

Electrical Engineering
Coursework

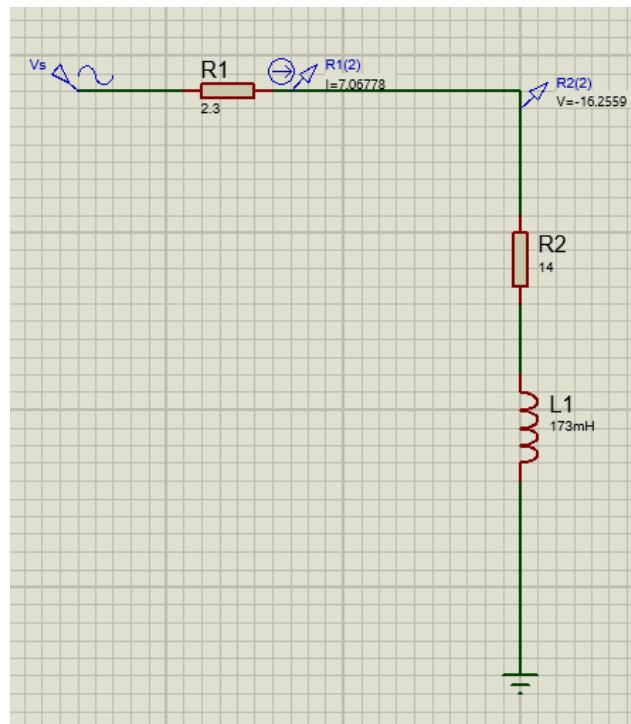
ACDA5001

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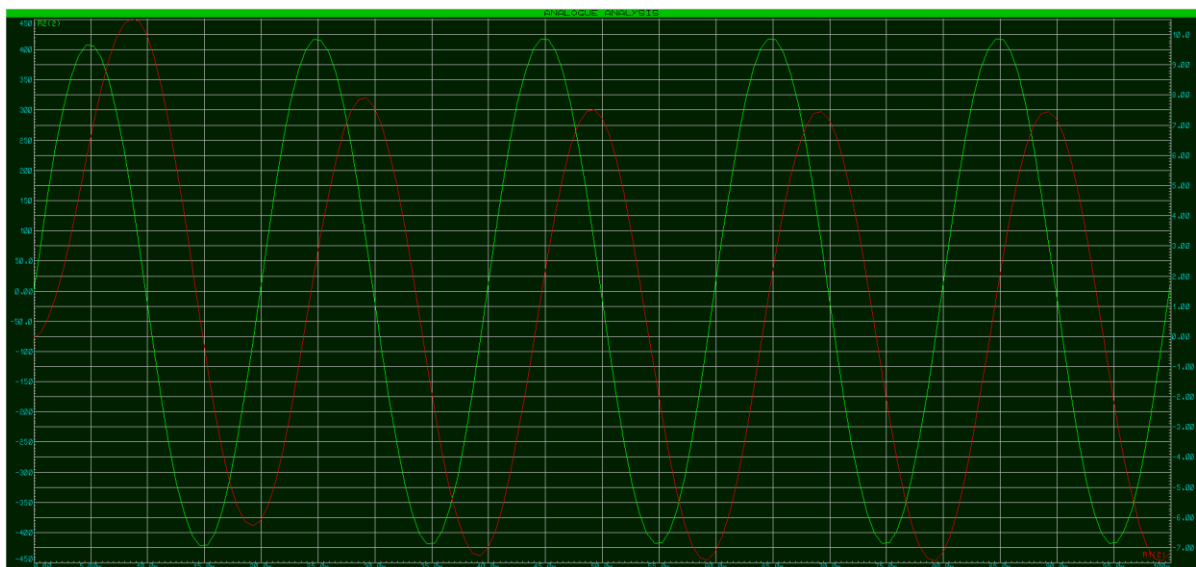
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Task 1

i) Initial model



ii) Graph and phase difference of circuit current and load voltage before power factor correction capacitor is added



Voltage peak time = 5.24ms

Current peak time = 9.09ms

$$\text{Phase difference} = \frac{t}{T} \times 360^\circ$$

$$t = 9.09 - 5.24 = 3.85\text{ms}$$

$$T = 45 - 25 = 20\text{ms}$$

$$\text{Phase difference} = \frac{3.85}{20} \times 360 = 69.3^\circ$$

iii)

$\frac{418}{\sqrt{2}} \cdot \frac{7.51}{\sqrt{2}}$	= 1 569.59 
$1\,569.59 \cdot \cos(69.3)$	= 554.81058
$1\,569.59 \cdot \sin(69.3)$	= 1 468.263596

Apparent power = 1.56959 kVA

Active power = 554.81 W

Reactive power = 1.46826 kVAR

iv)

$2\pi(50)(0.173)$	= 54.34955291
$\sqrt{14^2 + 54.34^2}$	= 56.11448654
$\frac{16.3}{56.11448654}$	= 0.2904775755

Power factor = 0.29

v)

$$2\pi(50)(0.173) = 54.34955291$$

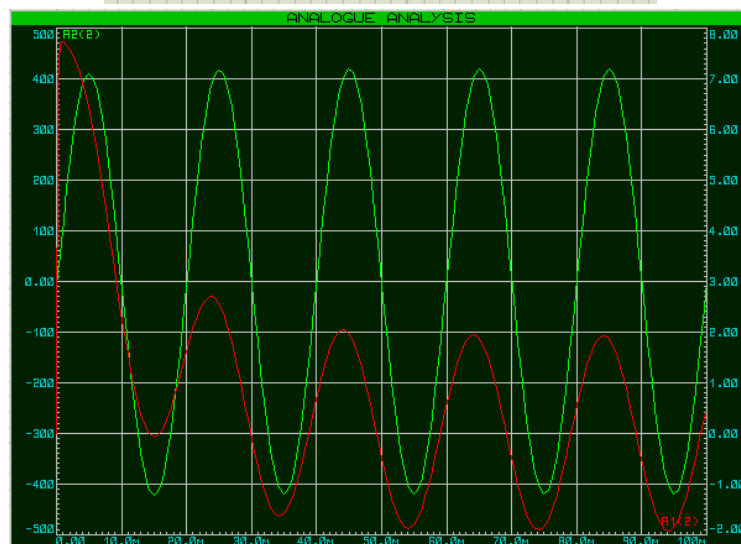
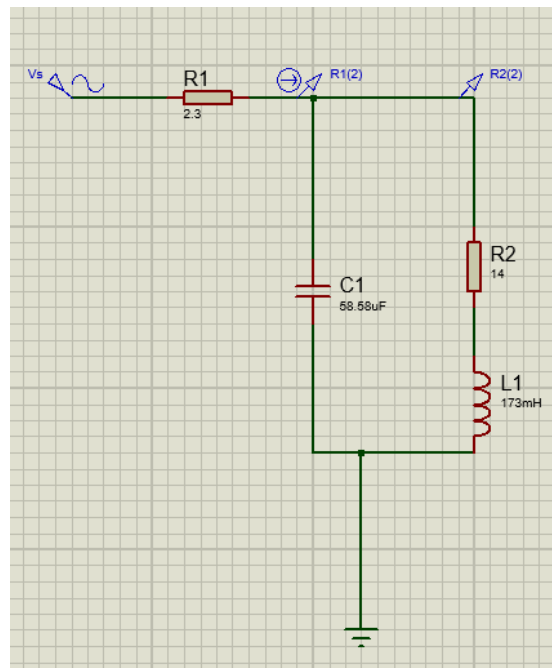
$$\frac{1}{2\pi(50)(54.34)} = 5.85774542 \times 10^{-5}$$

To cancel out inductive reactance:

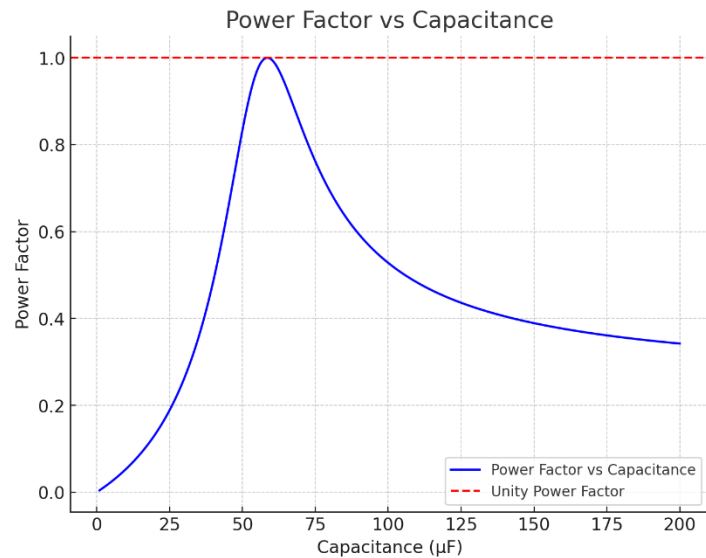
$$X_c = X_l = 54.35 \Omega$$

Capacitor value:

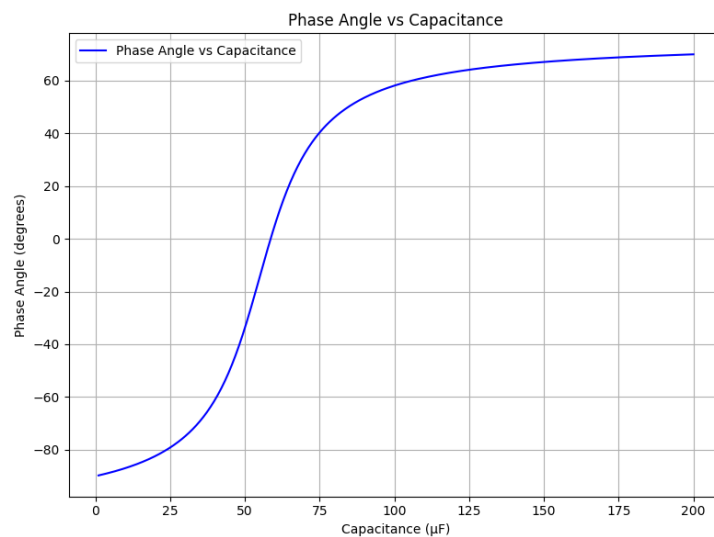
$$C = 58.58 \mu\text{F}$$



vi)



vii)



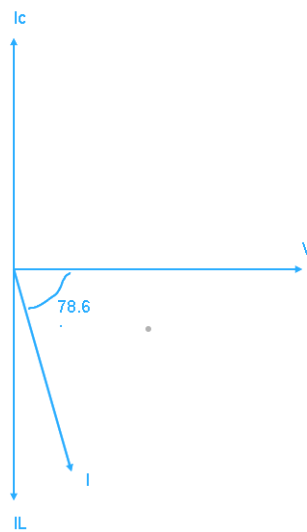
viii) As the power factor approaches 1, the voltage and current become nearly in phase, therefore the circuit behaves more like a resistive load. This means that the energy stored in the inductor and capacitor decreases by a significant amount. The capacitive and inductive reactance cancel each

other out as the power factor approaches 1. The energy stored in the inductor becomes negligible since the inductor has little effect on the circuit at this point.

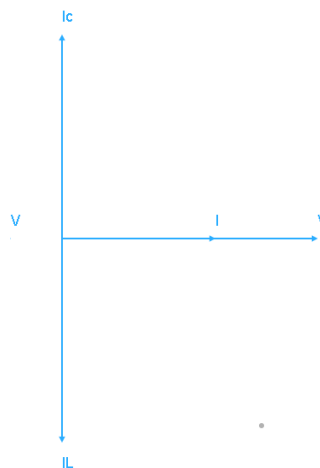
Task 2

The formula for resonant frequency is $\frac{1}{2\pi\sqrt{LC}}$. Using this formula, resonant frequency is 49.99451923, which is just rounded to 50 Hz as in most applications the difference is negligible. Hence there are only 3 phasor diagrams.

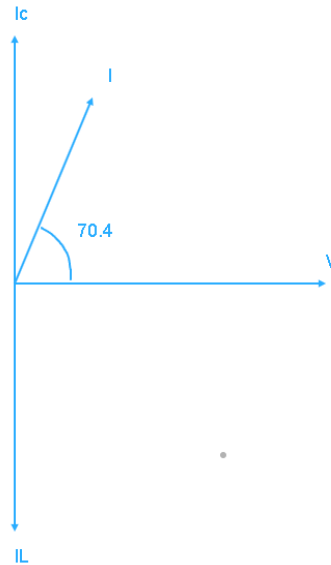
i)



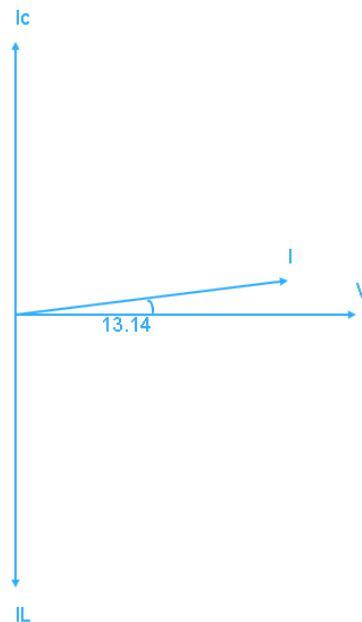
ii)



iii)



iv)



Below resonance, the circuit is inductively dominated, causing the current to lag the voltage by a phase angle of about 78.6° . At resonance, the circuit has the behaviour of a purely resistive one, resulting in maximum power transfer and energy efficiency. Above resonance, the capacitance causes the current to lead the voltage by a phase angle of about 70.4° .

In resonance, the voltage and current align, allowing efficient energy transfer, while below or above, reactive power reduces energy efficiency.

Task 3

- i) Since I added the capacitor in the circuit, it becomes less inductive which means it gets close to resonance, where impedance is minimised, and the power factor is improved. The power factor correction through the capacitor means the inductive reactance and capacitive reactance are now better balanced, which is something that is present in resonant circuits.
- ii) Adding the capacitor bring the circuit closer to resonance which improves the power factor. This enables the system to operate more efficiently and maximises power transfer to the load. With a power factor of 0.95 or over, most of the apparent power is now being used effectively; less energy is wasted through reactive power.
- iii) Since I corrected the power factor to 0.95, I have reduced the reactive power in the system. This results in lower apparent power for the same active power, which minimises losses in the system, which leads to a more efficient use of electrical energy.

Here are the following key calculations for apparent, active and reactive power:

$\frac{418}{\sqrt{2}} \cdot \frac{7.51}{\sqrt{2}}$	$= 1\,569.59$ 
$1\,569.59 \cdot \cos(69.3)$	$= 554.81058$
$1\,569.59 \cdot \sin(69.3)$	$= 1\,468.263596$

The formulas used were:

- Apparent power = $V_{rms} \times I_{rms} = VA$
- Active power = Apparent power $\times \cos(\text{phase angle}) = W$
- Reactive power = Apparent power $\times \sin(\text{phase angle}) = VAR$

Key calculations required for power factor:

$2\pi(50)(0.173)$	$= 54.34955291$
$\sqrt{14^2 + 54.34^2}$	$= 56.11448654$
$\frac{16.3}{56.11448654}$	$= 0.2904775755$

The formulas used were:

$$\text{Inductive impedance} = 2\pi fL$$

$$\text{Total impedance} = \sqrt{R^2 + X_L^2}$$

$$\text{Power factor} = R / Z$$

Last calculations for working out capacitor value required:

$2\pi(50)(0.173)$	$= 54.34955291$
$\frac{1}{2\pi(50)(54.34)}$	$= 5.85774542 \times 10^{-5}$

To cancel out inductive reactance:

$$X_C = X_L = 2\pi fL$$

Capacitor value:

$$C = \frac{1}{2\pi fL} = 58.58 \text{ uF}$$

Task 4

Abstract

This report evaluates the performance of an RLC circuit powered by a 300V RMS, 50Hz sinusoidal source (Vs). This circuit comprises of a 2.3 Ω source resistor (Rsrc), a 14 Ω load resistor (RL), a 173mH inductor(L) in series and a capacitor in parallel with the load resistor and inductor. The main tasks involved determining the capacitor value required to correct the power factor of the circuit to 0.95, analysing the voltage and current phase relationships at 25Hz below resonance, resonance, 25 Hz above resonance and 50Hz, while constructing phasor diagrams for each case. For this circuit, resonance was worked out to be 49.99451923 Hz, which is close enough to 50 Hz for most practical applications; the difference is negligible. Optimal transfer of energy occurs at 50Hz, while at non-resonant frequencies, reactive components influence phase angles and reduce energy efficiency.

Introduction

Alternating Current (AC) systems are the backbone of modern electrical engineering, providing power to residential, commercial and industrial facilities. Unlike direct current (DC), which flows in a single direction, AC alternates its direction in cycles, influenced by its frequency and amplitude. This allows AC to be transmitted efficiently over long distances, making it the standard for electrical power distribution. However, the influence of resistors, inductors and capacitors introduce complexities, such as phase shifts and reactive power, which affect overall system efficiency. [1]

RLC (Resistor, Inductor, Capacitor) circuits influence the behaviour of AC systems. Resistors dissipates energy as heat, inductors store energy in a magnetic field and capacitors store energy in an electric field. The combination of these components creates a frequency-dependent impedance (the total opposition to current in the circuit). The impedance is made up of reactance caused by the capacitors and inductors, as well as resistance. [2] At a specific frequency known as the resonant frequency, the inductive and capacitive reactance cancel each other, leaving only resistive components to determine the impedance. This frequency is given by $\frac{1}{2\pi\sqrt{LC}}$. [3]

The power factor in AC circuits is a measure of how effectively electrical power is converted into useful work. It is the ratio of active power (real power) to apparent power. A power factor of 1 suggests that there is a perfect alignment between the voltage and the current, maximising efficiency. However, in practical systems with inductive and capacitive loads, the power factor is often less than 1 due to phase shifts caused by reactive components. Low power factors result in energy losses, reduced system capacity and higher operational costs, which means corrective measures such as adding capacitors to cancel the inductive effects must be taken. [4]

Resonance occurs when the inductive and capacitive reactance are equal in magnitude but opposite in phase, resulting in a purely resistive impedance. At resonance, the circuit shows a few unique features, such as the current and voltage being in phase, eliminating reactive power, as well as its impedance being at its minimum, leading to maximum current flow for a given voltage. These properties make resonance crucial in the design of filters, oscillators and other important circuits in daily scenarios. [5]

Maximum power is delivered to the load when the load impedance matches the source impedance, according to the maximum power transfer theorem. In AC systems, this includes both the resistive and reactive components. However, in power systems, achieving exact impedance matching is less practical than improving the power factor to ensure as efficient of power delivery as possible. [6]

Research and practical applications showcase how important optimising AC circuit performance is. Studies have shown that power factor significantly reduces energy losses in industrial facilities. Furthermore, resonance principles are used in wireless power transfer systems and radio frequency circuits. By understanding these basic concepts, engineers can design efficient and reliable systems. [7]

This report explores the behaviour of an RLC circuit under various conditions, outlining the role of power factor correction in getting efficient energy transfer.

Method

Firstly, the initial circuit without the power factor correction capacitor was wired up. The source voltage was sinusoidal at 300Vrms and 50Hz frequency. It was then connected to a 2.3Ω source resistor, which was then wired up to a 14Ω load resistor and 173mH inductor, in series. This was all wired to ground too.

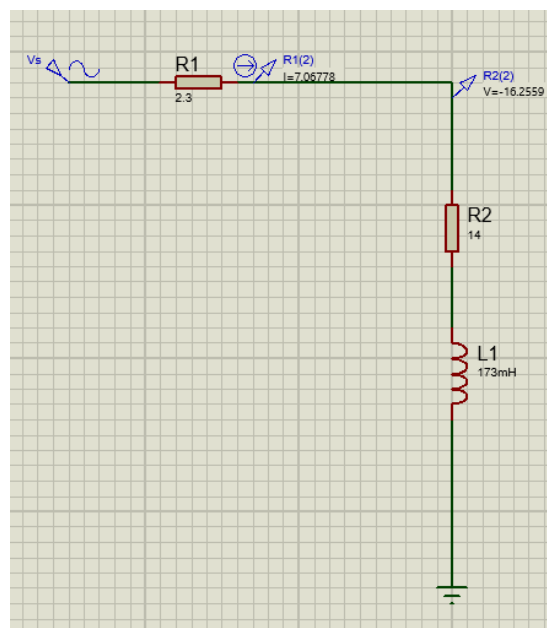


Figure 1: Initial out-of-phase circuit

Afterwards, using circuit current probe R1(2) and load voltage probe R2(2), an analogue analysis graph was created, with a stop time of 100ms.

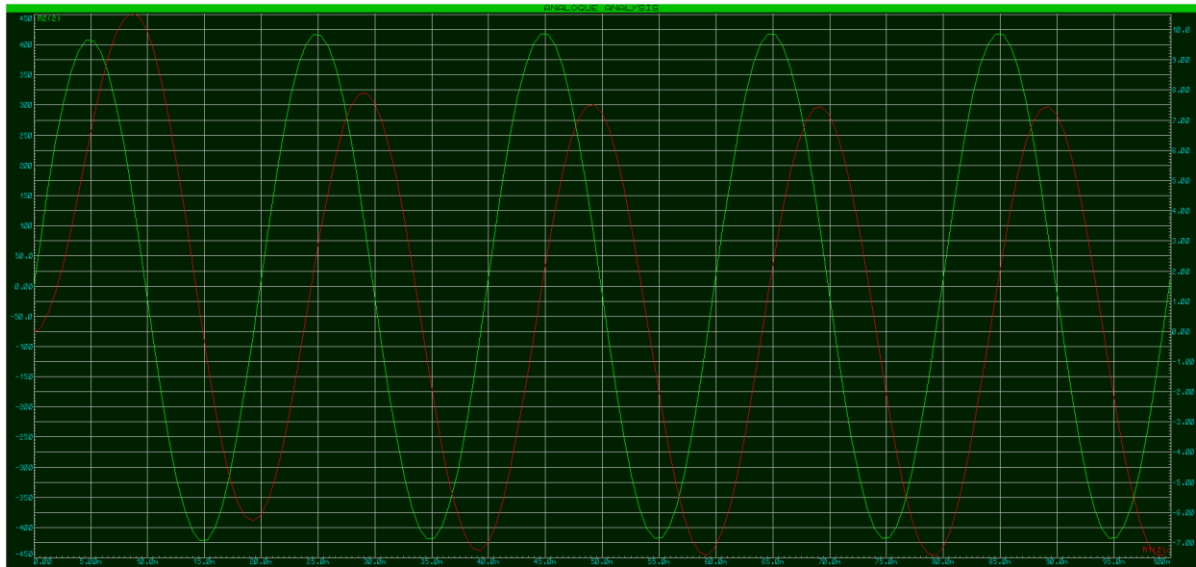


Figure 2: Voltage load (left axis) and circuit current (right axis) analogue analysis graph

Using the graph, the phase angle (in degrees) was calculated using the following formula [8]:

$$\text{Formula 1: Phase angle} = \frac{t}{T} \times 360^\circ$$

The phase angle worked out to be 69.3° . The time delay was 3.85ms, the length of one cycle was 20ms.

In the next step, the apparent, active and reactive power was calculated using the following formulas [9]:

$$\text{Formula 2: Apparent power}(S) = V_{rms} \times I_{rms}$$

$$\text{Formula 3: Active power}(P) = S \times \cos(\theta)$$

$$\text{Formula 4: Reactive power}(Q) = S \times \sin(\theta)$$

Apparent power worked out to be 1.56959 kVA, active power worked out to 554.81 W, reactive power worked out to 1.46826 kVAR.

The power factor was then worked out using the total impedance from load resistor and inductor reactance. The formulas used were[10]:

$$\text{Formula 5: Inductor reactance } (X_L) = 2\pi fL$$

$$\text{Formula 6: Total impedance } (Z) = \sqrt{R^2 + X_L^2}$$

$$\text{Formula 7: Power factor } (PF) = \frac{R}{Z}$$

Using the formulas, the inductor reactance worked out to be 54.35Ω , the impedance worked out to be 56.11Ω . The power factor worked out to be 0.29.

Afterwards, the power factor correction capacitor value was calculated using the following rearranged formula [10]:

$$\text{Formula 8: Capacitor value}(C) = \frac{1}{2\pi X_c}$$

“ X_c ” is the capacitor reactance. This was set to be the same value as the inductor reactance, so the capacitor value required for cancelling the effects of the inductor was calculated. The capacitor value worked out to be $58.58\mu\text{F}$.

This was then drawn up in proteus to ensure that the circuit was power factor corrected to a value of over 0.95.

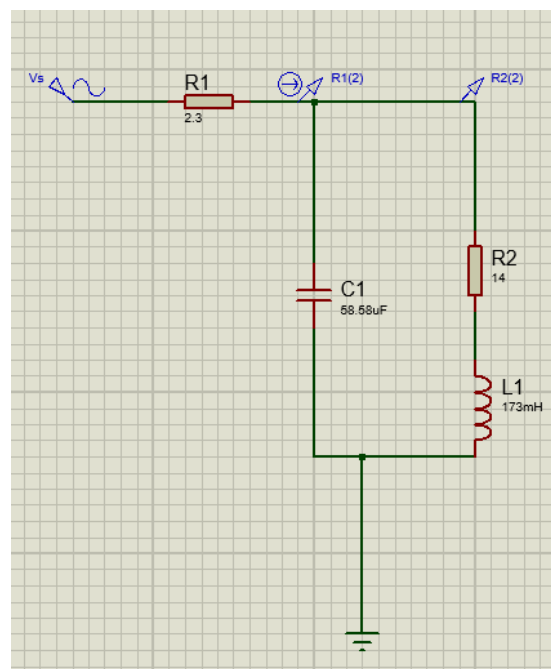


Figure 3: Power factor capacitor added in parallel to RL branch

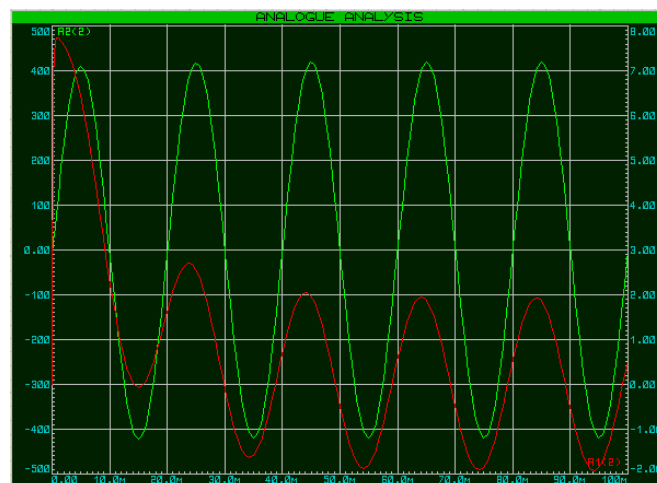


Figure 4: Power factor corrected graph

To test that the circuit was indeed power factor corrected, the graph was used again to work out the phase angle. The time delay was 0.73, cycle time was 20ms, so using the formula from earlier the phase angle worked out to be 13.14° . This time, the following formula was used for power factor[10]:

$$\text{Formula 9: } PF = \cos(\theta)$$

The power factor worked out to be 0.97.

The resonant frequency was then worked out to be used later for the phasor diagrams.

The formula for resonant frequency is as follows:

$$\text{Formula 10: } \text{Resonant frequency}(fr) = \frac{1}{2\pi\sqrt{LC}}$$

Using the formula, the resonant frequency worked out to be about 49.99Hz.

Using the formulas from earlier plus the formula from below, the following values were worked out:

$$\text{Formula 11: } I_{rms} = \frac{V_{rms}}{Z}$$

1. At 25 Hz, $X_L = 27.14\Omega$, $X_C = 108.91\Omega$, $Z = 83.11$, $I_{rms} = 3.60$ A, phase angle -78.6°
2. At resonance, $X_L = 54.35\Omega$, $X_C = 54.35\Omega$, $Z = 16.3\Omega$, $I_{rms} = 18.40$ A, phase angle 0°
3. At 75 Hz, $X_L = 81.63\Omega$, $X_C = 36.3$, $Z = 48.94$, $I_{rms} = 6.13$ A, phase angle 70.4°
4. At 50hz, phase angle was 13.14° .

Results

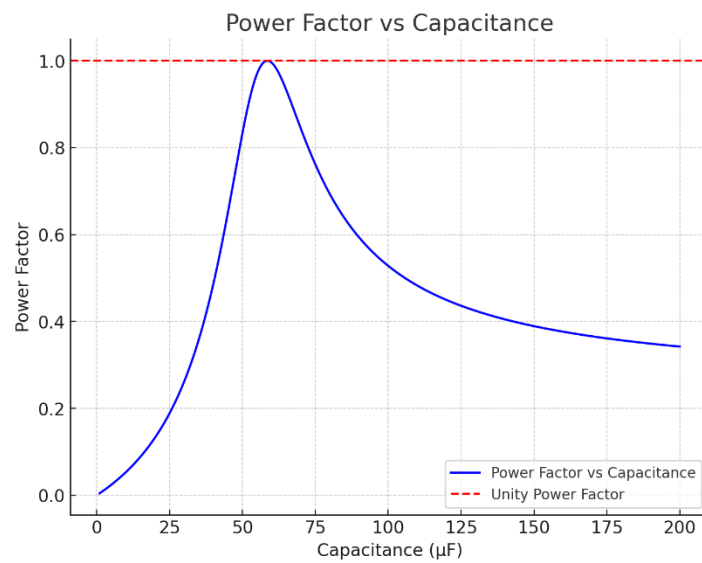


Figure 6: Power factor against capacitance graph

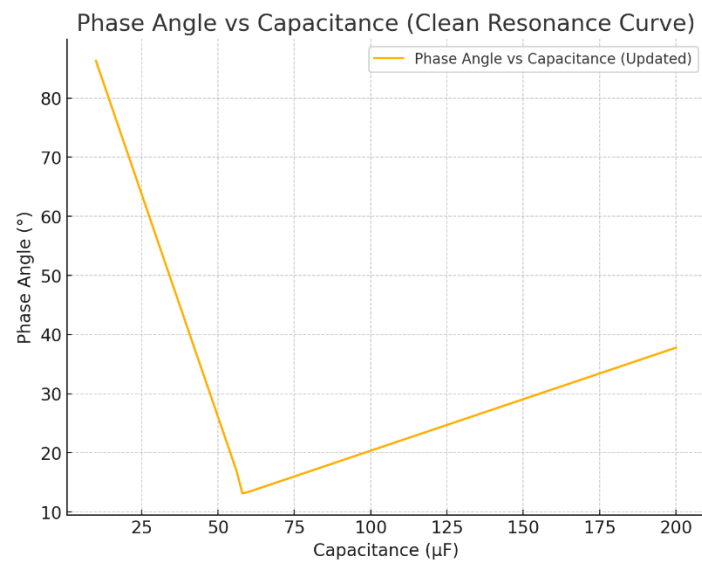


Figure 7: Phase angle against capacitance graph

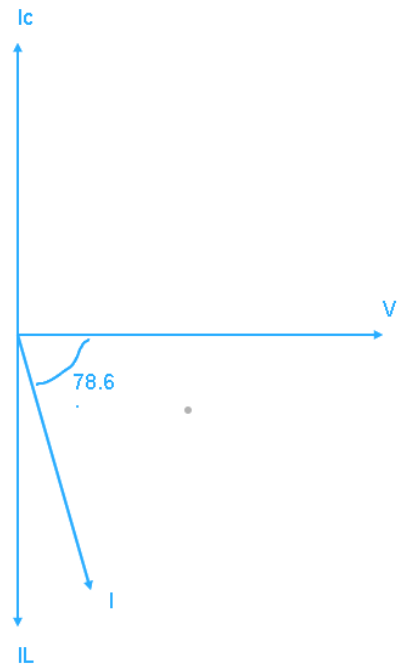


Figure 8: Phasor diagram below 25Hz resonance

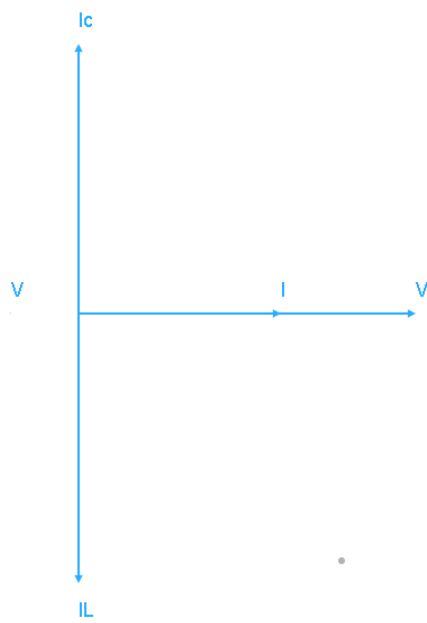


Figure 9: Phasor diagram at resonance

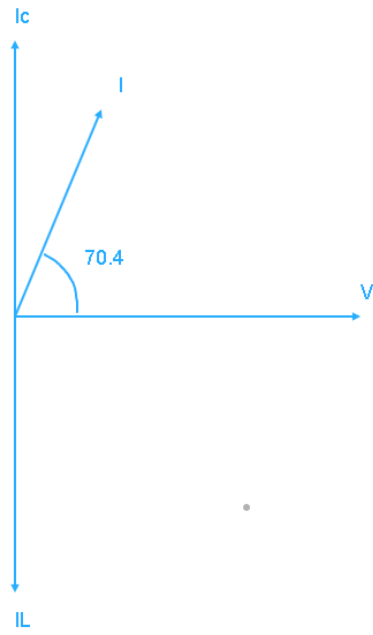


Figure 10: Phasor diagram at 25Hz above resonance

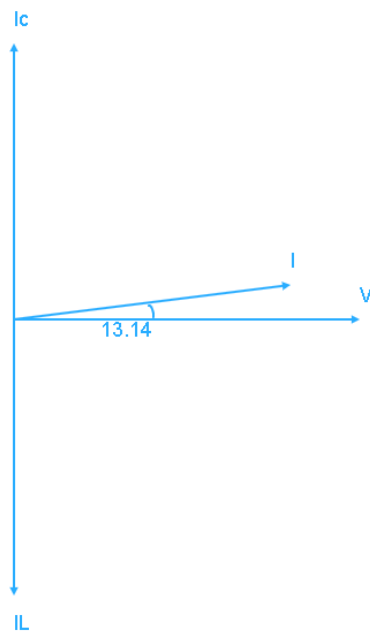


Figure 11: Phasor diagram at 50 Hz

Discussion

Resonance occurs when the inductive reactance and capacitive reactance are equal, meaning they cancel each other out. [5] For this circuit, the resonance frequency is 49.9945 Hz. At this point, the current is 18.4A and the total impedance of the inductor and capacitor becomes purely resistive. Furthermore, the circuit operates with minimal reactive power, and the total impedance will be at its minimum.

At 25Hz below resonance, as seen in figure 8 the phase angle is -78.6° , which indicates that the current lags the voltage, which is typical capacitive behaviour [11]. Furthermore, the total impedance of the system increases because the inductor and capacitor introduce significant reactance. This leads to a lower current (3.6A) compared to resonance. The increased impedance leads to the circuit's power output and energy efficiency being reduced. Additionally, the capacitance of the circuit at 25Hz below resonance leads to a phase mismatch, lowering the power factor to 0.19. This can lead to the source power not being used as efficiently as it can be.

At resonance, since the circuit achieves maximum power transfer from the source to the load, reactive power is absent which means that all the power supplied to the circuit is converted into useful work. Energy efficiency is maximised since no energy is cyclically stored and released by the inductor and capacitor.

At 25Hz above resonance, as seen in figure 10, the phase angle is 70.4° ; the inductive reactance is dominating, resulting in a net inductive behaviour in the circuit [12]. This leads to the current being decreased to 6.13A; total impedance is increased. Also, the phase angle becomes positive, indicating that the current leads the voltage, which is typical inductive behaviour. Furthermore, the inductive nature of the circuit at this frequency decreases the power factor due to the phase difference between the voltage and the current. Energy efficiency declines as reactive power increases, and less of the power source is converted into useful work.

At 50 Hz, since the frequency is a tiny bit above the resonant frequency, it leads to the phase angle to be 13.14° , the current still slightly decreases; the effects are not as severe as the other frequencies since the frequency difference is very minimal.

Furthermore, as seen in figure 7, it is evident that increasing capacitance reduces the phase angle. The graph shows a smooth decline in phase angle as the capacitance increases towards resonance. On the left side of the graph (lower capacitance), the phase angle is larger, which corresponds to capacitive behaviour, where the current lags the voltage. The steep slope in the region reflects the significant effect of capacitance changes on the phase angle, as the capacitive reactance counteracts the inductor's reactance. On the right side, the phase angle increases, which showcases inductive behaviour.

Conclusion

Initially, without the capacitor, the circuit showcases a low power factor (0.29) due to the inductive reactance. Calculations show that the inductive reactance at 50Hz is about 54.34Ω , leading to a high total impedance. By selecting a $58.58\mu\text{F}$ capacitor (which provides equal reactance to the inductor's reactance), the power factor improves to 0.97, indicating that the voltage and current are nearly in phase. Consequently, more of the input power is converted into useful work.

The resonant frequency, calculated at 49.9945Hz, is very close to the circuit's operating frequency. At resonance, the inductor and capacitor cancel each other out, leaving only the resistive component of the circuit. The current peaks at 18.4A and there is minimal reactive power. This maximises the energy transfer, supporting the principle that resonance enhances circuit efficiency.

Overall, the results highlight that ensuring resonance and proper power factor correction is critical for efficient energy transfer. By adjusting the capacitance, the circuit's phase angle and reactive power can be controlled, improving power quality and reducing losses.

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