{e^x - 1}} dx$$

$$e^{2x} = (u+1)^2$$

$$3 \int (u+1)(u^{\frac{1}{2}}) du$$

$$2u^{\frac{3}{2}}$$

$$u = e^x - 1$$

$$du = e^x dx$$

$$3 \left(2u^{\frac{3}{2}}(u+1) - \int (2u^{\frac{1}{2}}) \right)$$

$$3 \left(2u^{\frac{3}{2}} + 2u^{\frac{1}{2}} - 4u^{\frac{3}{2}} \right)$$

$$3 \left(2u^{\frac{3}{2}} + 2u^{\frac{1}{2}} \right)$$

$$\left[6u^{\frac{3}{2}} + 6u^{\frac{1}{2}} \right]_{\ln^2}$$

$$\left(6(e^{\ln^2} - 1)^{\frac{3}{2}} + 6(e^{\ln^2} - 1)^{\frac{1}{2}} \right) - \left(6(e^{-1} - 1)^{\frac{3}{2}} + 6(e^{-1} - 1)^{\frac{1}{2}} \right)$$

$$(48 + 12) - (6 + 6)$$

$$60$$

$$- 12$$

$$(48)$$

$$(ii) \int_1^e \ln x \, dx$$

$$\int_1^e \ln x \, dx$$

$$u = \ln x \quad v = x$$

$$\frac{du}{dx} = \frac{1}{x} \quad dv = dx$$

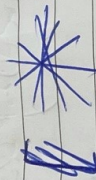
$$u = \ln x \quad v = x$$

$$\frac{du}{dx} = \frac{1}{x} \quad dv = dx$$

$$= x \ln x - \int \frac{x}{x} \, dx$$

$$= x \ln x - \int 1 \, dx$$

$$= x \ln x - x + C$$



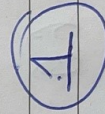
$$3. a) i) \left(\frac{1+i}{1-i} \right)^3$$

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$\left(\frac{1+2i+i^2}{1-i^2} \right)^3 = \left(\frac{1+2i-1}{1+1} \right)^3 = \left(\frac{2i}{2} \right)^3 = i^3$$

$$\int_0^1 [x \ln x - x] e^x \, dx = [e \ln e - e] - [1 \ln 1 - 1]$$

$$0 - -1$$



$$(i) \frac{3}{(1+i)(2+i)}$$

$$\frac{3}{2+3i+i^2} = \frac{3}{2+3i-1} = \frac{3}{1+3i}$$

$$(ii) \frac{1+i}{1-i} = \frac{(1+i)(2+2i) + 3-i}{1+i} = \frac{2+2i+2i+2i^2 + 3-i}{1+i} = \frac{2+2i+4i+4i^2 + 3-i}{1+i} = \frac{2+6i-4 + 3-i}{1+i} = \frac{-1-7i}{1+i}$$

$$(i) - \frac{(2+2i+4i+4i^2) + (1-2i)}{2+6i-4} = \frac{(2+2i+4i+4i^2) + (1-2i)}{2+6i-4}$$

$$i - \frac{(-2+6i) + (1-2i)}{1+i} = \frac{3-i}{1+i}$$

$$\frac{-1-7i}{1+i}$$

$$\frac{3-3i-i+i^2}{1-i^2}$$

$$(ii) \frac{2i(i-1) + (\sqrt{3}-i) + (1+i)(1+i)}{2i^2-2i + (\sqrt{3}-i) + (1+2i+i^2)}$$

$$\frac{-2-2i + (\sqrt{3}-i) + (1+2i+i^2)}{-2-2i + (\sqrt{3}-i) + (1+2i+i^2)}$$

$$\frac{-2-2i + (\sqrt{3}-i) + (1+2i+i^2)}{-2-2i + (\sqrt{3}-i) + (1+2i+i^2)}$$

$$\frac{-2+\sqrt{3}+i}{-2+\sqrt{3}+i}$$

$$\frac{-2+\sqrt{3}+i}{-2+\sqrt{3}+i}$$

$$b) \quad Z = (4 + 7i)(3 - 2i)$$

$$12 - 8i + 21i - 14i^2$$

$$12 + 13i + 14$$

$$26 + 13i$$

$$R = \sqrt{26^2 + 13^2}$$

$$R = 29.07$$

$$\tan^{-1}\left(\frac{13}{26}\right) = 26.57^\circ$$

$$\therefore Z = 29.07 \angle 26.57^\circ$$

$$c) \quad \frac{1+5i}{1+i}$$

$$\frac{1+3i}{1+i} \times \frac{1-i}{1-i} = \frac{1-i+3i-3i^2}{1-i^2}$$

$$\frac{1+2i+3}{1+i} = \frac{4+2i}{2} = \frac{2+i}{1}$$