

Engineering Science Guidance Notes

POTENTIAL AND KINETIC ENERGY KIT ES9

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Introduction

These Guidance Notes introduce you to the theory for a set of experiments in an engineering science topic. You fit different parts of your kit to a Work Panel to do an experiment. Figure 1 shows a typical experiment.

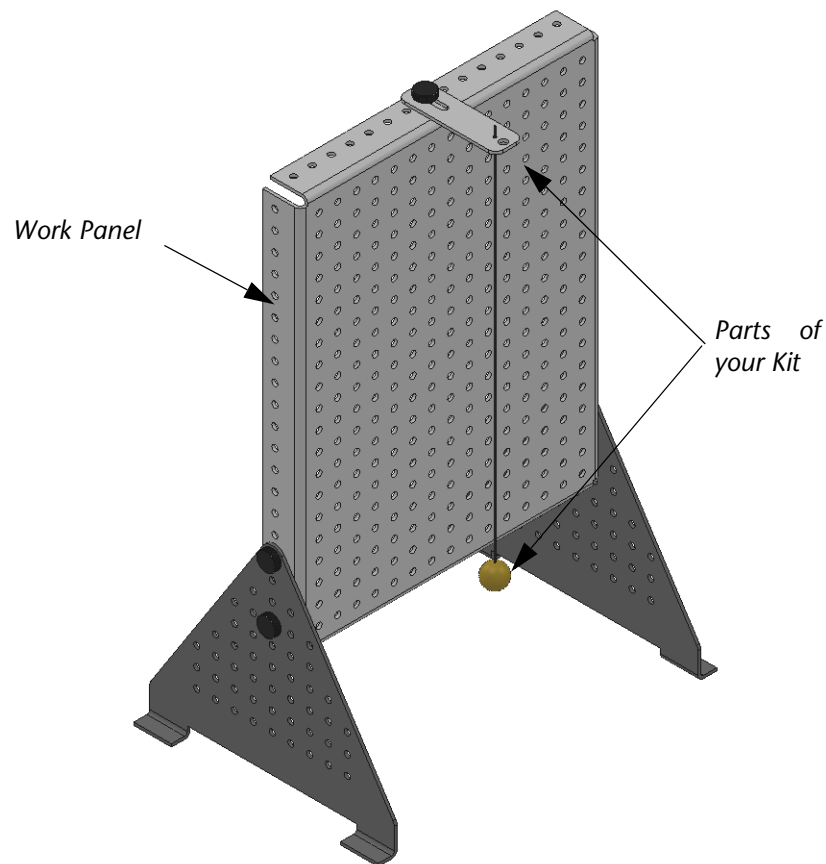


Figure 1 A Typical Experiment

Each kit can do one or more experiments and each experiment has Worksheets that tell you how to do the experiment. You must use the Worksheets with the Guidance Notes as they:

- Introduce the parts in the kit, and list the experiments that it can do.
- Give you important information you need to do the experiments or complete your Worksheet.

Your tutor may decide to ask you to do all the experiments or just a few. You must be sure what you need to do. To help you, Table 1 shows a list of the experiments for your kit.

The ES9 Kit - What is it and what can it do?

This kit helps you to understand potential and kinetic energy in pendulums, springs and flywheels. It shows you that:

- You can convert potential energy into kinetic energy and back - energy can be conserved
- You can store energy in different ways

Scientists and engineers used experiments to prove their theories for kinetic and potential energy. Textbooks show these experiments and the theory, giving them popular names. The experiments and theory for this kit try to follow these names (for example the 'energy in a flywheel' or 'elastic potential energy' experiments) so you can easily compare them with the information you will find in textbooks or in your classroom lessons.

The kit comes in a plastic box with a lid and contains all the parts you need to do the experiments shown in Table 1. Refer to the Parts List in your kit to see what parts are included.

Your tutor may decide to ask you to do all the experiments or just a few. You must be sure what you need to do.

List of Experiments

Experiment	Does my teacher need me to do this experiment?	Have I got the Worksheet?
1. Energy in a spring		
2. Energy in a pendulum		
3. Energy in a flywheel		

Table 1 List of Experiments

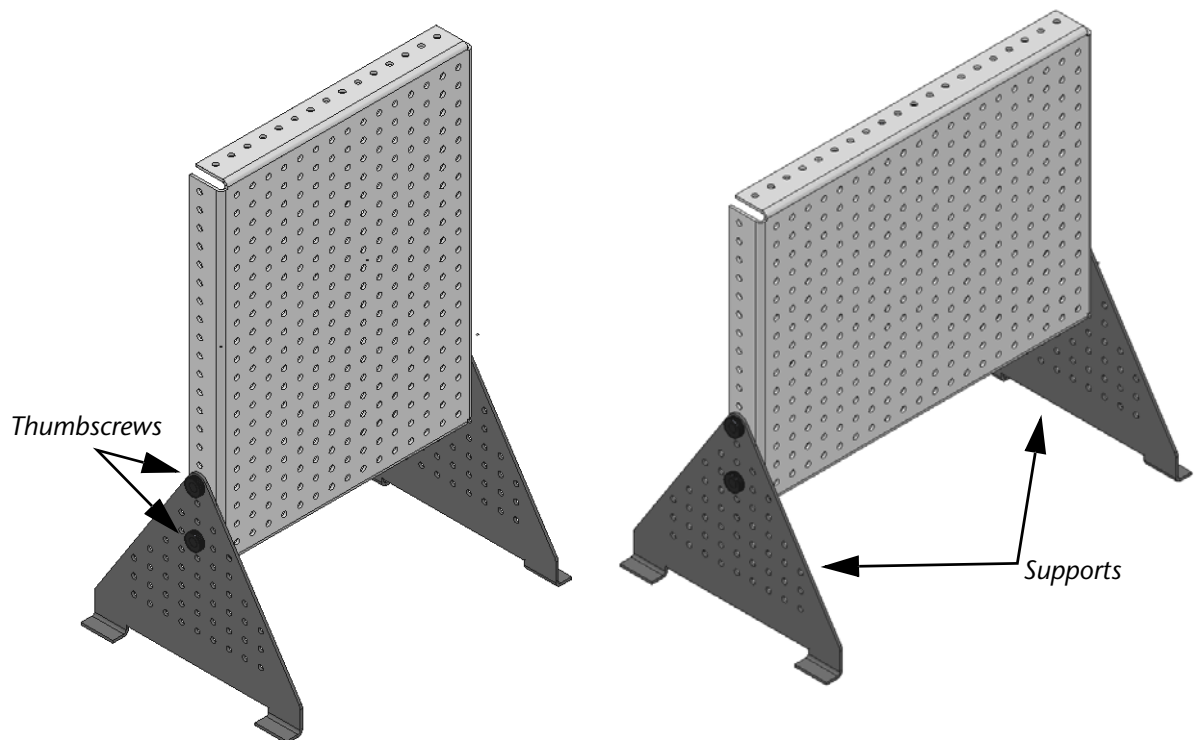
General Notes**The Work Panel**

Figure 2 The Work Panel Mounted in Two Typical ways (Portrait and Landscape)

The Work Panel mounts on its Supports in different ways as needed by each experiment. The Worksheets show you which way TecQuipment recommends you to fit the Supports but you may find an alternative that fits better on your desk. To change how the Supports hold the Work Panel, ask your Teacher or a classmate to help you hold the Work Panel while you change the Supports around. However you mount the Work Panel, you must always use two Thumbscrews and Thumbnuts to hold each Support to the Work Panel.

Weights, Masses, Weight Hangers and Forces

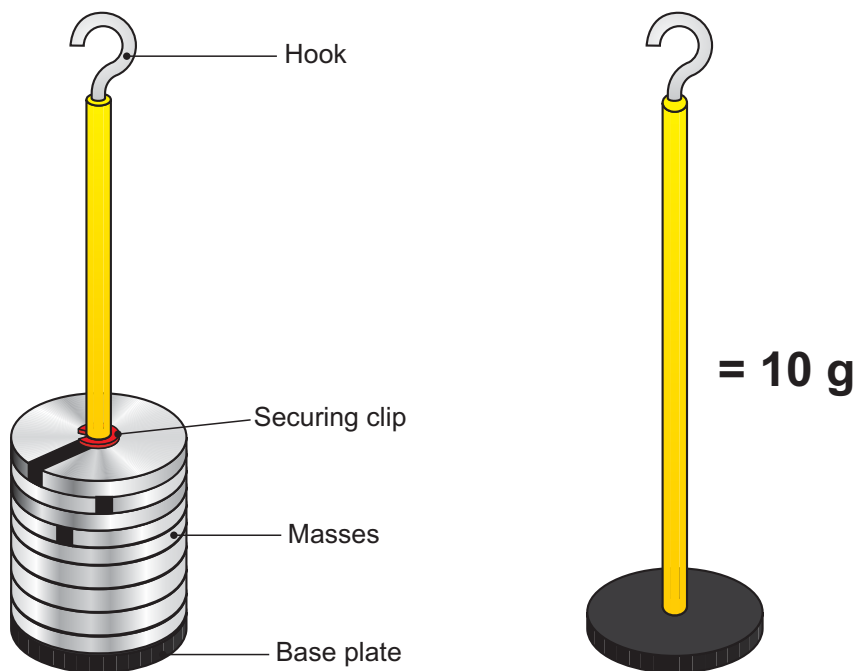


Figure 3 A Weight Hanger

The masses supplied with the Engineering Science Kits have markings in grams (or grammes). For your calculations, you use the unit of force (Newton - N) caused by the pull of gravity (downward) on the masses.

$$1 \text{ kg} = 9.81 \text{ N} \text{ or } 100 \text{ g} = 0.98 \text{ N}$$

Note: Each weight hanger itself weighs 10 g, so you only need to add 9 x 10 g masses to get 100 g.

Weights	Force (N)
1 x 10 g = 10 g	0.098
10 x 10 g = 100 g	0.98
20 x 10 g = 200 g	1.96
30 x 10 g = 300 g	2.94
40 x 10 g = 400 g	3.92
50 x 10 g = 500 g	4.90

Using the Weight Hanger Guide

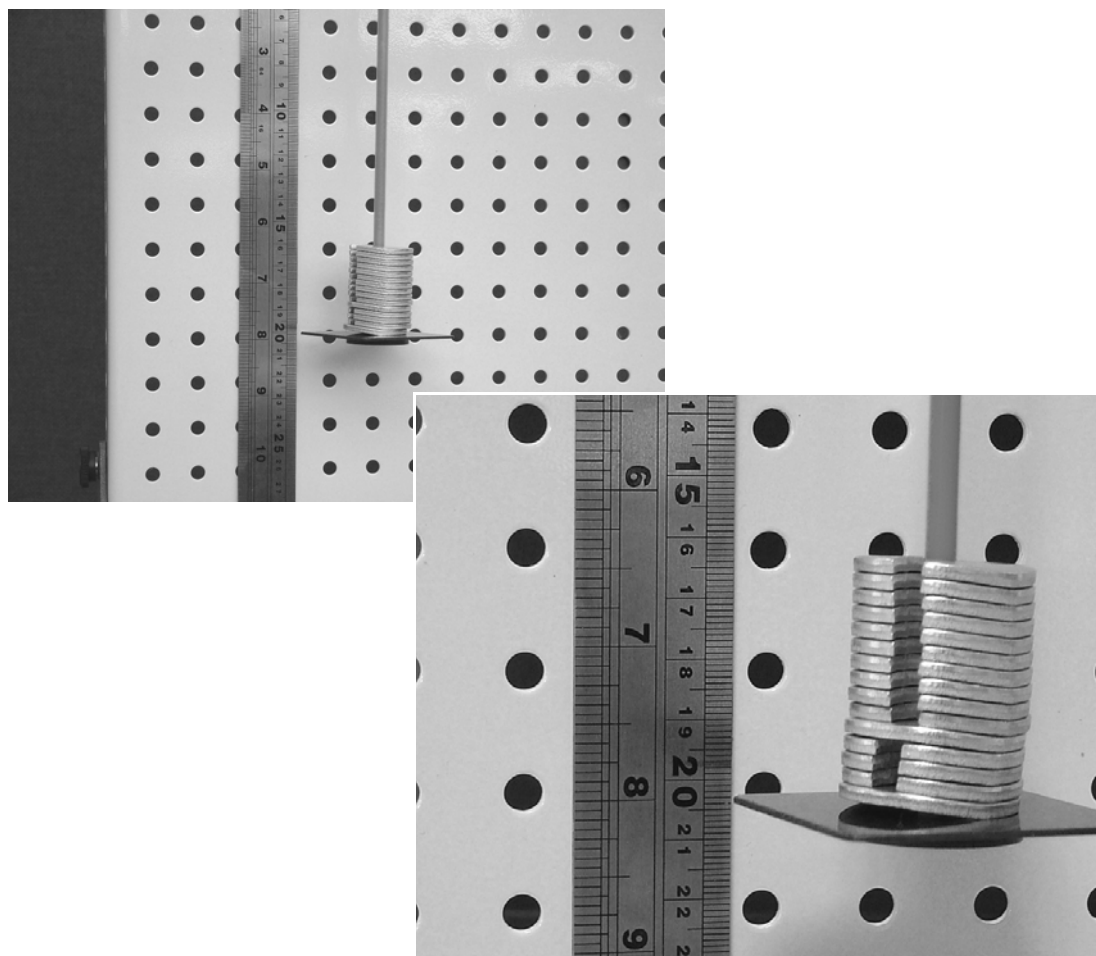


Figure 4 Using the Weight Hanger Guide

When you do the simple spring test for one of the experiments, fit the guide to the bottom of the Weight Hanger.

Now slide the magnetic rule so it is behind the guide. The guide then becomes your reference to help measure how far the spring extends downwards.

To reduce measuring errors, make sure your eye is at the same level as the Weight Hanger Guide.

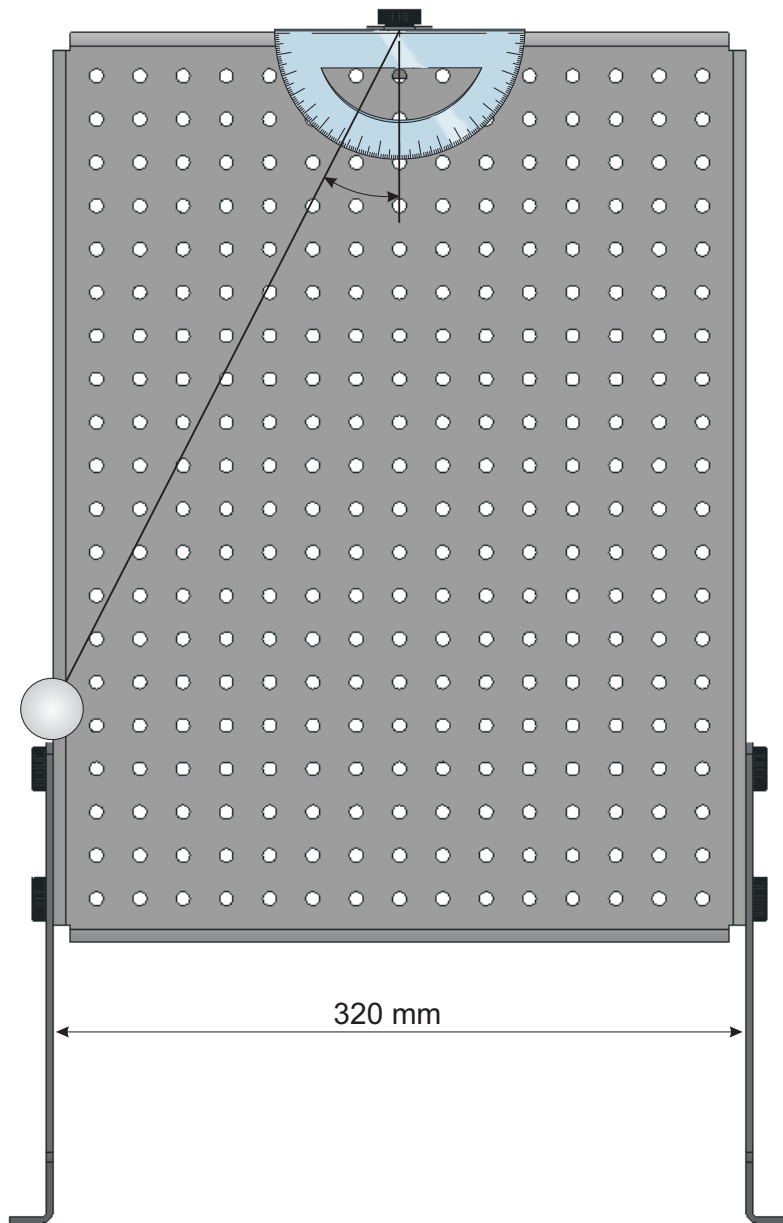
Measuring Pendulum Angles and Distances

Figure 5 Measuring Angles and Distances

The Work Panel and its holes help you to measure pendulum distances, as the holes are exactly 20 mm apart and the panel height and width are divisible by 20 at 420 mm and 320 mm. You need an accurate protractor to measure the swing angle from the vertical at the point where the string leaves the Plummet Arm.

Alternatively, you could use trigonometry to find the angle and use the protractor to double-check your calculations.

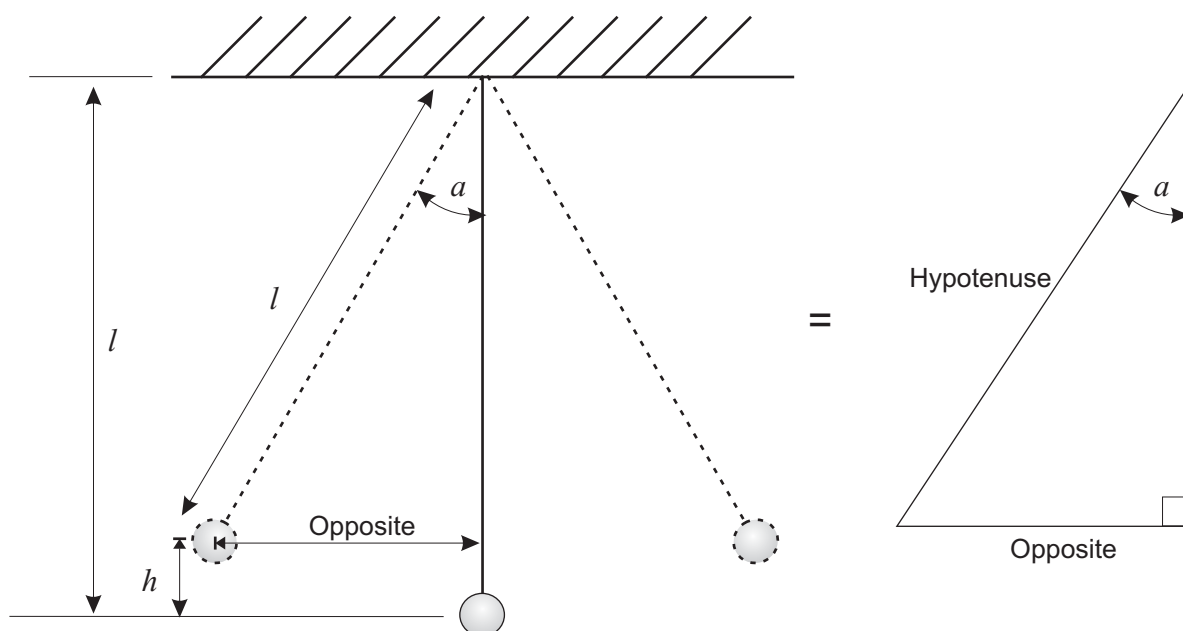


Figure 6 Using Trigonometry

See Figure 6. The pendulum at the highest part of its swing forms a right angle triangle with a hypotenuse and an opposite side. Its hypotenuse is the length l of the pendulum and you can measure the length of the opposite side by counting the holes in the Work Panel or using a rule. From this:

$$\sin a = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

So:

$$a = \arcsin \frac{\text{Opposite}}{\text{Hypotenuse}}$$

The Flywheel Brake

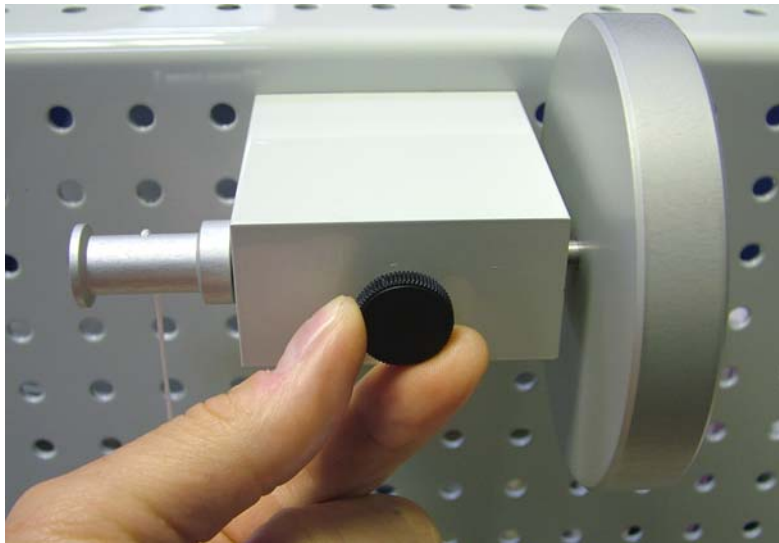


Figure 7 The Flywheel Brake

The Bearing Block has an extra thumbscrew to the front. This helps lock or 'brake' the flywheel shaft in position, which you may find useful during your experiments.

NOTE



Fully unscrew the brake by at least one turn to allow the flywheel to rotate, or your results will be wrong.

Nominal Details of Parts

You need to know the details of some of the parts used in the experiments. Table 2 gives the nominal values that you may use in your experiments. However, the weights may change slightly due to manufacturing tolerances. For more accuracy, you could ask your teacher or lecturer for an accurate weighing scale to measure the weights yourself.

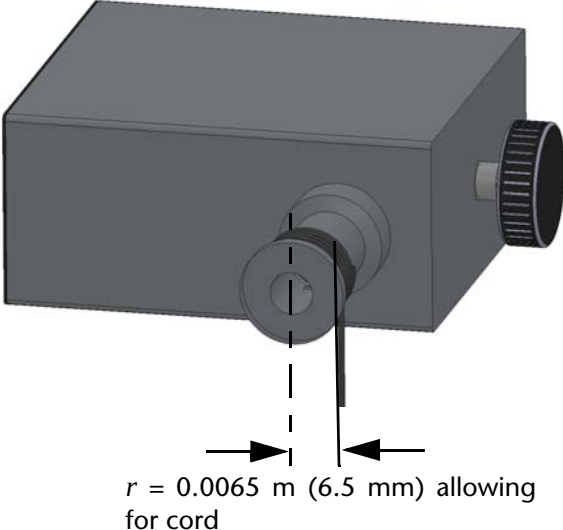
Part	Nominal Details
	330 g (0.330 kg) $I = 3.862 \times 10^{-4} \text{ kg.m}^2$
	60 g (0.06 kg)
	I for shaft and pulley $= 0.834 \times 10^{-4} \text{ kg.m}^2$ Allowing for the cord $r = 0.0065 \text{ m}$ $d = 0.013 \text{ m}$ $c = \pi \times 0.013 = 0.04084$

Table 2 Nominal Weights

Accurate Results

For best results:

- Do not rush your experiment.
- Double-check each reading.
- Repeat the experiment if you are not sure of your results.
- Unless the instructions say otherwise, gently tap the Work Panel before taking a reading. Sometimes, mechanical parts 'stick' against each other (caused by friction). This is often called 'stiction' or static friction. Tapping the Work Panel helps to reduce this.

Do not expect your results to be exactly as shown in the theory. Theory always shows 'perfect' or 'ideal' results, based on perfect scientific conditions. Your 'actual' results will be slightly different to theory, based on the accuracy of the equipment and how carefully you do your experiment.

You may learn more about your experiments by making and finding mistakes than getting things right first time!

Other Things You May Need

These are things that are not in the Kit but you may need to complete your experiments. You should already have these things as part of your normal student equipment (pencil case) or your teacher may supply them:

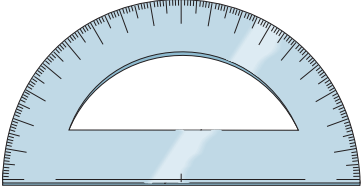
Part	Image
Calculator	
An accurate Protractor	

Table 3 Other Things You May Need

Theory**Notation**

Symbol	Meaning	Units
r	Radius	m
α	Acceleration	m.s^{-2}
L and l	Length	m
k	Spring rate	N.m^{-1}
π	Pi	3.142
c	Circumference	m
h	Height	m
PE and KE	Potential and Kinetic Energy	J
I	Moment of inertia	kg.m^2
F	Force	N
g	Acceleration due to gravity	9.81 m.s^{-2}
m	Masses	kg
m_f	Mass of flywheel	kg
K	Mass radius of gyration	-
τ	Torque	Nm
t	Time	s
ω	Angular velocity	rad.s^{-1}
v_a	Velocity	m.s^{-1}
v_m	Mean (straight line) velocity	m.s^{-1}

Table 4 Notation

Energy and Work Done

The work done on an object is simply the force to move the object multiplied by the displacement or distance over which the force acts.

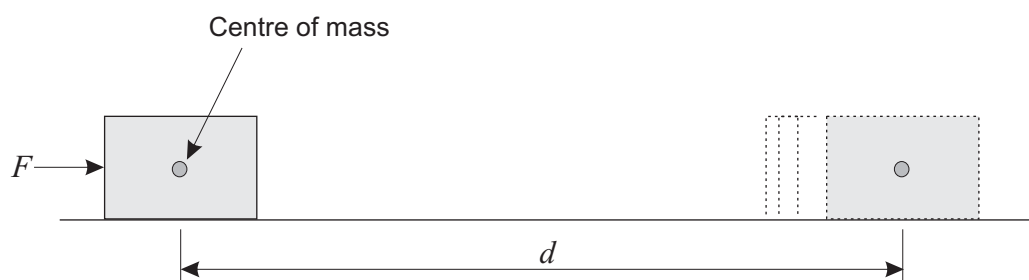


Figure 8 Energy and Work Done

A Joule (J) is the work done or energy used in moving an object using a force of 1 N over a distance of 1 metre. This can be shown in a graph, where the work done is simply the area under the force distance graph.

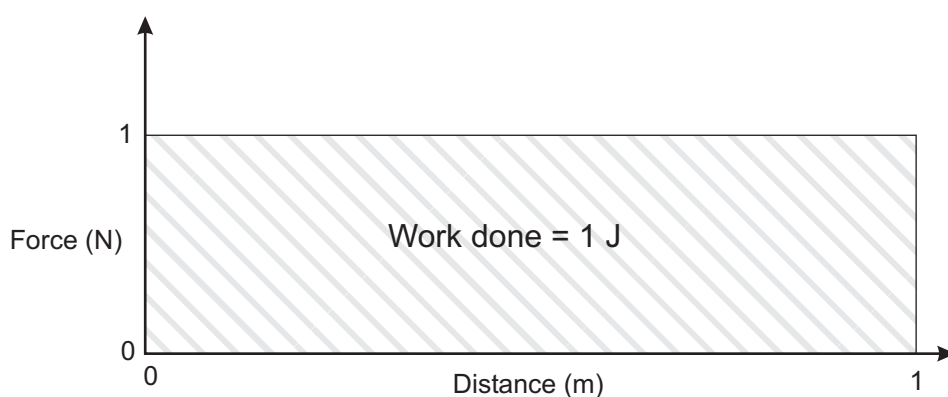


Figure 9 Force Distance Graph

Work Done by a Variable Force

Where the force remains constant, the work done is simply $F \times d$. When the force varies over the distance, this simple equation will not work. However, the work done is still the area under the graph.

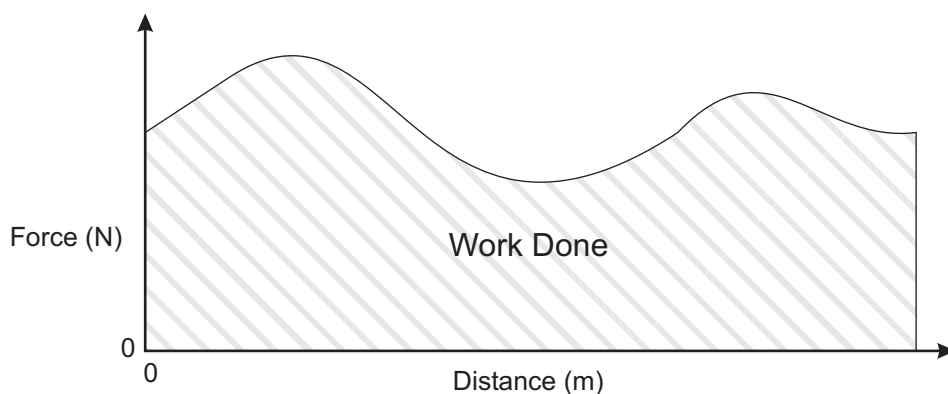


Figure 10 Force Distance Graph

Hooke's Law

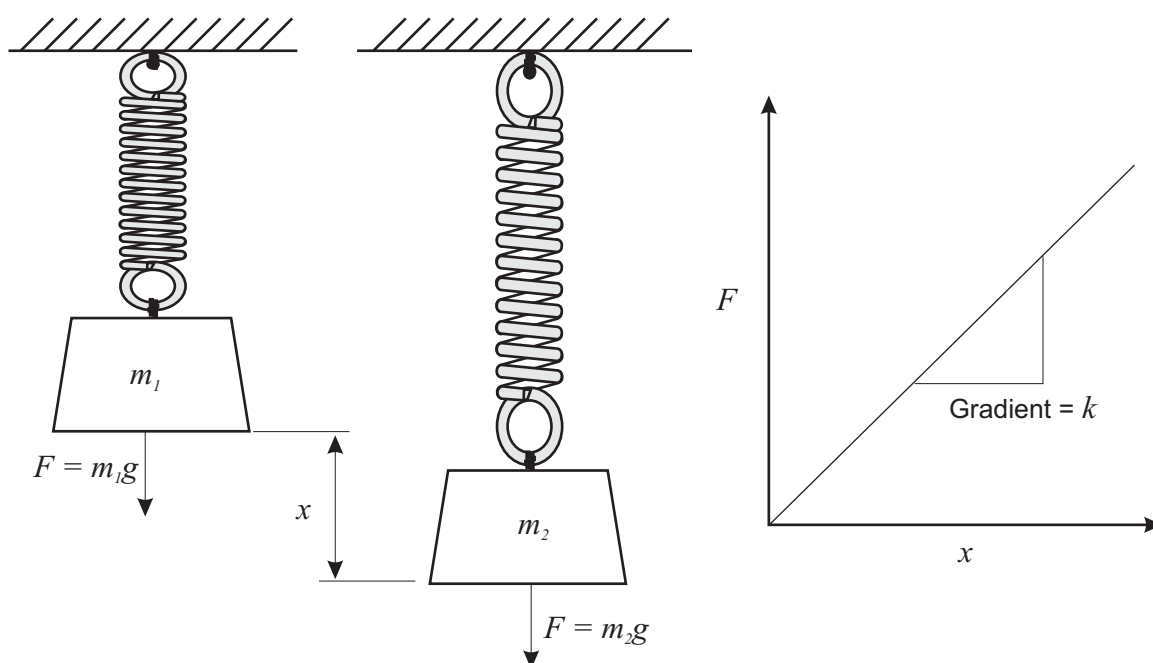


Figure 11 Hooke's Law

An English physicist - Robert Hooke experimented with wires and springs in the late 17th Century. He found that if you test a spring **in its normal elastic region**, its extension or compression (x) is directly proportional to the force (F) you apply to it.

$$F \propto x$$

From this, a test of spring extension or compression for a range of applied forces produces a chart with a gradient of a constant value called the 'spring rate' (k).

$$F = kx$$

If the units of force are Newtons (N) and the units of extension are in mm, then the spring rate is in units of N/mm or $\text{N}\cdot\text{mm}^{-1}$.

Since work is the area under the force displacement graph it follows that the energy (work) stored in a spring is the area under the force and extension (displacement) graph.

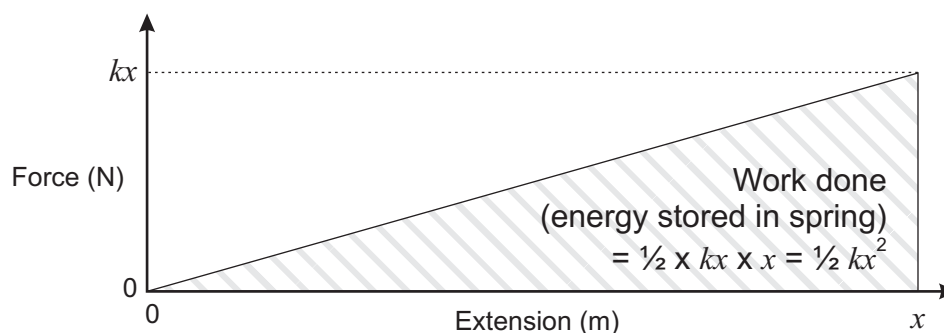


Figure 12 Energy Stored in a Spring

Spring Rate Test

To test the spring rate you need to do a simple load and extension test. The slope of a chart of your results gives the spring rate (k) (see Figure 13). The spring rate is normally in Newton per metre (N.m^{-1}) but some textbooks and manufacturers use Newton per millimetre (N.mm^{-1}). Try the other Engineering Science kits for a more detailed study of springs.

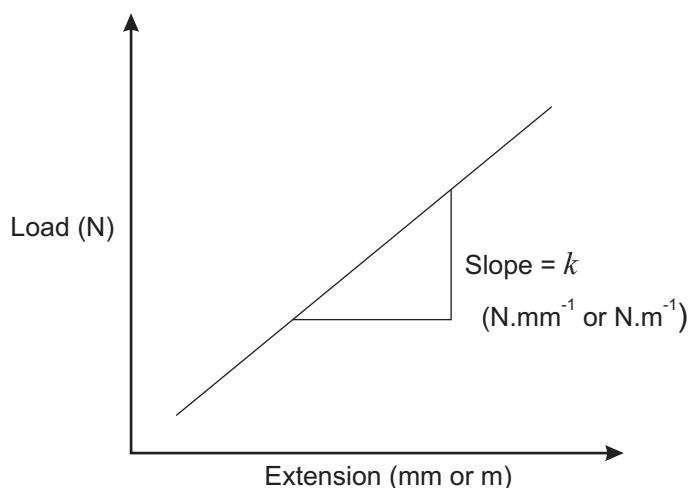


Figure 13 Spring Rate

Worked example of finding spring rate:

The mass used changes from 50 g to 350 g, giving a load change of 0.49 N to 3.40 N.

The spring extends from 11 mm to 77 mm, giving an extension in metres of 0.011 m to 0.077 m.

The slope of a straight line is the difference in the vertical divided by the difference of the horizontal.

This gives $(3.4 - 0.49)/(0.077 - 0.011) = 2.91/0.066 = 44.09$ or roughly 44 N.m^{-1} spring rate (k).

Worked example of finding energy stored for a given extension:

The spring rate $k = 44 \text{ N.m}^{-1}$

The extension $x = 0.077 \text{ m}$ (77 mm)

Potential Energy $\text{PE} = 0.5 \times 44 \times 0.077^2 = 22 \times 0.005929 = 0.13 \text{ J}$

Potential Energy

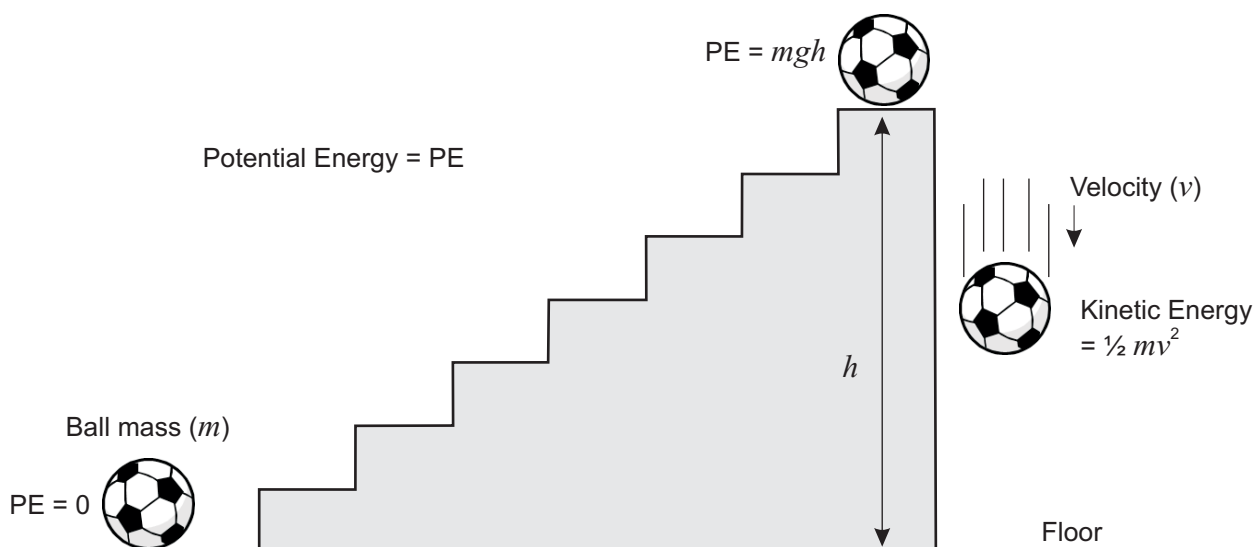


Figure 14 Potential and Kinetic Energy

Potential energy (PE) is the energy something has available due to its position, usually height from the ground. It can also describe the energy held by an object or material that has been compressed, stretched or expanded (see Energy in a Spring). Kinetic energy (KE) is the energy something has when it is moving.

See Figure 14. A ball sitting on the ground has no potential energy **with respect to the ground**. A ball on the highest step has the higher potential energy. As soon as the ball falls off the highest step, it moves downwards, changing its potential energy into kinetic energy, which will be at a maximum just before the ball hits the floor (the highest velocity).



NOTE

It is important to note the 'with respect to ground' statement. Your local 'ground' could be a high floor in high rise building, in which case the ball would have a high potential energy with respect to real ground and no potential energy with respect to the floor it sits on.

So, when the ball is on the top step, its:

Potential Energy = mgh

Kinetic Energy = 0, since the ball is stationary and $v = 0$.

When the ball is just about to hit the ground, its:

Potential Energy = 0, since $h = 0$

Kinetic Energy = $\frac{1}{2}mv^2$.

Since the laws of physics state that energy cannot be lost or destroyed, then for a perfect system:

$$PE = KE$$

In reality, some energy is lost to friction or other resistance to motion of some kind, so the transfer is never 100% efficient. However, the laws of physics state that energy is never 'lost', only transferred from one state to another. The energy lost to friction is simply another conversion of some of the original energy (often to heat).

Energy in a Pendulum

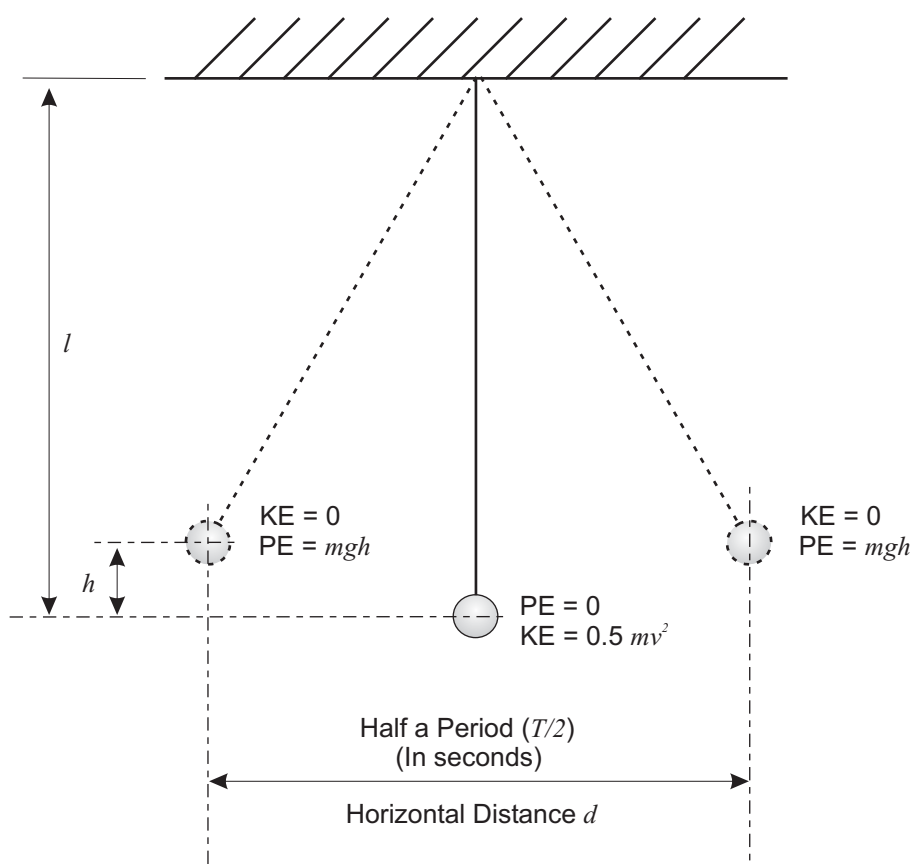


Figure 15 Energy in a Pendulum

$$PE = mgh$$

$$KE = 0.5mv^2$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Figure 15 and the Equations show the theory for a simple pendulum with an inelastic string (a string that doesn't stretch). The pendulum has a length (l) from its fixing point to the centre of gravity of the mass at its end. If you pull the mass to one side, it swings from one side to the other. The string or cord only has a tiny mass, so its effect is ignored.

The time to swing from one side to the other is half the swing 'period' ($T/2$). It completes a full swing or one period (T) when it returns to its starting position. Gravity (g) affects the swing, and along with the length, affects the time (period) it takes for the pendulum to swing. A longer pendulum or less gravity would give a longer period.

The pendulum shows a simple change of energy from potential energy (PE) at each end of its swing to kinetic energy (KE) as it moves from one side of its swing to the other. The pendulum transfers its potential energy to kinetic energy until it reaches the bottom of its swing. It then uses its kinetic energy to lift the pendulum up to the highest parts of its swing, changing back to potential energy. The maximum potential energy at each end of the swing is simply a product of the pendulum mass, relative height and the acceleration due to gravity. The maximum kinetic energy half way through the swing is half the product of the mass and the square of its velocity.

Remember that the potential energy is relative. The pendulum has no potential energy when its string is vertical with respect to its lowest possible state. However, the pendulum is usually above ground, so it does have some potential energy with respect to ground.

Good results depend on accurately measuring the distance from the fixing point to the centre of gravity of the mass, and the pendulum angles. You should not expect PE to be exactly the same as KE, due to the possible measurement errors, but results should be similar, confirming the theory.

In theory the pendulum will swing forever, but it loses energy to the air as it moves. The higher the angle you use (larger the oscillation), the more affect air resistance has on the experiment and gives poor results. So, use swing angles less than 40 degrees (from vertical) for best results.

Finding Velocity

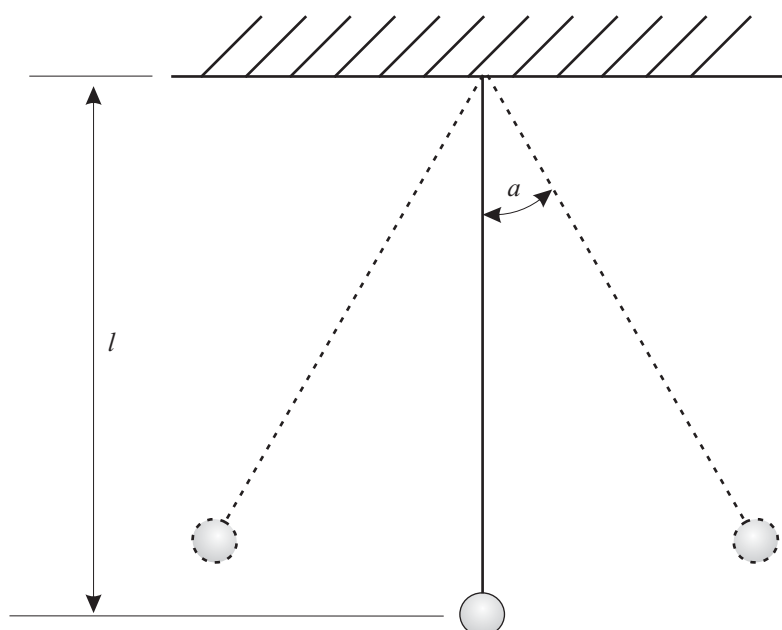


Figure 16 Pendulum Velocity

The pendulum does not follow a straight line as it moves from one side to the other of its swing. It moves through an angle. For this reason, you cannot use a simple distance and time calculation for its velocity. If you use a very long pendulum and very small angles, then the swing is almost straight and you can use the simple straight line method, but it will still be technically incorrect.

Using **straight line theory**, you find the average velocity (v_m) from the distance moved and the time taken to move over that distance:

$$v_m = \frac{d}{0.5T}$$

A more accurate calculation needs to use the angle of the swing (a) and the acceleration due to gravity (g), giving an angle based velocity (v_a):

$$v_a = \sqrt{2gl(1 - \cos a)}$$

Your experiment should help to prove that using the straight theory can cause extra errors in your theoretical calculations, especially for the shorter pendulums.

Energy in a Flywheel

Kinetic energy can be 'stored' in the moving parts of a machine (such as a flywheel). The larger the mass or speed of the moving object, the more kinetic energy it can store. For a flywheel, its kinetic energy is equal to half the product of its moment of inertia and its angular velocity squared.

$$KE = 0.5I_t\omega^2$$

The **moment of inertia** (I_f) of the flywheel is its resistance to a change in rotation. You find the value from

$$I_f = 0.5m_f r^2$$

However, the true total moment of inertia (I_t) of the flywheel and its connected parts is very difficult to find as it equal to the sum of the moment of inertia of the flywheel (I_f), and an additional amount for the metal shaft, the pulley and part of the bearing race that the shaft turns in. These Guidance Notes give a nominal value and you can find an average value from experiment (see later).

The **angular velocity** (ω) is the velocity of the flywheel in terms of the angle that it passes through in a given second, with the units of radians. It is similar to revolutions per minute or per second, but one radian is equal to 57.3 degrees, while one revolution is 360 degrees.

Therefore:

One revolution per second = 6.28 radians per second

$$1 \text{ rev.s}^{-1} = 6.28 \text{ rad.s}^{-1}$$

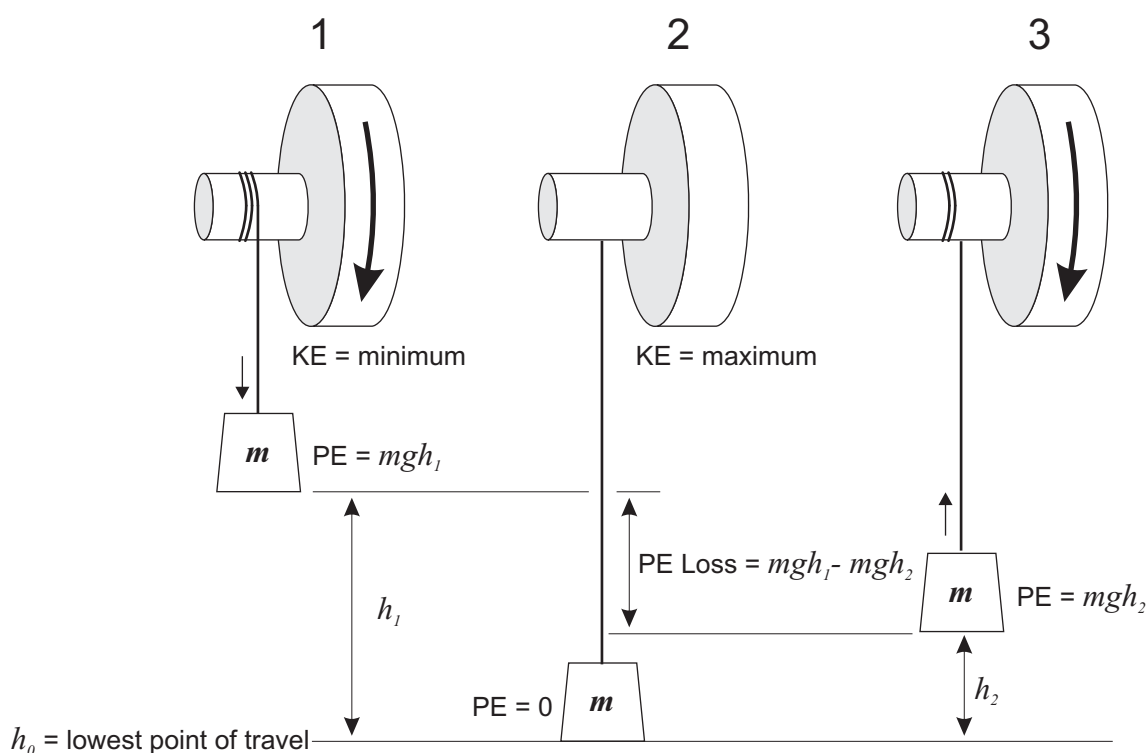


Figure 17 Energy in a Flywheel

See Figure 17. In step 1, the flywheel has not yet started to turn, so its Kinetic Energy $KE = 0$. The mass that turns the flywheel does so by giving up its initial potential energy (mgh_1) in turning the flywheel until it reaches the lowest point of its downwards travel (h_0) at step 2. At this point, the flywheel turns at its fastest

velocity and acceleration has reached a maximum. This is a conversion from potential energy into kinetic energy. Like the pendulum, the flywheel now continues using its kinetic energy to lift the mass back up (step 3).

If the mass returns back to its original height (h_1), then the flywheel will have given up all its kinetic energy back to the mass and $PE = KE$. However, some energy is lost in the bearings of the Bearing Block and other factors, so the mass will not return to its original height. Instead, it will reach a lower height (h_2). The loss is potential energy loss PE_{loss} .

$$PE_{loss} = mgh_1 - mgh_2$$

$$\text{or simplified } PE_{loss} = mg(h_1 - h_2)$$

The PE loss is the total loss through the downward and upward movement. To compare it more accurately with KE, which is for simply for the downward movement, you must multiply it by the height ratio, just as with the acceleration time (see later).

$$PE \text{ loss (adjusted)} = PE \text{ loss} \times h_1/(h_1+h_2)$$

Worked Example of Finding PE

Mass $m = 500 \text{ g}$ (0.5 kg)

$$g = 9.81 \text{ m.s}^{-2}$$

$$h_1 = 280 \text{ mm}$$
 (0.28 m)

$$PE = 0.5 \times 9.81 \times 0.28 = 1.37 \text{ J}$$

Worked Example of Finding PE loss

$$h_1 = 280 \text{ mm}$$
 (0.28 m) and $h_2 = 250 \text{ mm}$ (0.25 m)

$$m = 0.5 \text{ kg and } g = 9.81$$

$$PE \text{ loss} = 0.5 \times 9.81 \times (0.28 - 0.25) = 0.147 \text{ J}$$

$$PE \text{ loss (adjusted)} = 0.147 \times \{280/(280+250)\} = 0.078 \text{ J}$$

Acceleration and Velocity of the Mass and Flywheel

The acceleration of the falling mass (and therefore flywheel) reaches a maximum at step 2, where the mass has fallen through a distance h_1 , over the square of a given time t_f taken for the fall:

$$a = \frac{h_1}{t_f^2}$$

However, it is difficult to measure this 'falling time' t_f accurately, so for better results you measure the total time (t_t) taken to fall from step 1 and rise to step 3 and multiply by the ratio $h_1/(h_1+h_2)$. Therefore, for this kit:

$$t_f = t_t \times \frac{h_1}{h_1 + h_2}$$

The maximum linear velocity v of the falling mass is then simply a product of its maximum acceleration and time, so:

$$v = at_f$$

The falling mass **linear** velocity (v) is the same as the **angular** velocity (ω) of the perimeter of the pulley and of course the flywheel (they are all connected), but you must convert the linear velocity into angular velocity, so using the perimeter (circumference) of the pulley:

$$\omega = \frac{2\pi v}{c}$$

Where $c = 0.0408$ m (allowing for the diameter of the cord on the pulley)

Finding I_t from experiment

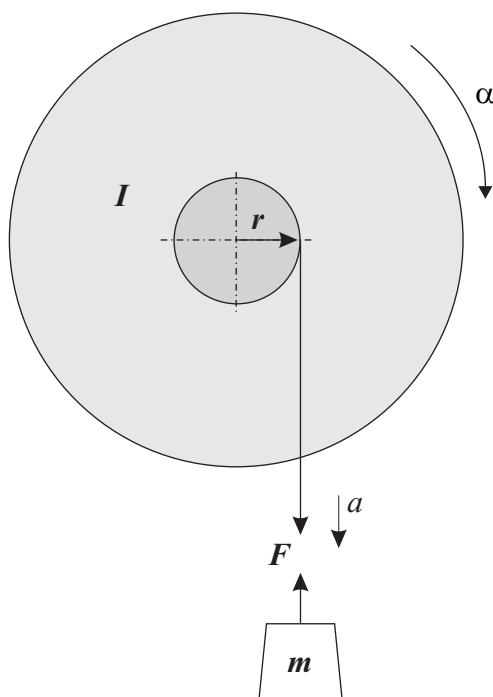


Figure 18 Acceleration and Inertia in a Flywheel

There are certain governing equations that the flywheel obeys, which you can use to find a reasonably accurate value for its total moment of inertia from experiment.

As shown in Figure 18, the force in the cord of a flywheel at any time is equal to the difference between the force created by the acceleration due to gravity on the mass (mg) and the force due to the accelerating mass (ma).

$$F = mg - ma$$

Assuming the cord does not stretch, then from simple moment theory, the torque in the cord is equal to the product of the force in the cord at the radius of the pulley.

$$\tau = Fr$$

$$\text{From this: } \tau = (mg - ma)r$$



The radius of the pulley must allow for the thickness of the cord used.

The torque due to the turning flywheel is equal to the product of the flywheel angular acceleration and the moment of inertia of the flywheel.

$$\tau = I\alpha$$

The angular acceleration in the flywheel is equal to the acceleration of the moving mass divided by the pulley radius.

$$\alpha = \frac{a}{r}$$

$$\text{From this: } \tau = I\frac{a}{r}$$

Therefore:

$$(mg - ma)r = I\frac{a}{r}$$

From this equation, following the general chart equation of $y = mx + c$, the gradient of the chart of $(mg - ma)r$ against a/r will give a good average value for I_t of the flywheel, based on actual results.

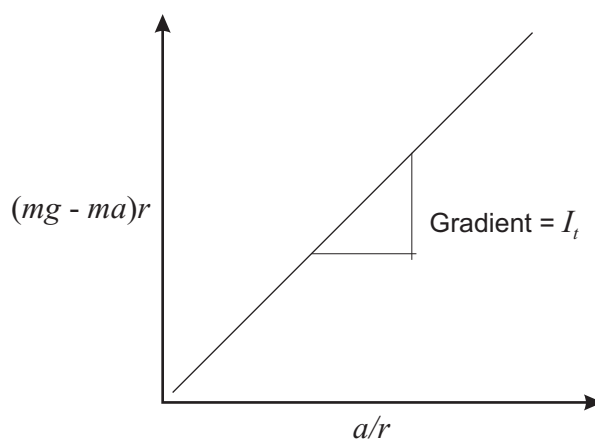


Figure 19 Chart to find Moment of Inertia

