

Elastic Failure and Plastic Deformation

1 Modes of Failure

There are many ways in which components can fail, making the designer's task a complex one. This complexity is illustrated by the following list of potential failure modes:

- 1 Mechanical overload/under-design
- 2 Elastic yielding - force and/or temperature induced
- 3 Fatigue
 - High cycle
 - Low cycle
 - Thermal
 - Corrosion
 - Impact
 - Surface
- 4 Brittle fracture
- 5 Creep
- 6 Combined creep and fatigue
- 7 Ductile rupture
- 8 Corrosion
 - Direct chemical
 - Galvanic
 - Pitting
 - Cavitation
 - Stress
 - Intergranular
 - Crevice
 - Selective leaching
 - Biological
 - Corrosion fatigue
- 9 Impact
 - Fracture
 - Fatigue
 - Deformation
 - Wear
- 10 Instability
 - Buckling
 - Creep buckling
 - Torsional instability
- 11 Wear
 - Adhesive
 - Abrasive
 - Corrosive
 - Impact
 - Deformation
 - Surface fatigue

- 12 Vibration
- 13 Environmental
 - Thermal shock
 - Radiation damage
 - Lubrication failure
- 14 Contact
 - Spalling
 - Pitting
 - Seizure
- 15 Stress rupture
- 16 Thermal rupture

2 Plastic Deformation in Metals

By definition, all flow processes involve a permanent change of shape. These shape changes are achieved by parts of a material sliding over one another. This is how glaciers and treacle flow and is what happened in the examples shown in Figure 1 below. Sliding is produced by shear forces acting on the plane on which the sliding occurs. The dark areas in the workpiece of Figure 1(a) have been etched relative to the white areas. These areas have been sheared into flow lines in the chip. The shape change becomes clear when you compare the shaded areas in Figure 1(b) before and after shearing. The predominant mechanism of plastic flow in crystalline materials, such as metals, is produced by the structural defect known as the **dislocation**.

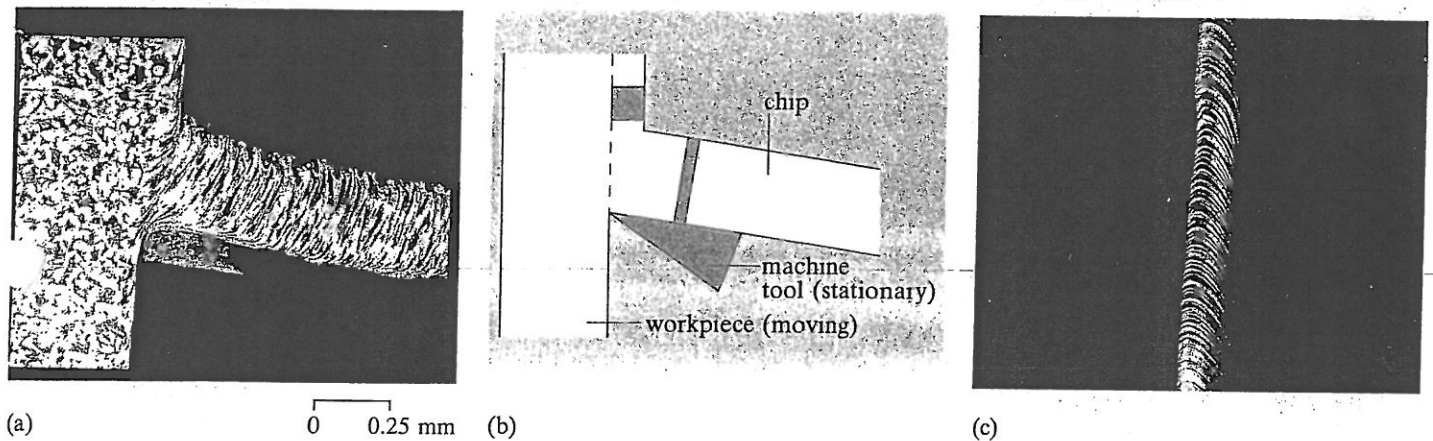


Figure 1: (a) Chip being machined from etched mild steel. (b) Geometry of the machining operation. (c) Single crystal of copper-aluminium alloy after being pulled along its axis.

2.1 Slip in Crystals

In Figure 1(c) different parts of the crystal have sheared over one another. It looks rather like a pack of playing cards that have been pushed sideways. This type of deformation is called **slip** and Figure 2 is a schematic representation of an incremental slip in a simple crystal structure, in which it is assumed that atoms are hard spheres packed together in a cubic array. Imagine that this is a minute volume of atoms plucked out of Figure 1(c) with the slip plane being the plane on which the single crystal has sheared.

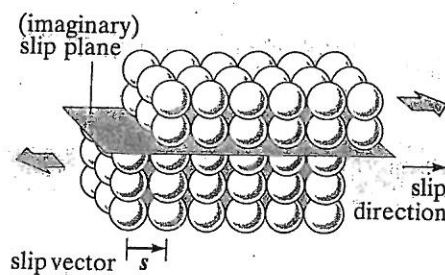


Figure 2: Slip in simple crystal structure

Now slip has a number of specific features, which can be defined with reference to Figure 2.

Slip planes - the (imaginary) plane between the planes of atoms on which slip occurs.

Slip step - the step produced on a crystal surface. They can be seen in Figure 1(c).

Slip direction - the direction in which slip occurs.

Slip vector - the direction and magnitude of an increment of slip.

Slip system - the combination of a slip plane and a slip direction.

So slip in crystals is, in fact, very different from the sliding of cards in a pack because the direction and amount of slip is specific. In some types of crystal it is possible for planes of atoms to be sheared over one another by distances less than that between atoms. This leads to a local change in the crystal structure. The process is called a **shear transformation**.

A slip system defines the geometry of slip. For instance, the crystal portrayed in Figure 2 has undergone slip on a particular plane and in a particular direction in that plane. This plane and direction constitute a slip system. Now a crystal does not slip in

any arbitrary plane or direction. It is very selective. The slip direction is always that in which atoms are closest together. This means that the slip vector is the smallest possible. So, in Figure 3(a), which shows the front face of a simple cubic structure, slip vector *a* is chosen rather than, say, *b*. The favoured slip plane is usually that on which the atoms are most closely packed. This is because the most closely packed planes are wider apart than others. In simple terms, on an atomic scale they are smoother and can slide over one another more easily- Figure 3(b).

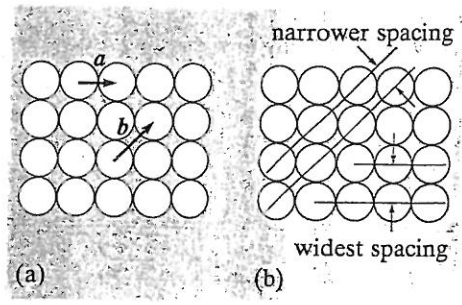


Figure 3

Since many crystals are highly symmetrical, they possess a number of equivalent slip planes and directions, and therefore slip systems. Under the appropriate shear stress, the crystal in Figure 2 could slip on the same plane but in a direction at 90° to the one shown. The crystal also has identical planes of atoms lying vertically and mutually perpendicular to one another (think of the other faces of a cube of atoms). So, how many different slip systems has this structure?

The answer is six. There are three equivalent sets of slip planes, each of which contains two slip directions.

Slip in polycrystalline materials is more complicated than in single crystals because the individual grains cannot behave as if they were single crystals. We know that polycrystalline metals can be deformed by large amounts - bars can be drawn, into wire, sheets pressed into car body panels, for example. This does not cause the metal to fall apart at the grain boundaries, so each grain must deform "in sympathy" with its neighbours, that is; in such a way that it remains in contact with them at its boundary. Each grain must slip on a number of slip systems.

On the assumption that in a perfect crystal slip occurs in a single movement by one whole plane of atoms sliding bodily over another, it is possible to calculate the theoretical shear stress (and hence yield stress) at which this should happen. The answer turns out to be of the order of 10^{10} Nm^{-2} ; this compares with measured yield stress of unalloyed metals of 10^6 Nm^{-2} to 10^7 Nm^{-2} . Clearly a different theoretical model is needed.

2.2 *The dislocation model of slip*

In real crystals, planes of atoms do not slip bodily over one another. Instead slip is an incremental process. A few neighbouring atoms in a plane suddenly undergo slip by jumping simultaneously into their next steps in the slip direction - pushing the atoms before them out of their normal positions. At this stage, slip has occurred over a small area of a plane relative to the neighbouring plane. This slipped area then grows by the atoms on its outside - those that were pushed out of position - undergoing slip, pushing atoms ahead of them out of position and so on. Eventually, all of the atoms in the plane will have slipped and only then will it look as if the whole plane had slipped bodily.

The simple analogy of trying to move a large heavy carpet may help here. Rather than trying to slide the carpet by pulling on one edge (that is, sliding a whole plane of atoms simultaneously), it is easier to put a ruck in the carpet and push the ruck across the carpet (Figure 4). Large movement can be achieved by repeating the process.

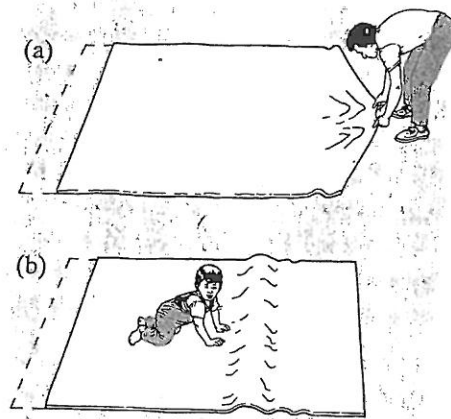


Figure 4: (a) The hard way. (b) The easy way.

Now picture, schematically, what happens. Imagine a block of crystals under a shear stress τ and containing a plane ABCD on which slip occurs by an increment described by the slip vector s . Figure 5(a) and (b) represent the position before and after slip occurs. Two sorts of model will be used to represent material undergoing slip: the "block" representation of Figure 5(a) and (b), which is trying to give an overall picture; and a model which represents the centres of atoms by small circles and in which, to illustrate the regularity of the structure, straight lines connect atoms that are near neighbours. Figure 5(c) is the atomic model of the slipped crystal of Figure 5(b). This way of representing atoms is used, rather than that of Figure 2, because it is easier to visualize changes.

Suppose that slip starts at a stress concentration at the point D and begins to spread across the plane ABCD; that is, that part of the top half of the block slips by an amount s relative to the bottom half. Figure 5(a) shows the situation when slip has occurred over the area DEF and hasn't yet occurred over the area FEABC.

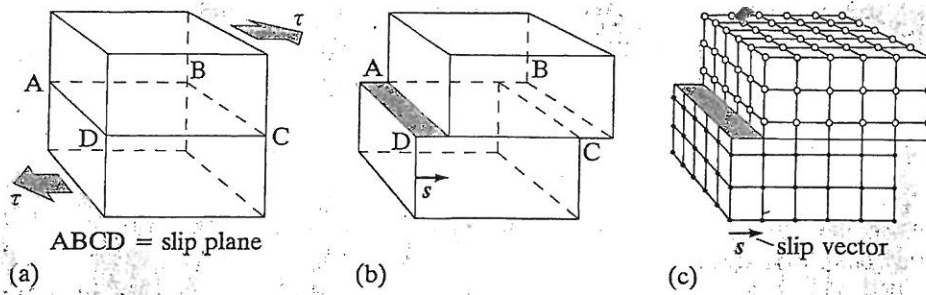


Figure 5: (a) Block model. (b) Atomic model.

The line in Figure 6(a) that marks the boundary between the slipped and unslipped areas is called a **dislocation line** (usually simply **dislocation**). In Figure 6(a) it is the line EF. As the slipped area grows, the dislocation line moves across the plane.

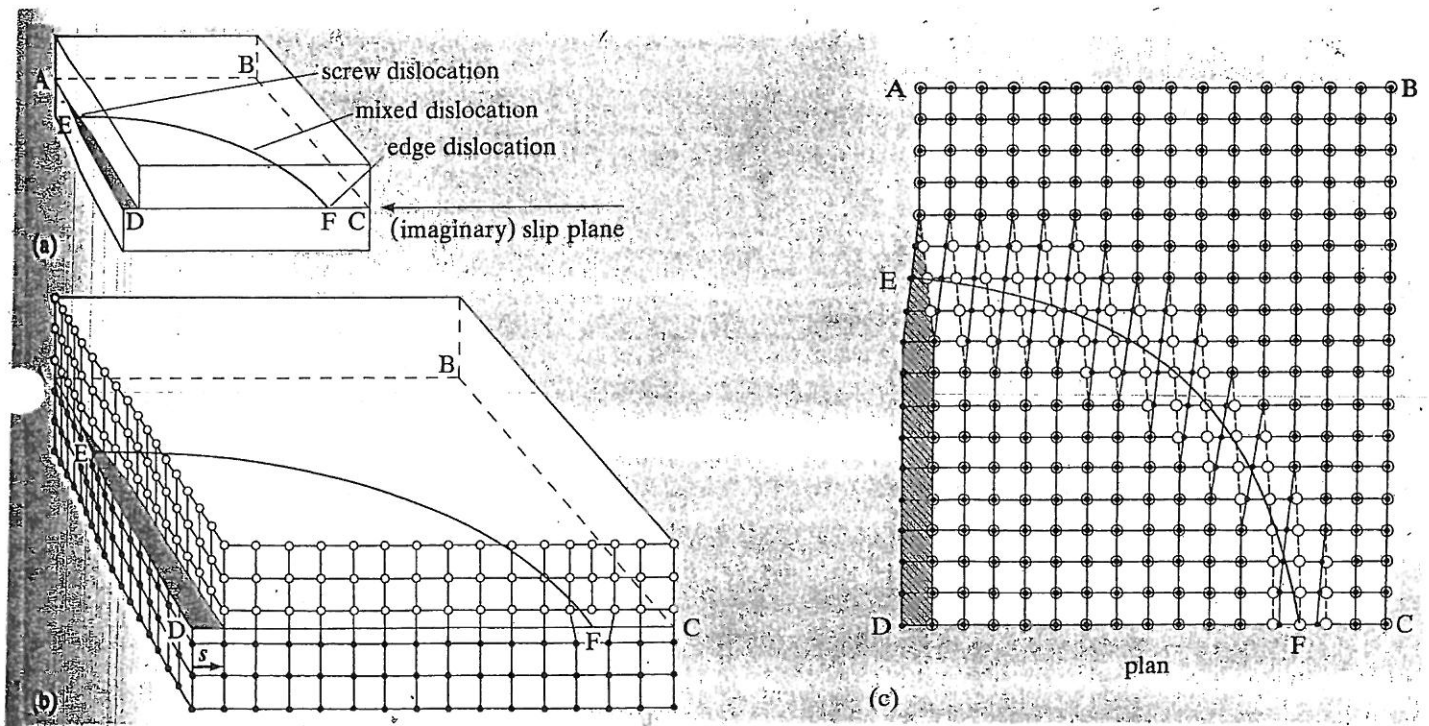


Figure 6: (a) Slip part way across the plane ABCD in block model. (b) and (c) show atomic model. In (c) atoms below the slip plane are represented by a dot. Those above are represented by a circle. A circle with a dot indicates atoms in register.

Now since nothing has yet happened on the far side of the crystal, the line EF must be a region of some disorder. And so it is, which is why it is called a dislocation. The plan view of the atomic model, Figure 6(c), shows clearly the slipped area, the unslipped area and the transition range in between. The disorder produced in the crystal depends on the angle the dislocation line makes with the slip vector s . Two extremes are possible:

- The dislocation line lies at 90° to the vector. Such a segment is called an **edge dislocation** and is the one that emerges at point F in Figure 6(a). The atomic disorder around it is illustrated in Figure 6(b).
- The dislocation line lies parallel to the vector. Such a segment is called a **screw dislocation** and emerges at point E in Figure 6(a).

Of course, most of the dislocation line EF is in some orientation other than edge and screw. Such orientations are called **mixed dislocation** because they can be considered as a mixture of small segments of edge and screw dislocation. As the dislocation EF continues to move across the slip plane, it will change its configuration. Figure 7 shows a later stage in its movement. It has moved nearly across the whole of the slip plane.

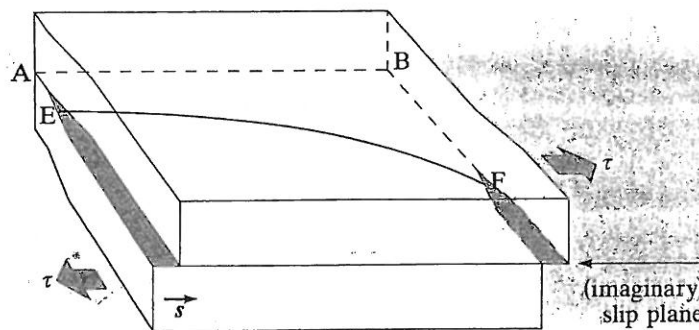


Figure 7.

Is the dislocation predominantly edge, screw or mixed?

It is mainly screw dislocation, that is with the dislocation line parallel to the slip vector. When the dislocation passes through points A and B, the block will have slipped to that shown in Figure 5(b).

Dislocations can be seen by **transmission electron microscopy**, and Figure 8 shows an example. Figure 8(a) shows dislocations moving on slip planes in stainless steel. The dislocations are the wiggly black lines. Figure 8(b) shows why they appear as short lines. Do not be misled by the illusion that the dislocations appear to end inside the material. The appearance of Figure 8(a) arises because the electron beam of the

microscope is fired through a very thin slice of the metal about 100 nm thick. Figure 8(b) illustrates, very simply, the geometry of the situation.

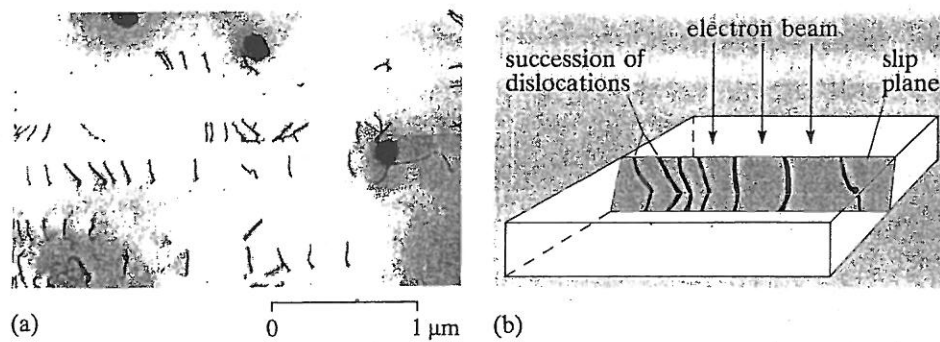


Figure 8: (a) Dislocations in stainless steel. (b) Orientation of slip plane in (a).

It is important to appreciate that, since a dislocation bounds a slipped area, it must be continuous within a crystal. Dislocations can exist as loops or join up with one another. The electron micrograph of Figure 9 shows these features in a plastically deformed specimen of copper containing small particles of silica. (The dislocations are lying on a plane at a different angle from the one in Figure 8(a)). Of course, dislocations can terminate at a surface.

Whether a dislocation is mainly edge or screw in character is important because the two types behave very differently.

3 Theories of Elastic Failure

When dealing with the design of structures or components the physical properties of the constituent materials are usually found from the results of laboratory experiments, which have only subjected the materials to the simplest stress conditions. The most usual test is the uniaxial tensile test in which the value of the stress at yield or at fracture (whichever occurs first) is easily determined. The strengths of materials under complex stress systems are not generally known except in a few particular cases. In practice it is these complicated systems of stress which are more often encountered, and therefore it is necessary to have some basis for determining allowable working stresses so that failure will not occur. Thus the function of the theories of elastic failure is to predict from the behaviour of materials in a simple tensile test when elastic failure will occur under *any* condition of applied stress.

A number of theoretical criteria have been proposed each seeking to obtain adequate correlation between estimated component life and that actually achieved under service load conditions for both brittle and ductile material applications. The five main theories are:

- (a) Maximum principal stress theory (Rankine)
- (b) Maximum shear stress theory (Guest-Tresca)
- (c) Maximum principal strain (Saint-Venant)
- (d) Total strain energy per unit volume (Haigh)
- (e) Shear strain energy per unit volume (Maxwell-Huber-von Mises)

In each case the value of the selected critical property implied in the title of the theory is determined for both the simple tensile test and a three-dimensional complex stress

In Table 1 σ_y is the stress at the yield point in the simple tension test, and σ_1 , σ_2 and σ_3 are the three principal stresses in the three-dimensional complex stress system in order of magnitude. Thus in the case of the maximum shear stress theory $\sigma_1 - \sigma_3$ is the greatest numerical difference between two principal stresses taking into account signs and the fact that one principal stress may be zero.

The Mohr's principal stress theory attempts to take into account the additional strength of brittle materials in compression.

3.1 Maximum principal stress theory

This theory assumes that when the maximum principal stress in the complex stress system reaches the elastic limit stress in simple tension failure occurs. The criterion of failure is thus:

$$\sigma_1 = \sigma_y$$

It should be noted, however, that failure could also occur in compression if the least principal stress σ_3 were compressive and its value reached the value of the yield stress in compression for the material concerned before the value of σ_y was reached in tension. An additional criterion is therefore

$$\sigma_3 = -\sigma_y \quad (\text{compressive})$$

Whilst the theory can be shown to hold fairly well for brittle materials, there is considerable experimental evidence that the theory should not be applied for ductile materials. For example, even in the case of the pure tension test itself, failure for ductile materials takes place not because of the direct stresses applied but in shear on planes at 45° to the specimen axis. Also, truly homogeneous materials can withstand very high hydrostatic pressures without failing, thus indicating that maximum direct stresses alone do not constitute a valid failure criterion for all loading conditions.

3.2 Maximum shear stress theory

This theory states that failure can be assumed to occur when the maximum shear stress in the complex stress system becomes equal to that at the yield point in the simple tensile test.

Since the maximum shear stress is half the greatest difference between two principal stresses the criterion of failure becomes

$$1/2(\sigma_1 - \sigma_3) = 1/2(\sigma_y - 0)$$

$$\sigma_1 - \sigma_3 = \sigma_y$$

the value of σ_3 being algebraically the smallest value, i.e. taking account of sign and the fact that one stress may be zero. This produces fairly accurate correlation with experimental results, particularly for ductile materials, and is often used for ductile materials in machine design. The criterion is often referred to as the **Tresca** theory and is one of the widely used laws of plasticity.

3.3 Maximum shear strain theory

This theory assumes that failure occurs when the maximum strain in the complex stress system equals that at the yield point in the tensile test,

$$\frac{\sigma_1}{E} - \nu \frac{\sigma_2}{E} - \nu \frac{\sigma_3}{E} = \frac{\sigma_y}{E}$$

$$\sigma_1 - \nu \sigma_2 - \nu \sigma_3 = \sigma_y$$

This theory is contradicted by the results obtained from tests on flat plates subjected to two mutually perpendicular tensions. The Poisson's ratio effect of each tension reduces the strain in the perpendicular direction so that, according to this theory, failure should occur at a higher load. This is not always the case. The theory holds reasonably well for cast iron but is not generally used in design procedures these days.

3.4 Maximum total strain energy per unit volume theory

The theory assumes that failure occurs when the total strain energy in the complex stress system is equal to that at the yield point in the tensile test. The criterion of failure is thus

$$\frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] = \frac{\sigma_y^2}{2E}$$

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \sigma_y^2$$

The theory gives fairly good results for ductile materials but is seldom used in preference to the theory below.

3.5 Maximum shear strain energy per unit volume (or distortion energy) theory

The strain energy of a stressed component can be divided into volumetric strain energy and shear strain energy components, the former being associated with volume change and no distortion, the latter producing distortion of the stressed elements. This theory states that failure occurs when the maximum shear strain energy component in the complex stress system is equal to that at the yield point in the tensile test,

$$\frac{1}{12G}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2] = \frac{\sigma_y^2}{6G}$$

$$\frac{1}{6G}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] = \frac{\sigma_y^2}{6G}$$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$$

This theory has received considerable verification in practice and is widely regarded as the most reliable basis for design, particularly when dealing with ductile materials. It is often referred to as the **von Mises** or **Maxwell** criterion and is probably the best theory of the five. It is also sometimes referred to as the **distortion energy** or **maximum octahedral shear stress theory**.

In the above theories it has been assumed that the properties of the material in tension and compression are similar. It is well known, however, that certain materials, notably concrete, cast irons, soils etc., exhibit vastly different properties depending on the nature of the applied stress. For brittle materials this has been explained by Griffith who introduced the principle of surface energy at microscopic cracks and shown that an existing crack will propagate rapidly if the available elastic strain energy release is greater than the surface energy of the crack. In this way Griffith indicates the greater seriousness of tensile stresses compared with compressive ones with respect to failure, particularly in fatigue environments. A further theory has been introduced by Mohr to predict failure of materials the strength of which are considerably different in tension and shear.

3.5.1 Brittle materials

Failure of brittle materials has been shown to be governed by the maximum principal tensile stress present in the three-dimensional stress system. This is thought to be due to the microscopic cracks, flaws or discontinuities which are present in most brittle materials and which act as local stress raisers. These stress raisers, or stress concentrations, have a much greater adverse effect in tension and hence produce the characteristic weaker behaviour of brittle materials in tension than in compression. Thus if the greatest tensile principal stress exceeds the yield stress, then failure occurs.

Theory	Value in tension test at failure	Value in complex stress system	Criterion for failure
Maximum principal stress (<i>Rankine</i>)	σ_y	σ_1	$\sigma_1 = \sigma_y$
Maximum shear stress (<i>Guest-Tresca</i>)	$\frac{1}{2}\sigma_y$	$\frac{1}{2}(\sigma_1 - \sigma_3)$	$\sigma_1 - \sigma_3 = \sigma_y$
Maximum principal strain (<i>Saint-Venant</i>)	$\frac{\sigma_y}{E}$	$\frac{\sigma_1 - \nu \sigma_2 - \nu \sigma_3}{E}$	$\sigma_1 - \nu \sigma_2 - \nu \sigma_3 = \sigma_y$
Total strain energy per unit volume (<i>Haigh</i>)	$\frac{\sigma_y^2}{2E}$	$\frac{1}{2}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$	$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \sigma_y^2$
Shear strain energy per unit volume Distortion energy theory (<i>Maxwell-Huber-von Mises</i>)	$\frac{\sigma_y^2}{6G}$	$\frac{1}{12G}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$	$\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = \sigma_y^2$
Modified shear Stress/Internal friction theory (<i>Mohr</i>)			$\frac{\sigma_2 + \sigma_3}{\sigma_{yt} \sigma_{yc}} = 1$

Table 1: Elastic failure theories - summary of failure criteria

3.7 *Limitations of failure theories*

It is important to remember that the theories introduced above are those of *elastic* failure, i.e. they relate to the "failure" which is assumed to occur under elastic loading conditions at an equivalent stage to that of yielding in a simple tensile test. If it is anticipated that loading conditions are such that the component may fail in service in a way which cannot easily be related to standard simple loading tests (e.g. under fatigue, creep, buckling, impact loading, etc.) then the above "classical" elastic failure theories should not be applied. A good example of this is the brittle fracture of steel under low temperature or very high strain rate (impact) conditions compared with simple ductile failure under normal ambient conditions. If any doubt exists about the relevance of the failure theories then, ideally, specially designed tests should be carried out on the component with loading conditions as near as possible to those expected in service. If, however, elastic failure can be assumed to be relevant it is necessary to consider which of the theories is the most appropriate for the material in question and for the service loading condition expected.

In most cases the Von Mises "distortion energy" theory is considered to be the most reliable and relevant theory with the following exceptions:

- (a) For brittle materials the maximum principal stress or Mohr "internal friction" theories are most suitable. (It must be noted, however, that the former is definitely unsafe for ductile materials). Some authorities also recommend the Mohr theory for extension of the theories to ductile *fracture* consideration as opposed to ductile yielding as assumed in the elastic theories.
- (b) All theories produce similar results in loading situations where one principal stress is large compared to another.
- (c) For bi-axial stress conditions, the Mohr modified theory is often preferred, provided that reliable test data are available for tension, compression and torsion.
- (d) In most general bi-axial and tri-axial stress conditions the Tresca maximum shear stress theory is the most conservative (i.e. the safest) theory and this, together with its easily applied and simple formula, probably explains its widespread use in industry.
- (e) The St. Venant maximum principal strain and Haigh total strain energy per unit volume theories are now rarely, if ever, used in general engineering practice.

3.8 *Effect of stress concentrations*

Whilst stress concentrations have their most significant effect under fatigue loading conditions and impact situations, nevertheless, there are also some important considerations for static loading applications, namely:

- (a) In the presence of ductile yielding, stress concentrations are relatively unimportant since the yielding which will occur at the concentrations, e.g. the tip of a notch, will merely redistribute the stresses and not necessarily lead to failure. If however, there is only marginal ductility, or in the presence of low

temperatures, then stress concentrations become more significant as the likelihood of brittle failure increases. It is wise, therefore, to keep stress concentration factors as low as possible.

- (b) For brittle materials like cast iron, internal stress concentrations arise within the material due to the presence of, for example, flaws, impurities or graphite flakes. These produce stress increases at least as large as those given by surface stress concentrations which, therefore, may have little or no effect on failure. A cast iron bar with a small transverse hole, for example, may not fracture at the hole when a tensile load is applied.

3.9 *Safety Factors*

When using elastic design procedures incorporating any of the failure theories it is normal to incorporate safety factors to take account of various imponderables which arise when one attempts to forecast accurately service loads or operating conditions or to make allowance for variations in material properties or behaviour from those assumed by the acceptance of "standard" values. "Ideal" application of the theories, i.e. a rigorous mathematical analysis, is thus rarely possible and the following factors indicate in a little more detail the likely sources of inaccuracy:

- 1 Whilst design may have been based upon nominally static loading, changing service conditions or misuse by operators can often lead to dynamic, fluctuating or impact loading situations which will produce significant increases in maximum stress levels.
- 2 A precise knowledge of the mechanical properties of the material used in the design is seldom available. Standard elastic values found in reference texts assume ideal homogeneous and isotropic materials with equal "strengths" in all directions. This rarely true in practice and the effect of internal flaws, inclusions or other weaknesses in the material may be quite significant.
- 3 The method of manufacture or construction of the component can have a significant effect on service life, particularly if residual stresses are introduced by, for example, welding or straining beyond the elastic limit during the assembly stages.
- 4 Complex designs often give rise to difficult analysis problems which even after time-consuming and expensive theoretical procedures, at best yield only a reasonable estimate of maximum service stresses.

Despite these problems and the assumptions which are often required to overcome them, it has been shown that elastic design procedures can be made to agree with experimental results within a reasonable margin of error provided that appropriate safety factors are applied.

It has been shown that alternative definitions are used for the safety factor depending upon whether it is based on the tensile strength of the material used or its yield strength, i.e. either

$$\text{Safety factor } n = \frac{\text{tensile strength}}{\text{allowable working stress}}$$

$$\text{Safety factor } n = \frac{\text{yield strength (or proof stress)}}{\text{allowable working stress}}$$

Clearly, it is important when quoting safety factors to state which definition has been used.

Safety values vary depending on the type of industry and the area of application of the component being designed. National codes of practice (e.g. British Standards) or other external authority regulations often quote mandatory values to be applied and some companies produce their own guideline values.

Table 2 shows the way in which the various factors outlined above contribute to the overall factor of safety for some typical service conditions. These values are based on the yield stress of the materials concerned.

Application	Nature of stress (a)	Nature of load (b)	Type of service (c)	Overall safety factor (a) x (b) x (c)
Steelwork in buildings	1	1	2	2
Pressure vessels	1	1	3	3
Transmission shafts	3	1	2	6
Connecting rods	3	2	1.5	9

Table 2: Typical safety factors

It should be noted, however, that the values given in the "type of service" column can be considered to be conservative and severe misuse or overload could increase these (and, hence, the overall factors) by as much as five times.

Legislative issues such as *Product Liability* and *Health and Safety at Work* will undoubtedly be a consideration when determining appropriate safety factors, and may lead to the adoption of higher values. Since this could well result in uneconomic utilisation of materials, such a trend would be regrettable. However, the advent of computational techniques such as *Finite Element Analysis* means that this trend is, to an extent, counteracted by the ability to use the software to identify those critical areas of a design which need to be strengthened and those less critical areas where material can be saved. Designs can thus be optimised. It must be noted, however, that in most instances computational models are still validated with a certain amount of experimental testing in order to give confidence in the results. It is anticipated that the amount of testing required will diminish in time, as software techniques improve.

Theories of Elastic Failure

Examples

Example 1

A material subjected to a simple tension test shows an elastic limit of 240 MN/m^2 . Calculate the factor of safety provided if the principal stresses set up in a complex two-dimensional stress system are limited to 140 MN/m^2 tensile and 45 MN/m^2 compressive. The appropriate theories of failure on which your answer should be based are:

- (a) the maximum shear stress theory;
- (b) the maximum shear strain energy theory.

Example 2

A steel tube has a mean diameter of 100 mm and a thickness of 3 mm. Calculate the torque which can be transmitted by the tube with a factor of safety of 2.25 if the criterion of failure is (a) maximum shear stress; (b) maximum strain energy; (c) maximum shear strain energy. The elastic limit of the steel in tension is 225 MN/m^2 and Poisson's ratio ν is 0.3.

Example 3

A mild steel shaft of 50 mm diameter is subjected to a bending moment of 1.9 kNm. If the yield point of the steel in simple tension is 200 MNm^{-2} , find the maximum torque that can also be applied according to: (a) the maximum shear stress; (b) the shear strain energy theories of yielding.

(Answer: (a) 1.55 kNm (b) 1.79 kNm)

Example 4

A thin walled steel cylinder of 2 m diameter is subjected to an internal pressure of 2.5 MNm^{-2} . Using a safety factor of 2 and a yield stress in simple tension of 400 MNm^{-2} , calculate the wall thickness on the basis of the Tresca and Von Mises yield criteria. It may be assumed that the radial stress in the wall is negligible.

(Answer: (a) 12.5 mm (b) 10.8 mm)