

ELECTRICAL ENGINEERING

Assignment 1 – April 2023

Tomos Hall
2219188@student.uwtsd.ac.uk

Abstract

This report details the influence of capacitance in an RL circuit and the results of modifying frequency. A current lagging inductive circuit was analysed to determine the phase difference. Active, reactive, and apparent power were calculated resulting in a power factor computation.

A desired power factor of 0.95 was used to calculate the resulting phase angle. A capacitor value was selected to achieve the required phase difference. The consequences of a higher power factor were explored.

It was found that a capacitor value $8.67\mu\text{F}$ was required for a power factor of 0.95. Phasor diagrams were produced illustrating all currents, voltages, and phase angles at various frequencies. Resonant frequency was concluded to be 99.596 Hz, at this frequency the load current was in phase with the supply voltage.

Introduction

In industry, many processes are carried out to ensure machinery, or manufacturing processes run as efficiently as possible. It's the task of an electrical engineer to accurately calculate how an electrical process can be made more efficient. Decreasing energy consumption of an industrial operation can save companies on energy costs and prolong the lifespan of electrical equipment [1].

An inductive load, such as an AC induction motor, causes the supply current and voltage to become out of phase. The current lags the voltage, which lowers the power factor of the circuit. A power factor of one would be considered a purely resistive load where the circuit is neither capacitive nor inductive, producing unity, drawing an in-phase supply [2]. This is the most efficient operation.

Power factor is the ratio between real power and apparent power. Apparent power is the total power provided by the energy company and real power, or active power, is the power utilised by the load [1] [3]. The difference between these values is wasted energy, known as reactive power. This means that for the load to achieve its required power level, a larger power supply is required due to the lost reactive power [1].

Power factor correction is a process used to achieve a power factor closer to unity, meaning less apparent power is required to deliver the necessary active power to the load. In an inductive scenario, such as an induction motor, capacitor banks are connected in parallel to the load to help limit the phase shift between supply voltage and current [1]. The capacitors generate reactive current, supplying reactive power to the circuit and limiting the reactive power required from the supply. This means the supplied apparent power can be lower, saving costs and energy, providing a more efficient procedure [2]. The capacitor creates a leading current which counteracts the lagging current created by the induction motor's magnetic field current. This brings the current and voltage closer to phase [1].

This project explores a simplified version of this industrial problem, creating the same scenario in the form of an RLC circuit. An RLC circuit is made up of a resistor, an inductor, and a capacitor. The resistor represents the motor windings, the inductor represents the inductive load of a motor, and the capacitor represents the power factor correction capacitor bank.

In this report, an RLC circuit will be simulated using proteus and values measured for calculations to be carried out. The apparent, active, and reactive power will be calculated as well as the power factor of the circuit. This information will then be utilised to calculate the required capacitor value to achieve a power factor of 0.95, completing power factor correction. Capacitor values will be graphically analysed against power factor and phase angles, determining the resulting correlation. Phasor diagrams will be produced for the circuit at different frequencies representing the relationship between component's currents and voltages.

Method

An RL circuit was simulated in proteus, representing an induction motor connected to an industrial supply. In figure one, supply voltage settings and component values can be observed as set in the assignment brief.

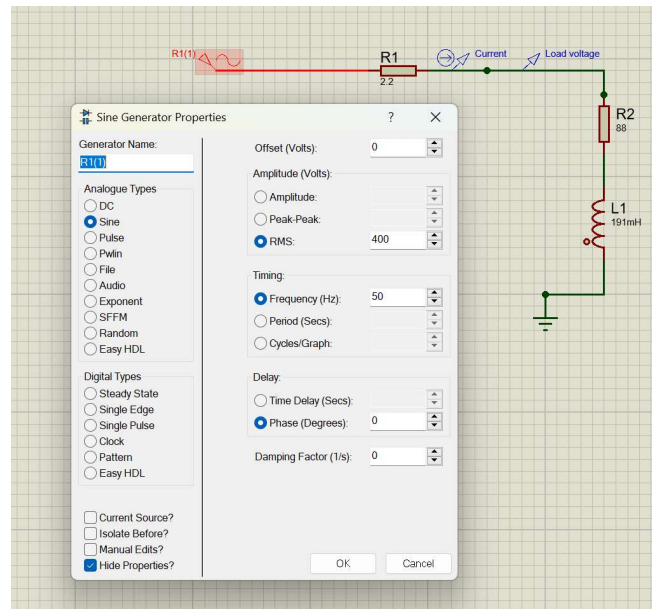


Figure 1 - Simulated circuit before addition of power factor correction

The measured voltage and current were plotted in a graph against time to measure the phase difference.

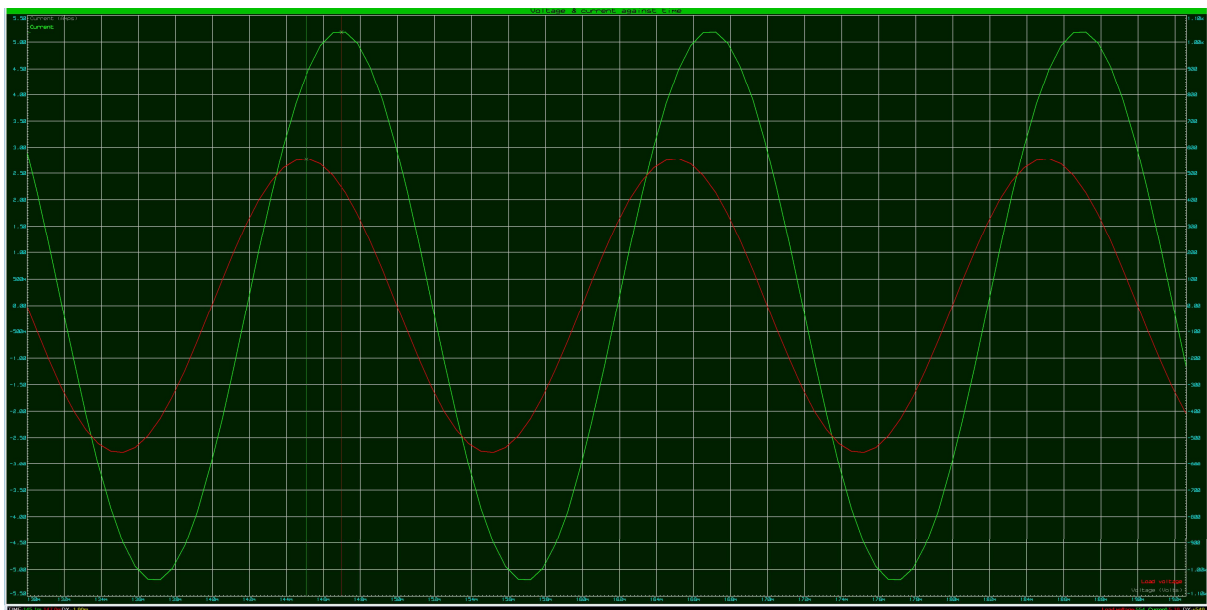


Figure 2 - Load voltage and circuit current plotted against time.

From figure two it was observed that the peak load voltage was 554V and the peak circuit current was 5.19A. This was converted to RMS values as:

$$I_{RMS} = Peak / \sqrt{2}$$

$$= 5.19 / \sqrt{2}$$

$$= 3.66 \text{ A}$$

$$V_{RMS} = Peak / \sqrt{2}$$

$$= 554 / \sqrt{2}$$

$$= 391.7 \text{ V}$$

The Current waveform lagged the voltage by 1.90mS allowing for the phase difference to be calculated using the formula:

$$\phi = 360^\circ \times f \times \Delta t \text{ [4] [5]}$$

As one full cycle of the sinusoidal wave equates to 360° this can be multiplied by the frequency and then by the difference in time (t) between the two waveforms to produce the phase difference Φ .

$$\phi = 360^\circ \times 50\text{Hz} \times 1.90\text{mS}$$

$$\phi = 34.2^\circ$$

For an increased accuracy and to check the measured results, the phase angle can be calculated using the following formula [6, pp. 213, 216] [7, pp. 226, 229]:

$$\tan \phi = XL/R$$

$$\phi = \tan^{-1} XL/R$$

$$XL = 2\pi \times f \times L$$

From the brief, frequency is 50Hz, L is 191mH and R is 88Ω.

$$XL = 2\pi \times 50 \times 191 \times 10^{-3}$$

$$XL = 60.004 \Omega$$

$$\phi = \tan^{-1} 60.004 / 88$$

$$\phi = 34.29^\circ$$

This calculation confirms the accuracy of the graphical measurement.

The active, reactive, and apparent power of the circuit was then calculated using the following formulas [6, p. 232] [7, p. 242] :

$$\text{Apparent power } S = V \cdot I \text{ (Voltamperes)}$$

$$\text{Active power } P = V \cdot I \cdot \cos(\phi) \text{ (Watts)}$$

$$\text{Reactive power } Q = V \cdot I \cdot \sin(\phi) \text{ (Voltamperes Reactive)}$$

From these results the power factor was calculated using the formula [6, p. 232] [7, p. 242]:

$$\text{Power factor} = \text{Active power } P / \text{Apparent power } S$$

To improve the power factor, the new required phase angle (ϕ) was calculated for power factor correction [7, p. 258].

$$\text{Power Factor} = \cos(\phi)$$

$$\phi = \cos^{-1}(\text{Power factor})$$

The required reactive power was calculated, using the power triangle in figure 3 and basic trigonometry the formula was produced.

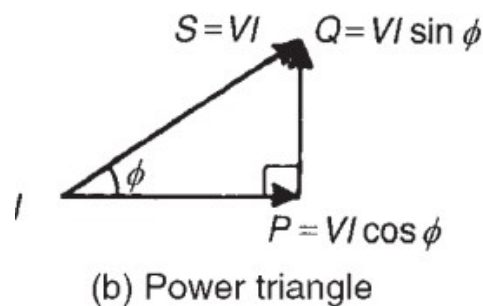


Figure 3 - Power triangle [7, p. 242]

As apparent power Q is the opposite and active power P is the adjacent [7, p. 242]:

$$\tan(\phi) = \text{Opposite} / \text{Adjacent}$$

$$\tan(\phi) = \text{Reactive power } Q / \text{Active power } P$$

As the phase angle and active power were previously calculated, the formula can be rearranged leaving the required reactive power Q as the unknown.

$$\text{Required reactive power } Q = \tan(\phi) \times \text{Active Power}$$

The required Q was then subtracted from the original Q leaving the required capacitive Q . Rearranging the following formula calculated the required reactance X [8].

$$Q = V^2 / X$$

$$X = V^2 / Q$$

Using the formula for capacitive reactance, the required capacitor, for a power factor of 0.95, could be determined [6] [7].

$$X = 1 / 2 \pi f C$$

$$C = 1 / 2 \pi f X$$

To calculate the resonant frequency of the circuit the following formula was used [7]:

$$F_r = (1 / 2\pi) \times (\sqrt{1/LC - R^2 / L^2})$$

Phasors diagrams were created illustrating the currents and voltages for each component of the circuit. This was done by calculating the load impedance, current, and phase angle using the following formulas [7].

$$X_C = 1 / 2\pi f C$$

$$X_L = 2\pi f L$$

$$Z_{RL} = \sqrt{R^2 + X_L^2}$$

$$I_{RL} = V / Z_{RL}$$

$$I_C = V / X_C$$

$$\Phi_{RL} = \tan^{-1} (X_L/R)$$

$$\Phi_{\text{supply}} = \cos^{-1}((I_{RL}\cos\Phi_{RL})/I)$$

To determine whether the supply current was leading or lagging the voltage, the capacitor current was compared with the equation:

$$I_{RL}\sin\Phi_{RL}$$

If I_C was greater than the equation, the supply current would be leading the voltage. If I_C was less than the equation, the supply current would be lagging the voltage. If they were equivalent, the supply voltage and current would be in parallel resonance [7].

Results

As explained in the method section, the active reactive and apparent power were calculated to determine the power factor.

Active power

$$\text{Active power } P = V \cdot I \cdot \cos(\phi) \text{ (Watts)}$$

$$P = 391.7 \times 3.66 \times \cos(34.29)$$

$$P = 1184.45 \text{ W}$$

Reactive power

$$\text{Reactive power } Q = V \cdot I \cdot \sin(\phi) \text{ (Voltamperes Reactive)}$$

$$Q = 391.7 \times 3.66 \times \sin(34.29)$$

$$Q = 807.68 \text{ VAr}$$

Apparent power

$$\text{Apparent power } S = V \cdot I \text{ (Voltamperes)}$$

$$S = 391.7 \times 3.66$$

$$S = 1433.62 \text{ VA}$$

Power factor

$$\text{Power factor} = \text{Active power } P / \text{Apparent power } S$$

$$\text{Power factor} = 1184.45 / 1433.62$$

$$\text{Power factor} = 0.83$$

As a power factor of 1 is unity, the calculated power factor of 0.83 proves the circuit current to be significantly lagging the voltage, requiring power factor correction to improve efficiency. As detailed in the method, the required capacitor value for a power factor of 0.95 was calculated.

Power factor correction

$$\phi = \cos^{-1}(\text{Power factor})$$

$$\phi = \cos^{-1}(0.95)$$

$$\phi = 18.195^\circ$$

Required reactive power $Q = \tan(\phi) \times \text{Active Power}$

$$Q = \tan(18.195) \times 1184.45$$

$$= 389.313 \text{ VAR}$$

$$\text{New } Q = \Delta Q$$

$$\text{New } Q = 807.68 - 389.313$$

$$\text{New } Q = 418.367 \text{ VAR}$$

$$X = V^2 / Q$$

$$X = 391.7^2 / 418.367$$

$$X = 366.733$$

$$C = 1 / 2 \times \pi \times f \times X$$

$$C = 1 / 2 \times \pi \times 50 \times 366.733$$

$$C = 8.68 \mu\text{F}$$

According to the calculation, the circuit requires an 8.68μF capacitor in parallel with the load. Using these formulas and excel, the calculations were repeated for ten increments of power factor from 0.5 to 1.0. This calculated the required capacitor for each power factor level. The graph in figure four was then produced, depicting the correlation between power factor and capacitor value. As the power factor lowers the requirements for capacitance become negative. Another graph was produced depicting phase angle against capacitance value, shown in figure 5. As the phase angle increases the capacitance requirements decrease.

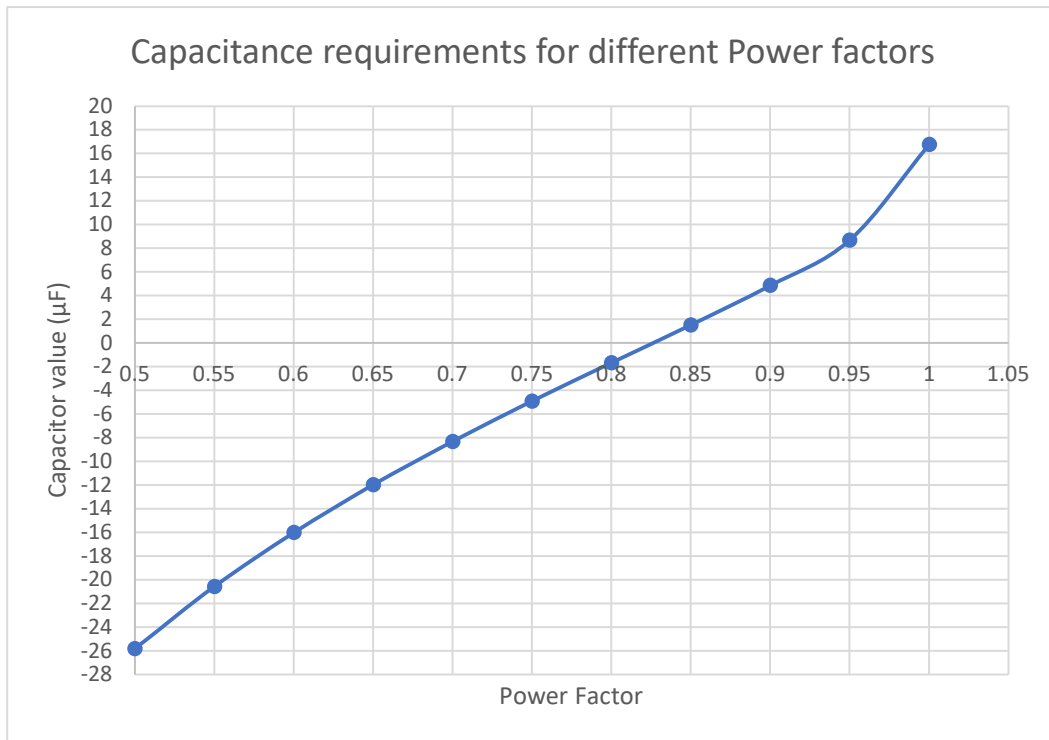


Figure 4 - Graph showing the correlation between power factor and capacitance requirements.

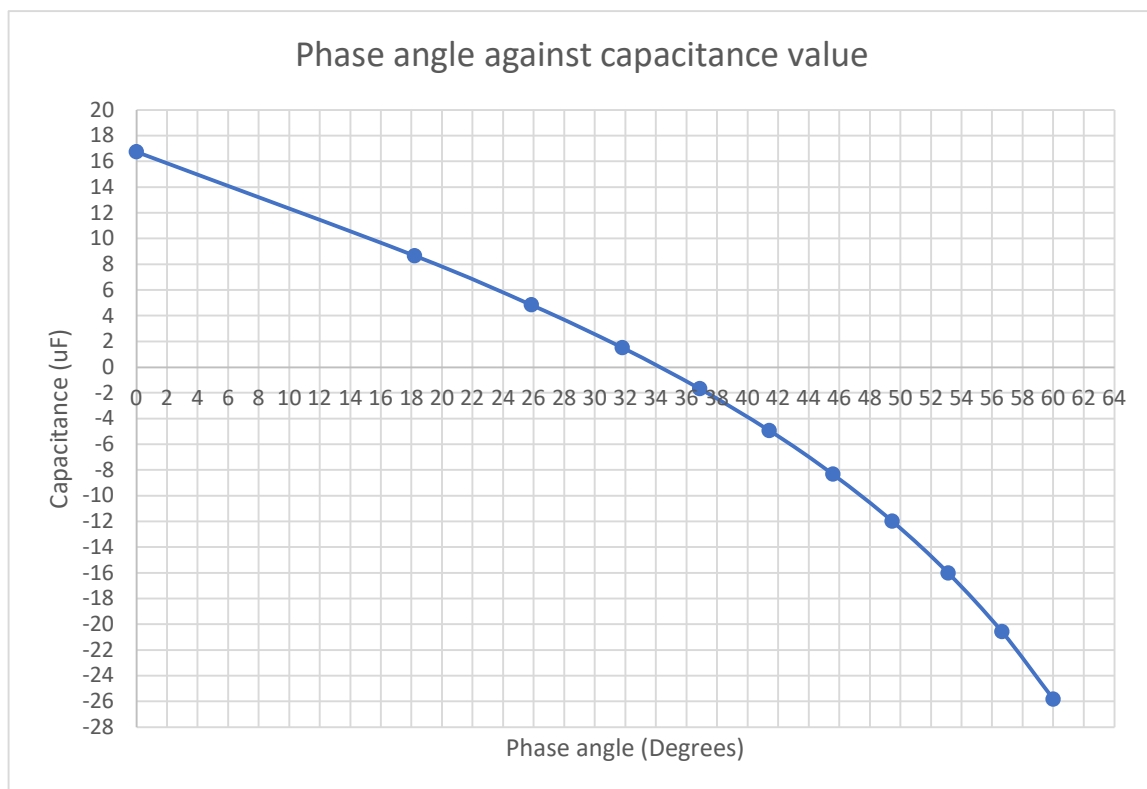


Figure 5 - Graph to show the relationship between phase angle and capacitor value.

The resonant frequency was calculated, and phasor diagrams were constructed at resonance, 25Hz below and above resonance and at 50Hz.

Resonant frequency

$$F_r = (1 / 2\pi) \times (\sqrt{1/LC - R^2 / L^2})$$

$$F_r = (1 / 2\pi) \times (\sqrt{1/191 \times 10^{-3} \times 8.67 \times 10^{-6} - 88^2 / [191 \times 10^{-3}]^2})$$

$$F_r = 99.596 \text{ Hz}$$

Phasor calculations

At Resonant frequency

Voltage RMS = 395.98 Vrms (taken from proteus simulation)

Current RMS = 1.577 Arms (taken from proteus simulation)

Capacitive reactance

$$X_C = 1 / 2\pi \times f \times C$$

$$= 1 / 2\pi \times 99.596 \times 8.67 \mu\text{F}$$

$$= 184.314 \Omega$$

Inductive reactance

$$X_L = 2\pi \times f \times L$$

$$= 2\pi \times 99.596 \times 191 \text{mH}$$

$$= 119.524 \Omega$$

Load impedance

$$Z_{RL} = \sqrt{R^2 + X_L^2}$$

$$\sqrt{88^2 + 119.524^2}$$

$$= 148.425 \Omega$$

Load current

$$I_{RL} = V/Z_{RL}$$

$$= 395.98/148.425$$

$$= 2.668 \text{ A}$$

Phase angle of load

$$\Phi_{RL} = \tan^{-1} (X_L/R)$$

$$= \tan^{-1} (119.524 / 88)$$

$$\Phi_{RL} = 53.638^\circ$$

Capacitor current

$$\begin{aligned} I_C &= V / X_C \\ &= 395.98 / 184.314 \\ &= 2.148 \end{aligned}$$

Supply condition

$$I_{RL} \sin \phi_{RL} = 2.148$$

I_C is equivalent to $I_{RL} \sin \phi_{RL}$ meaning the circuit is in parallel resonance.

Phasor diagram for resonant frequency

The phase diagram at resonant frequency was constructed as shown in figure 6. As capacitor current lead voltage by 90° the phase angle was simply 90° [7]. As the circuit is at resonance, the supply current is in phase with the voltage supply, resulting in a phase angle of zero [7].

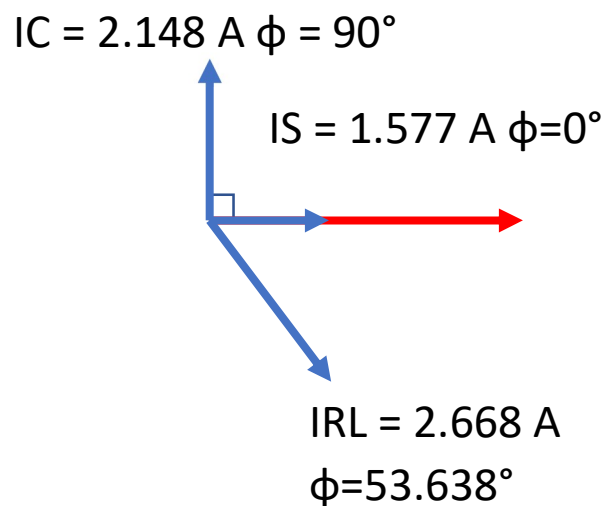


Figure 6 - Phasor diagram for resonant frequency (Voltage not to scale)

At 25Hz below Resonant frequency

Voltage RMS = 393.858 Vrms (taken from proteus simulation)

Current RMS = 2.284 Arms (taken from proteus simulation)

Capacitive reactance

$$\begin{aligned}X_C &= 1 / 2\pi \times f \times C \\&= 1 / 2\pi \times 74.596 \times 8.67\mu\text{F} \\&= 246.086 \Omega\end{aligned}$$

Inductive reactance

$$\begin{aligned}X_L &= 2\pi \times f \times L \\&= 2\pi \times 74.596 \times 191\text{mH} \\&= 89.522 \Omega\end{aligned}$$

Load impedance

$$\begin{aligned}Z_{RL} &= \sqrt{R^2 + X_L^2} \\&= \sqrt{88^2 + 89.522^2} \\&= 125.532 \Omega\end{aligned}$$

Load current

$$\begin{aligned}I_{RL} &= V / Z_{RL} \\&= 393.858 / 125.532 \\&= 3.138 \text{ A}\end{aligned}$$

Phase angle of load

$$\begin{aligned}\Phi_{RL} &= \tan^{-1} (X_L / R) \\&= \tan^{-1} (89.522 / 88) \\&= 45.491^\circ\end{aligned}$$

Capacitor current

$$\begin{aligned}I_C &= V / X_C \\&= 393.858 / 246.086 \\&= 1.60\end{aligned}$$

Supply current phase angle

$$\begin{aligned}\Phi &= \cos^{-1}((I_{RL} \cos \phi_{RL}) / I) \\&= \cos^{-1}((3.138 \cos 45.491) / 2.284) \\&= 15.605^\circ\end{aligned}$$

Supply condition

$$I_{RL}\sin\phi_{RL} = 2.238$$

IC is less than $I_{RL}\sin\phi_{RL}$ meaning the current lags voltage in the circuit.

Phasor diagram for 25Hz below resonant frequency

$$I_C = 1.60 \text{ A } \phi = 90^\circ$$

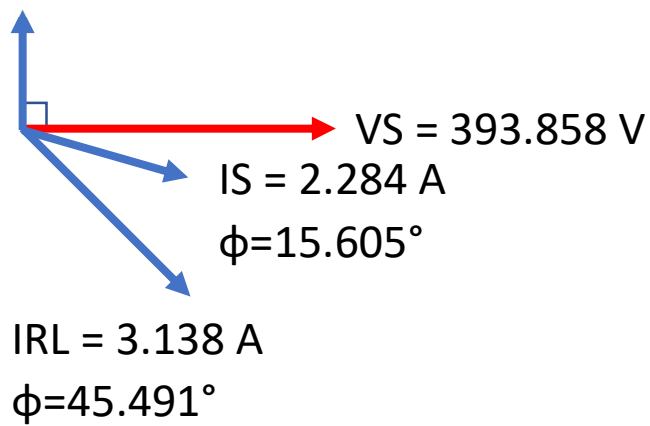


Figure 7 - Phasor diagram for 25Hz below resonant frequency (Voltage not to scale)

At 25Hz above Resonant frequency

Voltage RMS = 395.98 Vrms (taken from proteus simulation)

Current RMS = 1.372 Arms (taken from proteus simulation)

Capacitive reactance

$$\begin{aligned}X_C &= 1 / 2\pi \times f \times C \\&= 1 / 2\pi \times 124.596 \times 8.67\mu\text{F} \\&= 147.332 \Omega\end{aligned}$$

Inductive reactance

$$\begin{aligned}X_L &= 2\pi \times f \times L \\&= 2\pi \times 124.596 \times 191\text{mH} \\&= 149.526 \Omega\end{aligned}$$

Load impedance

$$\begin{aligned}Z_{RL} &= \sqrt{R^2 + X_L^2} \\&= \sqrt{88^2 + 149.526} \\&= 173.499 \Omega\end{aligned}$$

Load current

$$\begin{aligned}I_{RL} &= V / Z_{RL} \\&= 395.98 / 173.499 \\&= 2.282 \text{ A}\end{aligned}$$

Phase angle of load

$$\begin{aligned}\Phi_{RL} &= \tan^{-1} (X_L / R) \\&= \tan^{-1} (149.526 / 88) \\&= 59.522^\circ\end{aligned}$$

Capacitor current

$$\begin{aligned}I_C &= V / X_C \\&= 395.98 / 147.332 \\&= 2.688\end{aligned}$$

Supply current phase angle

$$\begin{aligned}\Phi &= \cos^{-1}((I_{RL} \cos \phi_{RL}) / I) \\&= \cos^{-1}((2.282 \cos 59.522) / 1.372) \\&= 32.476^\circ\end{aligned}$$

Supply condition

$$I_{RL}\sin\phi_{RL} = 1.967$$

I_C is greater than $I_{RL}\sin\phi_{RL}$ meaning the current leads voltage in the circuit.

Phasor diagram for 25Hz above resonant frequency

$$I_C = 2.688 \text{ A } \phi = 90^\circ$$

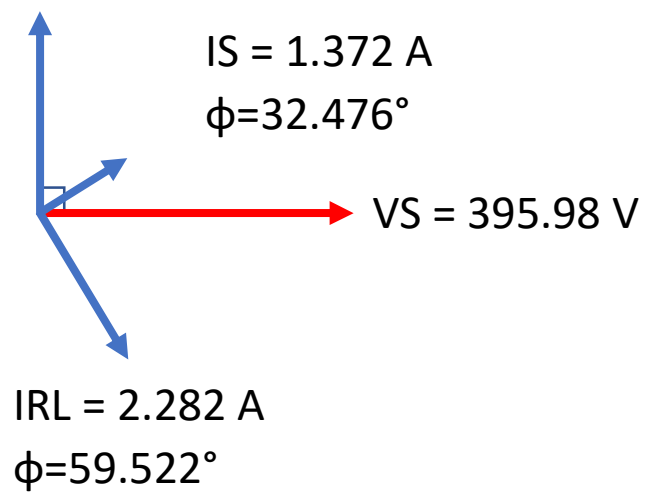


Figure 8 - Phase diagram for 25Hz above resonant frequency (Voltage not to scale)

At 50Hz

Voltage RMS = 393.151 Vrms (taken from proteus simulation)

Current RMS = 3.196 Arms (taken from proteus simulation)

Capacitive reactance

$$\begin{aligned}X_C &= 1 / 2\pi \times f \times C \\&= 1 / 2\pi \times 50 \times 8.67\mu\text{F} \\&= 367.139 \Omega\end{aligned}$$

Inductive reactance

$$\begin{aligned}X_L &= 2\pi \times f \times L \\&= 2\pi \times 50 \times 191\text{mH} \\&= 60.004 \Omega\end{aligned}$$

Load impedance

$$\begin{aligned}Z_{RL} &= \sqrt{R^2 + X_L^2} \\&= \sqrt{88^2 + 60.004} \\&= 106.51 \Omega\end{aligned}$$

Load current

$$\begin{aligned}I_{RL} &= V / Z_{RL} \\&= 393.151 / 106.51 \\&= 3.691 \text{ A}\end{aligned}$$

Phase angle of load

$$\begin{aligned}\Phi_{RL} &= \tan^{-1} (X_L / R) \\&= \tan^{-1} (60.004 / 88) \\&= 34.289^\circ\end{aligned}$$

Capacitor current

$$\begin{aligned}I_C &= V / X_C \\&= 393.151 / 367.139 \\&= 1.071\end{aligned}$$

Supply current phase angle

$$\begin{aligned}\Phi &= \cos^{-1}((I_{RL} \cos \phi_{RL}) / I) \\&= \cos^{-1}((3.691 \cos 34.289) / 3.196) \\&= 17.413^\circ\end{aligned}$$

Supply condition

$$I_{RL} \sin \phi_{RL} = 2.079$$

I_C is less than $I_{RL} \sin \phi_{RL}$ meaning the current lags voltage in the circuit.

Phasor diagram for 50Hz

$$I_C = 1.071 \text{ A } \phi = 90^\circ$$

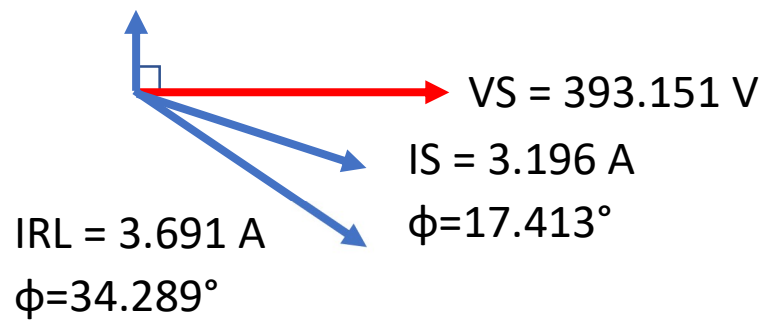


Figure 9 - Phasor diagram for 50HZ (voltage not to scale)

Discussion

Power factor correction is an important process in industry. The power calculations for the RLC circuit demonstrated the savings made by the addition of a parallel capacitor. The apparent power delivered to the circuit was calculated to be 1433.62 VA, whereas the active power was 1148.45 W. This meant that although the circuit only required 1148.45 W to operate, a total of 1433.62 VA was required from the supply. This was because 807.68 VAR was wasted by the reactive components of the circuit [1]. From these results the power factor for the original circuit was calculated to be only 0.83. A power factor of 1 is considered unity, meaning the supply current and voltage is in phase. If unity were to be achieved, the reactive power of the circuit would be reduced to zero. This would not be desirable in an RL circuit, as the load, L, of the circuit would not produce a magnetic field as is required by an inductor [1]. A power factor (PF) of 0.95 would deliver a more efficient circuit, whilst still allowing a small phase difference, as required to produce reactive power.

The required capacitor value to achieve 0.95 PF was calculated to be $8.67\mu\text{F}$. Once applied to the circuit, the phase difference was lowered to 18.2° . As the power factor nears unity, the circuit becomes closer to resonance. Parallel resonance occurs when the current and voltage are in phase, as previously discussed, at this point there is no reactive power in the circuit meaning the inductor cannot induce a magnetic field. This causes oscillation within the parallel branches, as the inductor draws current from the capacitor to create its magnetism until the capacitor is discharged. The current then drains from the inductor to recharge the capacitor producing the oscillating effect. This oscillation can cause current magnification measured as a ratio called Q-factor [7]. Resonance can cause overheating of electrical machinery such as motors and transformers, which could potentially cause fatal damage to windings [9, p. 190]. Circuit harmonics can also cause problems at resonance, if the harmonic source has a matching impedance to the capacitor, harmonic resonance occurs. This causes very high harmonic currents and voltages which could be catastrophic for the capacitor, destroying the power factor correction [10] [11].

At power factors nearing unity such as at 0.95, the active power is increased, and reactive power is decreased. This is the necessary combination to achieve maximum power transfer. Maximum power transfer can be achieved when a load is purely resistive, such as at resonance when there can be no reactive energy [6] [7].

Using the power factor calculations, graphs were plotted illustrating capacitor value against power factor and against phase angle. Against power factor, figure 4 showed that for a lower power factor, a negative capacitance would be required. This supports the theory as a negative capacitance would mean a more inductive load, which would produce a greater phase difference. This is also evident in figure 5 where the graph illustrates that as phase angle increases, the capacitance requirement decreases.

The phasor diagrams created, accurately exhibited the circuit voltages, currents, and phase angles. Whether the current lagged or lead the voltage can be clearly observed. The calculated currents and angles for each diagram can be proven correct as the supply current is the resultant diagonal if a parallelogram were to be constructed using the branch currents [7, p. 218].

At resonant frequency the phasor illustrates the supply current to be in phase with the supply voltage, as expected. For 25Hz below resonance the current lags voltage. This occurs as the lower than resonant frequency produces a greater X_C than X_L resulting in I_C lagging V . The same outcome can also be observed at 50Hz. At 25Hz above resonance, the current leads voltage as X_L is greater than X_C [7, p. 251].

Conclusion

The main findings of this project were the benefits of power factor correction. The addition of capacitance to a reactive circuit allowed for a more efficient transfer of power as the difference in phase between supply voltage and load current was lowered. It was however determined that increasing the capacitance too much would result in a purely resistive circuit, unable to produce reactance. This circuit would not function as required and would be fixed in oscillation.

As well as the effect of capacitance, the effects of frequency were also determined. Frequency directly influenced the phase difference between load current and supply voltage, ruling whether the current would lag or lead voltage. This influence was due to its impact on the resultant of capacitive reactance and inductive reactance.

Bibliography

- [1] J. Ware, "Achieving compliance with the building (scotland) regulations 2004," *IEE Wiring matters*, vol. Spring, no. 18, pp. 22-24, Spring 2006.
- [2] S. Khanchi and V. K. Garg, "Power Factor Improvement of Induction Motor," *International Journal of Engineering Trends and Technology*, vol. 4, no. 7, pp. 2967, 2968, July 2013.
- [3] M. H. Shwehdi and M. R. Sultan, "Power Factor Correction Capacitors; Essentials and Cautions," IEEE, Dhahran, 2000.
- [4] J. Papiewski, "How to Calculate the Phase Shift," Leaf Group Ltd, 13 03 2018. [Online]. Available: <https://sciencing.com/calculate-phase-shift-5157754.html>. [Accessed 10 04 2023].
- [5] Electronics Projects Focus, "What is Phase Difference : Formula & Its Equation," Electronics Projects Focus, 10 04 2023. [Online]. Available: <https://www.elprocus.com/phase-difference-formula-equation/>. [Accessed 10 04 2023].
- [6] J. Bird, *Electrical circuit theory and technology*, Revised second edition ed., Oxford: Newnes, 2003.
- [7] J. Bird, *Electrical and Electronic Principles and technology*, Fourth ed., Oxford: Newnes, 2010.
- [8] W. Storr, "Power Factor Correction," Electronics tutorials, 10 04 2023. [Online]. Available: <https://www.electronics-tutorials.ws/ac/circuits/power-factor-correction.html>. [Accessed 10 04 2023].
- [9] R. C. Dugan, M. F. McGranaghan, S. Santoso and H. W. Beaty, *Electrical power systems quality*, New york: McGraw-Hill, 2002.
- [10] D. J. Carnovale, "Power Factor Correction and harmonic resonance," in *PQ corner*, EC&M, 2003, pp. 16-19.
- [11] S. Chaladying, A. Charlangsut and N. Rugthaichareoncheep, "Parallel Resonance Impact on Power Factor," Rajamangala University of Technology, Bangkok, 2015.