

John Foster 2004393

i.

$$\begin{aligned}\epsilon_x &= 23 \times 10^{-6} & \epsilon_x &= \frac{\sigma_x}{E} \\ \epsilon_y &= -19 \times 10^{-6} & \epsilon_y &= \frac{\sigma_y}{E} \\ \nu &= 0.28 \\ E &= 170 \times 10^9 \text{ Pa}\end{aligned}$$

$$\epsilon_{max} \sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2)$$

$$\sigma_1 = \frac{170 \times 10^9}{1-(0.28)^2} (23 \times 10^{-6} + (0.28 \times -19 \times 10^{-6}))$$

$$\sigma_1 = 3.26 \text{ MPa} \therefore (\text{Tensile})$$

$$\sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1)$$

$$\sigma_2 = \frac{170 \times 10^9}{1-(0.28)^2} (-19 \times 10^{-6} + (0.28 \times 23 \times 10^{-6}))$$

$$\sigma_2 = -2.32 \text{ MPa} \therefore (\text{compressive})$$

ii.

$$\begin{aligned}\sigma_x &= 89 \text{ MPa } 89 \times 10^6 \text{ Pa} = \sigma_1 \\ \sigma_y &= -43 \times 10^6 \text{ Pa} = \sigma_2\end{aligned}$$

$$\epsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2) \quad \epsilon_1 = \frac{1}{170 \times 10^9} (89 \times 10^6 - (0.28 \times -43 \times 10^6))$$

$$\epsilon_1 = 4594.35 \times 10^{-6}$$

$$\epsilon_2 = \frac{1}{E} (\sigma_2 - \nu \sigma_1)$$

$$\epsilon_2 = \frac{1}{170 \times 10^9} (-43 \times 10^6 - (0.28 \times 89 \times 10^6))$$

$$\epsilon_2 = -399.53 \times 10^{-6}$$

2.

i.

$$\sigma_x = 103 \times 10^6 \text{ Pa}$$

$$\sigma_y = 209 \times 10^6 \text{ Pa}$$

$$\tau_{xy} = 125 \times 10^6 \text{ Pa}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\sigma_1 = \frac{103 \times 10^6 + 209 \times 10^6}{2} + \frac{\sqrt{(103 \times 10^6 - 209 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

$$\sigma_1 = 291.77 \times 10^6 \text{ Pa}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\sigma_2 = \frac{103 \times 10^6 + 209 \times 10^6}{2} - \frac{\sqrt{(103 \times 10^6 - 209 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

$$\sigma_2 = 20.23 \times 10^6 \text{ Pa}$$

ii.

$$\tau_{max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\tau_{max} = \frac{\sqrt{(103 \times 10^6 - 209 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

$$\tau_{max} = 135.77 \times 10^6 \text{ Pa}$$

$$\tau_{min} = \frac{-\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\tau_{min} = \frac{-\sqrt{(103 \times 10^6 - 209 \times 10^6)^2 + 4(125 \times 10^6)^2}}{2}$$

$$\tau_{min} = -135.77 \times 10^6 \text{ Pa}$$

iii.

~~Point 1~~

Point 1

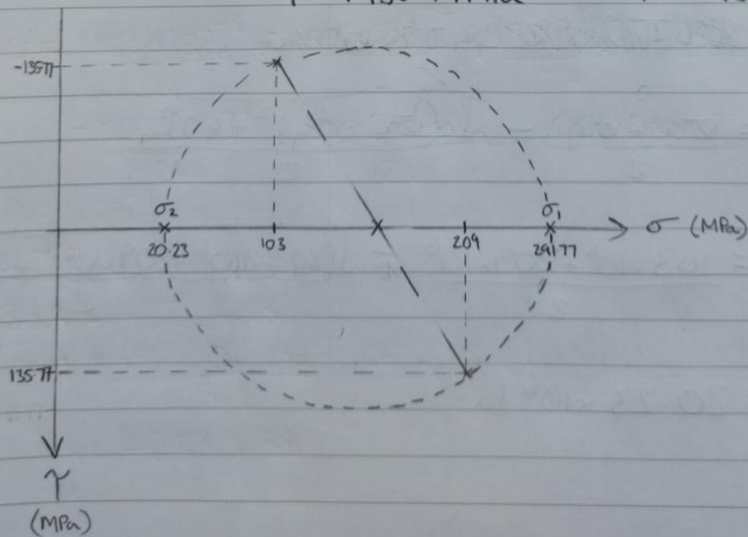
$$\sigma = +209 \text{ MPa}$$

$$\tau = +135.77 \text{ MPa}$$

Point 2

$$\sigma = +20.23 \text{ MPa}$$

$$\tau = -135.77 \text{ MPa}$$

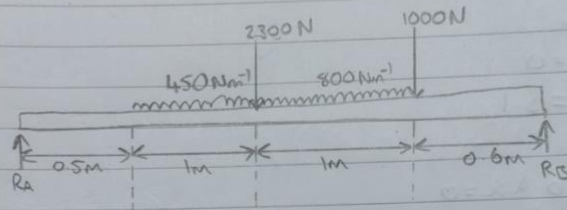


$$\tan 2\theta_P = \frac{2\gamma}{\sigma_x - \sigma_y} = \frac{2 \times 135.77 \times 10^6}{103 \times 10^6 - 209 \times 10^6}$$

$$\theta_P = \tan^{-1} \left(\frac{2 \times 135.77 \times 10^6}{103 \times 10^6 - 209 \times 10^6} \right)$$

$$\theta_P = 34.34^\circ$$

3.



$$R_A = 1969.4 \text{ N}$$

$$R_B = 2580.6 \text{ N}$$

$$EI = 1.39 \times 10^6 \text{ Pa}$$

~~XXXXXXXXXXXXXXXXXXXX~~

$$\text{i. } EI \frac{d^2 y}{dx^2} = 1969.4[x] - \frac{450[x-0.5]^2}{2} + \frac{450[x-1.5]^2}{2} - 2300[x-1.5] - \frac{800[x-1.5]^2}{2} + \frac{800[x-2.5]^2}{2} - 1000[x-2.5]$$

$$\text{ii } EI \frac{dy}{dx} = \frac{1969.4[x]^2}{2} - \frac{450[x-0.5]^3}{6} + \frac{450[x-1.5]^3}{6} - 2300[x-1.5]^2 - \frac{800[x-1.5]^3}{6} + \frac{800[x-2.5]^3}{6} - \frac{1000[x-2.5]^2}{2} + A$$

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iii

$$EI\eta = \frac{1969.4[x]^3}{6} - \frac{450[x-0.5]^4}{24} + \frac{450[x-1.5]^4}{24} - \frac{2300[x-1.5]^3}{6} - \frac{800[x-1.5]^4}{24} + \frac{800[x-2.5]^4}{24} - \frac{1000[x-2.5]^3}{6} + Ax + B$$

iv.

Boundary Conditions

$$y=0 \text{ when } x=0$$

$$y=0 \text{ when } x=3.1$$

$$\text{When } x=0, y=0 \text{ \& } x=0$$

$$0 = 0 - 0 + 0 - 0 - 0 + 0 - 0 + 0 + B$$

$$\therefore B=0$$

$$\text{when } y=0 \text{ \& } x=3.1$$

$$0 = \frac{1969.4[3.1]^3}{6} - \frac{450[3.1-0.5]^4}{24} + \frac{450[3.1-1.5]^4}{24} - \frac{2300[3.1-1.5]^3}{6} - \frac{800[3.1-1.5]^4}{24} + \frac{800[3.1-2.5]^4}{24} - \frac{1000[3.1-2.5]^3}{6} + 3.1A$$

$$0 = 7224.18 + 3.1A$$

$$3.1A = -7224.18$$

$$A = \frac{-7224.18}{3.1}$$

$$A = -2330.38$$

V. @ Centre $x = 1.55 \text{ m}$

$$1.39 \times 10^6 \eta = \frac{1969.4 [1.55]^3}{6} - \frac{450 [1.05]^4}{24} + \frac{450 [0.05]^4}{24} - \frac{800 [0.05]^4}{24} - 2330.38 [1.55]$$

$$1.39 \times 10^6 \eta =$$

$$1.39 \times 10^6 \eta = -2412.58$$

$$\eta = \frac{-2412.58}{1.39 \times 10^6}$$

$$\eta = -1.74 \times 10^{-3} \text{ m}$$

4.

i.

ds

$$D = 600 \times 10^{-3} \text{ m} \quad 280 \times 10^{-3} \text{ m}$$

$$L = 600 \times 10^{-3} \text{ m}$$

$$\eta_L = 0.59$$

$$\eta_C = 0.63$$

$$P = 14.3 \times 10^5 \text{ Pa} \cdot \text{Nm}^{-2}$$

$$t = ?$$

$$\sigma = 150 \times 10^6 \text{ Pa}$$

Circumfer. Circ.

$$\sigma_C = \frac{pD}{2t\eta_C} = \frac{14.3 \times 10^5 \times 280 \times 10^{-3}}{2t \times 0.63}$$

$$t = \frac{pD}{2\eta_C \sigma_C} = \frac{14.3 \times 10^5 \times 280 \times 10^{-3}}{2 \times 0.59 \times 150 \times 10^6}$$

$$t = 2.26 \times 10^{-3} \text{ m}$$

ii

Long.

$$\sigma_L = \frac{pD}{4t\eta_L} \quad t = \frac{pD}{4\eta_L \sigma_L} \quad t = \frac{14.3 \times 10^5 \times 280 \times 10^{-3}}{4 \times 0.63 \times 150 \times 10^6}$$

$$t = 1.06 \times 10^{-3} \text{ m}$$

iii. $t_{\min} = 2.26 \times 10^{-3} \text{ m}$

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5.
i.

$$\begin{aligned}L &= 1.39 \text{ m} \\ d_o &= 19.05 \times 10^{-3} \text{ m} \\ d_i &= 16.65 \times 10^{-3} \text{ m} \\ E &= 200 \times 10^9 \text{ Pa}\end{aligned}$$

$$A = \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$A = \frac{\pi}{4} ((19.05 \times 10^{-3})^2 - (16.65 \times 10^{-3})^2)$$

$$A = 6.73 \times 10^{-5} \text{ m}^2$$

$$I = \frac{\pi}{64} (d_o^4 - d_i^4)$$

$$I = \frac{\pi}{64} ((19.05 \times 10^{-3})^4 - (16.65 \times 10^{-3})^4)$$

$$I = 2.69 \times 10^{-9} \text{ m}^4$$

ii.

$n = 0.25$ as it is fixed on 1 end and free on the other

iii.

$$F_{cr} = \frac{n\pi^2 EI}{L^2} = \frac{0.25 \times (\pi)^2 \times 200 \times 10^9 \times 2.69 \times 10^{-9}}{1.39^2}$$

$$F_{cr} = 687.06 \text{ N}$$

iv.

$$SR = \frac{L}{K}$$

$$K = \frac{I}{A}$$

$$K = \frac{2.69 \times 10^{-9}}{6.73 \times 10^{-5}}$$

$$K = 4 \times 10^{-5}$$

$$SR = \frac{1.39}{4 \times 10^{-5}}$$

$$SR = 34750$$

8.