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LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 4

LEFEL 4

EXAMINATION PERIOD
MAY 2026

CYFNOD ARHOLIADAU
MAI 2026

MODULE CODE: GOEN4001

MATHEMATICS TEST 2

SOLUTIONS

TIME: 2 HOURS

ANSWER ALL QUESTIONS

Equipment or materials required – calculators & formula book

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Question One (33 marks)

a) Given the following complex numbers:

$$Z_1 = 3 + 7j \quad Z_2 = 4 + 5j \quad Z_3 = 2 + 6j$$

Calculate the following, expressing your answers in rectangular form:

1. $Z_1 + Z_2$ [1]

$$Z_1 + Z_2 = 3 + 7j + 4 + 5j = \mathbf{7 + 12j}$$

2. $Z_2 + 3Z_3$ [2]

$$Z_2 + 3Z_3 = (4 + 5j) + 3(2 + 6j)$$

$$= 4 + 5j + 6 + 18j = \mathbf{10 + 23j}$$

3. $Z_2^2 + Z_3$ [3]

$$Z_2^2 + Z_3 = (4 + 5j)(4 + 5j) + (2 + 6j)$$

$$= 16 + 20j + 20j - 25 + 2 + 6j = \mathbf{-7 + 46j}$$

4. $Z_1 Z_2 Z_3$ [4]

$$Z_1 Z_2 Z_3 = (3 + 7j)(4 + 5j)(2 + 6j) = (12 + 15j + 28j - 35)(2 + 6j)$$

$$= (-23 + 43j)(2 + 6j) = -46 - 138j + 86j - 258 \\ = \mathbf{-304 - 52j}$$

5. $\frac{Z_1}{Z_2}$ [5]

$$\frac{Z_1}{Z_2} = \frac{3 + 7j}{4 + 5j}$$

$$= \frac{3 + 7j}{4 + 5j} \times \frac{4 - 5j}{4 - 5j} = \frac{12 - 15j + 28j + 35}{16 + 25} \quad [3]$$

$$\frac{Z_1}{Z_2} = \frac{47 + 13j}{41} \quad [1]$$

$$\frac{Z_1}{Z_2} = \frac{47}{41} + \frac{13}{41} j \quad [1]$$

6. Simplify the following expression in algebraic form only:

[3]

$$\frac{Z_1^2 - Z_2^2}{Z_1^2 - Z_1 Z_2}$$

$$Z_1^2 - Z_2^2 = (Z_1 - Z_2)(Z_1 + Z_2)$$

$$Z_1^2 - Z_1 Z_2 = Z_1(Z_1 - Z_2)$$

$$\frac{(Z_1 - Z_2)(Z_1 + Z_2)}{Z_1(Z_1 - Z_2)} = \frac{Z_1 + Z_3}{Z_1}$$

b) Solve the following quadratic equation, expressing your answers in Polar Form:

$$2x^2 - 6x + 9 = 0$$

Hence, verify one of your solutions by substitution into the original equation.

[9]

$$2x^2 - 6x + 9 = 0$$

$$a = 2 \quad b = -6 \quad c = 9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(9)}}{2(2)} \quad [1]$$

$$x = \frac{6 \pm \sqrt{-36}}{4} = \frac{6 \pm 6j}{4} \quad [1]$$

$$x = \frac{6 + 6j}{4} = \frac{3}{2} + \frac{3}{2}j$$

Or

$$x = \frac{6 - 6j}{4} = \frac{3}{2} - \frac{3}{2}j \quad [1]$$

$$x = 1.5 + 1.5j$$

$$|1.5 + 1.5j| = \sqrt{1.5^2 + 1.5^2} = 2.12$$

$$\phi = \tan^{-1}\left(\frac{1.5}{1.5}\right)$$

$$\phi = 45^\circ$$

$$x = 1.5 + 1.5j$$

$$x = 2.12 \angle 45^\circ \quad [2]$$

$$x = 1.5 - 1.5j$$

$$|1.5 - 1.5j| = \sqrt{1.5^2 + 1.5^2} = 2.12$$

$$\phi = \tan^{-1}\left(\frac{-1.5}{1.5}\right)$$

$$\phi = 45^\circ \text{ not a solution}$$

$$\phi = 360^\circ - 45^\circ$$

$$\phi = 315^\circ$$

$$x = 1.5 - 1.5j \text{ rectangular form}$$

$$x = 2.12 \angle 315^\circ \quad [2]$$

Verification (substitute one root)

Let:

$$x = 1.5 + 1.5j$$

$$2x^2 - 6x + 9$$

$$x^2 = (1.5 + 1.5j)^2 = 4.5j$$

$$2x^2 = 9j$$

$$-6x = -9 - 9j$$

$$2x^2 - 6x + 9 = 9j - 9 - 9j + 9 = 0 \quad [2]$$

Verified

Final Answers $x = 2.12 \angle 45^\circ$ or $x = 2.12 \angle 315^\circ$

- c) Use de Moivre's theorem to calculate $(5 + 3j)^6$, giving your answers in rectangular form.

[6]

$(5 + 3j)$ in polar form:

$$|(5 + 3j)| = \sqrt{5^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34} = 5.83 \quad [1]$$

$$\phi = \tan^{-1}\left(\frac{3}{5}\right)$$

$$\phi = 30.96^\circ \quad [1]$$

$$5 + 3j = 5.83 \angle 30.96^\circ$$

$$(5 + 3j) = 5.83(\cos 30.96^\circ + j \sin 30.96^\circ) \quad [1]$$

$$\text{calculate } [5.83(\cos 30.96^\circ + j \sin 30.96^\circ)]^6$$

$$\text{Applying de Moivre's theorem gives } 5.83^6(\cos 6 \times 30.96^\circ + j \sin 6 \times 30.96^\circ) \quad [1]$$

$$39265.5(\cos 185.76^\circ + j \sin 185.76^\circ) \quad [1]$$

$$(5 + 3j)^6 = -39067.25 - j3940.75 \quad [1]$$

Question Two (33 marks)

a) If $A = \begin{bmatrix} 1 & 3 & 3 \\ 0 & 5 & 4 \\ 5 & 2 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 2 & -2 \\ 1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 4 \\ -2 & 6 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 2 & 2 \\ 3 & 5 & 1 \\ 5 & 1 & 4 \end{bmatrix}$

Find:

1. The Determinant of A

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 0 & 5 & 4 \\ 5 & 2 & 6 \end{bmatrix} = 1(30 - 8) - 3(0 - 20) + 3(0 - 25) = 22 + 60 - 75 = 7$$

[3]

2. The Determinant of C

$$C = \begin{vmatrix} 5 & 4 \\ -2 & 6 \end{vmatrix} = 30 + 8 = 38$$

[1]

3. AB

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 5 & 4 \\ 5 & 2 & 6 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 2 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ 14 & -6 \\ 25 & 27 \end{bmatrix}$$

[3]

4. AD

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 5 & 4 \\ 5 & 2 & 6 \end{bmatrix} \begin{bmatrix} 0 & 2 & 2 \\ 3 & 5 & 1 \\ 5 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 24 & 20 & 17 \\ 35 & 29 & 21 \\ 36 & 26 & 36 \end{bmatrix}$$

[4]

5. CD

$$\begin{bmatrix} 5 & 4 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 0 & 2 & 2 \\ 3 & 5 & 1 \\ 5 & 1 & 4 \end{bmatrix} = \text{Not Possible}$$

[1]

6. Given the following matrix:

$$\omega = \begin{bmatrix} x & 3 & -4 \\ 4 & 4 & 2 \\ -2 & 5 & (x+2) \end{bmatrix}$$

Solve for x if the determinant of $\omega = 4$.

[5]

$$\det(\omega) = x(4x - 2) - 3(4x + 12) - 4(28)$$

$$\det(\omega) = 4x^2 - 14x - 148$$

$$\det(\omega) = 4$$

$$4x^2 - 14x - 148 = 4$$

$$4x^2 - 14x - 152 = 0$$

$$2x^2 - 7x - 76 = 0$$

$$x = \frac{7 \pm \sqrt{657}}{4}$$

$$x = \frac{7 + 25.63}{4} \approx 8.16$$

$$x = \frac{7 - 25.63}{4} \approx -4.66$$

b) Find the Inverse of the following matrix

$$A = \begin{bmatrix} 6 & 3 & 1 \\ 2 & 0 & 5 \\ 4 & 2 & 3 \end{bmatrix}$$

[10]

$$A = \begin{bmatrix} 6 & 3 & 1 \\ 2 & 0 & 5 \\ 4 & 2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$|A| = 6(0 - 10) - 3(6 - 20) + 1(4 - 0)$$

$$|A| = -60 + 42 + 4$$

$$|A| = -14 \neq 0$$

[2]

Calculations for Minors,

$$1. (0 \times 3) - (5 \times 2) = -10$$

$$2. (2 \times 3) - (5 \times 4) = -14$$

$$3. (2 \times 2) - (0 \times 4) = 4$$

4. $(3 \times 3) - (2 \times 1) = 7$
5. $(6 \times 3) - (4 \times 1) = 14$
6. $(6 \times 2) - (4 \times 3) = 0$
7. $(3 \times 5) - (1 \times 0) = 15$
8. $(6 \times 5) - (1 \times 2) = 28$
9. $(6 \times 0) - (3 \times 2) = -6$

$$\begin{bmatrix} -10 & -14 & 4 \\ 7 & 14 & 0 \\ 15 & 28 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \rightarrow \begin{bmatrix} -10 & 14 & 4 \\ -7 & 14 & 0 \\ 15 & -28 & -6 \end{bmatrix} \quad [6]$$

[6 marks (-1) for each wrong answer]

$$\text{adj } A = \begin{bmatrix} -10 & -7 & 15 \\ 14 & 14 & -28 \\ 4 & 0 & -6 \end{bmatrix} \quad [1]$$

$$A^{-1} = \frac{1}{-14} \begin{bmatrix} -10 & -7 & 15 \\ 14 & 14 & -28 \\ 4 & 0 & -6 \end{bmatrix} \quad [1]$$

c) Use Cramer's Rule to solve the following system of simultaneous equations

$$2x - y - 2z = -2$$

$$-x + y + z = 0$$

$$x - 2y + z = 8$$

[6]

$$2x - y - 2z = -2$$

$$-x + y + z = 0$$

$$x - 2y + z = 8$$

$$\begin{bmatrix} 2 & -1 & -2 \\ -1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 8 \end{bmatrix} \quad [1]$$

$$\Delta = \begin{vmatrix} 2 & -1 & -2 \\ -1 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 2(1+2) + 1(-1-1) - 2(2-1)$$

$$\Delta = \mathbf{2} \quad [1]$$

$$\Delta_x = \begin{vmatrix} -2 & -1 & -2 \\ 0 & 1 & 1 \\ 8 & -2 & 1 \end{vmatrix} = -2(1+2) + 1(0-8) - 2(0-8)$$

$$\Delta_x = \mathbf{2} \quad [1]$$

$$\Delta_y = \begin{vmatrix} 2 & -2 & -2 \\ -1 & 0 & 1 \\ 1 & 8 & 1 \end{vmatrix} = 2(0-8) + 2(-1-1) - 2(-8-0)$$

$$\Delta_y = \mathbf{-4} \quad [1]$$

$$\Delta_z = \begin{vmatrix} 2 & -1 & -2 \\ -1 & 1 & 0 \\ 1 & -2 & 8 \end{vmatrix} = 2(8+0) + 1(-8-0) - 2(2-1)$$

$$\Delta_z = \mathbf{6} \quad [1]$$

$$\mathbf{x} = \frac{\Delta_x}{\Delta} = \frac{\mathbf{2}}{\mathbf{2}} = \mathbf{1}$$

$$\mathbf{y} = \frac{\Delta_y}{\Delta} = \frac{\mathbf{-4}}{\mathbf{2}} = \mathbf{-2}$$

$$\mathbf{z} = \frac{\Delta_z}{\Delta} = \frac{\mathbf{6}}{\mathbf{2}} = \mathbf{3}$$

[1]

Question Three (34 marks)

a) Integrate the following:

1. $\int x(2x^3 + 3x^2 - 3x + 5) dx$

[3]

$$\begin{aligned}\int x(2x^3 + 3x^2 - 3x + 5) dx &= \int (2x^4 + 3x^3 - 3x^2 + 5x) dx \\&= \frac{2x^5}{5} + \frac{3x^4}{4} - \frac{3x^3}{3} + \frac{5x^2}{2} + C \\&= \frac{2}{5}x^5 + \frac{3}{4}x^4 - x^3 + 2.5x^2 + C\end{aligned}\quad [3]$$

(−1 mark for each wrong answer)

2. $\int (\sin 3x + \cos 5x + (3x + 1)(4x + 3)) dx$

[3]

$$\begin{aligned}&\int (\sin 3x + \cos 5x + (3x + 1)(4x + 3)) dx \\&= \int (\sin 3x + \cos 5x + 12x^2 + 9x + 4x + 3) dx \\&= \int (\sin 3x + \cos 5x + 12x^2 + 13x + 3) dx \\&= \frac{-\cos 3x}{3} + \frac{\sin 5x}{5} + 12\frac{x^3}{3} + 13\frac{x^2}{2} + 3x + C \\&= -\frac{1}{3}\cos 3x + \frac{1}{5}\sin 5x + 4x^3 + \frac{13}{2}x^2 + 3x + C\end{aligned}\quad [3]$$

(−1 mark for each wrong answer)

3. $\int_0^3 (4x^3 + 3x^2 + 2x - 3) dx$

[3]

$$\begin{aligned} & \int_0^3 (4x^3 + 3x^2 + 2x - 3) dx \\ & \left[= 4 \frac{x^4}{4} + 3 \frac{x^3}{3} + 2 \frac{x^2}{2} - 3x \right]_0^3 = [x^4 + x^3 + x^2 - 3x]_0^3 \\ & = [3^4 + 3^3 + 3^2 - 3(3)] - [0^4 + 0^3 + 0^2 - 3(0)] = \mathbf{108} \end{aligned} \quad [3]$$

(−1 mark for each wrong answer)

4. $\int_1^2 (e^{2x} + 2x^3 + 7) dx$

[3]

$$\begin{aligned} & \int_1^2 (e^{2x} + 2x^3 + 7) dx = \left[\frac{e^{2x}}{2} + 2 \frac{x^4}{4} + 7x \right]_1^2 = \left[\frac{e^{2x}}{2} + \frac{x^4}{2} + 7x \right]_1^2 \\ & = \left[\frac{e^{2 \times 2}}{2} + \frac{2^4}{2} + 7 \times 2 \right] - \left[\frac{e^{2 \times 1}}{2} + \frac{1^4}{2} + 7 \times 1 \right] = \mathbf{38.104} \end{aligned} \quad [3]$$

(−1 mark for each wrong answer)

5. $\int_1^4 (\sqrt{x} - 3)(\sqrt{x} + 5) dx$

[4]

$$\begin{aligned} & \int_1^4 (\sqrt{x} - 3)(\sqrt{x} + 5) dx \\ & (\sqrt{x} - 3)(\sqrt{x} + 5) = x + 2\sqrt{x} - 15 \end{aligned} \quad [1]$$

$$\int (x + 2\sqrt{x} - 15) dx = \frac{x^2}{2} + \frac{4}{3}x^{3/2} - 15x \quad [1]$$

$$\left[\frac{x^2}{2} + \frac{4}{3}x^{3/2} - 15x \right]_1^4 = \left(\frac{4^2}{2} + \frac{4}{3}(4)^{3/2} - 15(4) \right) - \left(\frac{1^2}{2} + \frac{4}{3}(1)^{3/2} - 15 \right)$$

$$\left[\frac{x^2}{2} + \frac{4}{3}x^{3/2} - 15x \right]_1^4 \approx -28.17 \quad [2]$$

6. Evaluate

$$\int_0^2 \frac{x+1}{x^2+3x+2} dx$$

[4]

$$\begin{aligned} \int_0^2 \frac{x+1}{x^2+3x+2} dx \\ \frac{x+1}{(x+1)(x+2)} &= \frac{1}{x+2} \\ \int_0^2 \frac{1}{x+2} dx &= [\ln |x+2|]_0^2 \\ &= \ln(4) - \ln(2) = \ln\left(\frac{4}{2}\right) = \ln 2 \end{aligned}$$

b) Using the substitution method, let $u = 4x^2 - 5$ find

$$\int x(4x^2 - 5)^3 dx$$

[6]

$$u = 4x^2 - 5$$

$$\frac{du}{dx} = 8x \quad du = 8x dx \quad dx = \frac{du}{8x} \quad [2]$$

$$\int x(4x^2 - 5)^3 dx = \int x \cdot u^3 \frac{du}{8x} = \frac{1}{8} \int u^3 du \quad [2]$$

$$\frac{1}{8} \int u^3 du = \frac{1}{8} \frac{u^4}{4} + C = \frac{1}{32} (4x^2 - 5)^4 + C \quad [2]$$

c) Use Integration 'by Parts' to find $\int x^2 e^x dx$.

Hence find the value of $\int_0^1 x^2 e^x dx$

[8]

$$\int x^2 e^x dx$$

$$u = x^2 \quad \Rightarrow \quad du = 2x dx$$

$$dv = e^x dx \quad \Rightarrow \quad v = e^x$$

$$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx$$

[3]

$$\int 2x e^x dx$$

$$u = 2x \quad \Rightarrow \quad du = 2 dx$$

$$dv = e^x dx \quad \Rightarrow \quad v = e^x$$

$$\int 2x e^x dx = 2x e^x - \int 2 e^x dx$$

$$= 2x e^x - 2 e^x$$

$$\int x^2 e^x dx = x^2 e^x - (2x e^x - 2 e^x)$$

$$\int x^2 e^x dx = e^x (x^2 - 2x + 2) + C$$

[3]

$$\begin{aligned} & \int_0^1 x^2 e^x dx \\ &= [e^x (x^2 - 2x + 2)]_0^1 \end{aligned}$$

At $x = 1$

$$e^1 (1 - 2 + 2) = e(1) = e$$

At $x = 0$

$$e^0 (0 - 0 + 2) = 2$$

$$\int_0^1 x^2 e^x dx = e - 2$$

[2]

END