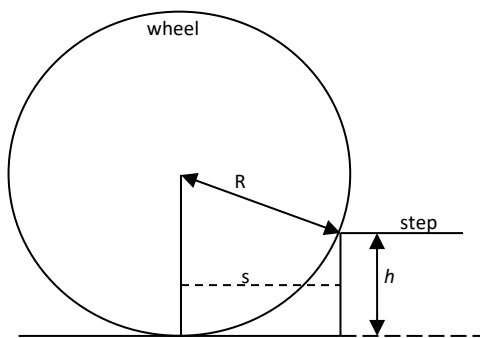


Moving up a Step

When a motorcycle moves up a step a reaction to the suspension occurs. Theoretically the following equation calculates this, but it is specifically for a motorcycle without suspension.

$$s = \sqrt{h(2R - h)}$$

The method of getting to the s value is taken from the following diagram;



$$\cos\theta = \frac{R - h}{R}$$

$$\sin\theta = \frac{s}{R}$$

$$s = R\sin\theta$$

$$s = R\sqrt{1 - \cos^2\theta}$$

$$s = R\sqrt{1 - \frac{(R - h)^2}{R^2}}$$

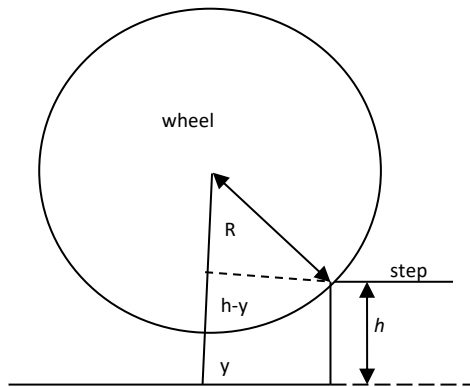
$$s = \sqrt{R^2 - (R - h)^2}$$

$$s = \sqrt{R^2 - R^2 + 2Rh - h^2}$$

$$s = \sqrt{2Rh - h^2}$$

$$s = \sqrt{h(2R - h)}$$

At some distance up the step, we get:



$$\cos\theta = \frac{R - (h - y)}{R}$$

$$\sin\theta = \frac{s}{R}$$

$$s = R\sin\theta$$

$$s = R\sqrt{1 - \cos^2\theta}$$

$$s = R\sqrt{1 - \frac{(R - (h - y))^2}{R^2}}$$

$$s = \sqrt{R^2 - (R - (h - y))^2}$$

$$s = \sqrt{R^2 - R^2 - 2R(h - y) - (h - y)^2}$$

$$s = \sqrt{-(h^2 + y^2 - 2hy - 2Rh + 2Ry)}$$

$$s = \sqrt{-(h - y)(-2R + h - y)}$$

$$s = \sqrt{(h - y)(2R - h + y)}$$

As a motorcycle travels forward the velocity it travels at changes at the point in which the wheel is moving up the step. The horizontal force is transferred into a vertical force as the wheel moves up the step, distance s and then back to a horizontal force when the wheel has reached the top of the step.

Consider a motorcycle with no suspension. The time the wheel takes to climb the step is a ratio;

$$\Delta t = \frac{s}{V}$$

so

$$s = \sqrt{(h-y)(2R-h+y)}$$

The square root is a pain, so:

$$s^2 = (h-y)(2R-h+y)$$

$$s^2 = 2hR - h^2 + hy - 2yR + hy - y^2$$

Velocity is

$$V = \frac{ds}{dt}$$

So differentiating, we get:

$$2s \frac{ds}{dt} = -2R \frac{dy}{dt} + h \frac{dy}{dt} - 2y \frac{dy}{dt} + h \frac{dy}{dt}$$

$$2sV = 2\{h-y-R\} \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{sV}{h-y-R}$$

$$\frac{dy}{dt} = \frac{V\sqrt{(h-y)(2R-h+y)}}{h-y-R}$$

$$\frac{dy}{dt} = V \sqrt{\frac{(h-y)(2R-h+y)}{(h-y-R)^2}}$$

$$\frac{dy}{dt} = V \sqrt{\frac{2hR - h^2 + hy - 2yR + hy - y^2}{\{R - (h - y)\}^2}}$$

$$\frac{dy}{dt} = V \sqrt{\frac{-h^2 - y^2 + 2hy + 2Rh - 2Ry}{\{R - (h - y)\}^2}}$$

$$\frac{dy}{dt} = V \sqrt{\frac{2R(h - y) - (h - y)^2}{\{R - (h - y)\}^2}}$$

In Vittore's book – He has:

$$\frac{dy}{dt} = V \sqrt{\left\{\frac{R}{R - (h - y)}\right\}^2 - 1}$$

So we need to get that!

$$\frac{dy}{dt} = V \sqrt{\frac{R^2 - R^2 - 2R(h - y) - (h - y)^2}{\{R - (h - y)\}^2}}$$

$$\frac{dy}{dt} = V \sqrt{\frac{R^2 - (R - (h - y))^2}{\{R - (h - y)\}^2}}$$

$$\frac{dy}{dt} = V \sqrt{\frac{R^2}{\{R - (h - y)\}^2} - \frac{(R - (h - y))^2}{\{R - (h - y)\}^2}}$$

$$\frac{dy}{dt} = V \sqrt{\left\{\frac{R}{R - (h - y)}\right\}^2 - 1}$$

Now – What about the acceleration?

Starting from:

$$2sV = 2\{h - y - R\} \frac{dy}{dt}$$

$$sV = \{h - y - R\} \frac{dy}{dt}$$

$$sV = -\{R - (h - y)\} \frac{dy}{dt}$$

$$V \frac{ds}{dt} = -\{R - (h - y)\} \frac{dy}{dt}$$

$$V \frac{ds}{dt} = -\{R - (h - y)\} \frac{d^2y}{dt^2} - \left(\frac{dy}{dt}\right)^2$$

$$V^2 + \left(\frac{dy}{dt}\right)^2 = -\{R - (h - y)\} \frac{d^2y}{dt^2}$$

Substitute in the dy/dt from earlier:

$$\frac{dy}{dt} = V \sqrt{\left\{\frac{R}{R - (h - y)}\right\}^2 - 1}$$

So:

$$\left(\frac{dy}{dt}\right)^2 = V^2 \left\{\frac{R}{R - (h - y)}\right\}^2 - 1^2$$

Stick that in:

$$V^2 + V^2 \left\{\frac{R^2}{\{R - (h - y)\}^2} - 1\right\} = -\{R - (h - y)\} \frac{d^2y}{dt^2}$$

$$V^2 \left\{ 1 + \frac{R^2}{\{R - (h - y)\}^2} - 1 \right\} = -\{R - (h - y)\} \frac{d^2 y}{dt^2}$$

$$\frac{V^2 R^2}{\{R - (h - y)\}^2} = -\{R - (h - y)\} \frac{d^2 y}{dt^2}$$

$$\frac{-V^2 R^2}{\{R - (h - y)\}^2} = \{R - (h - y)\} \frac{d^2 y}{dt^2}$$

$$\frac{-V^2 R^2}{\{R - (h - y)\}^3} = \frac{d^2 y}{dt^2}$$

$$\frac{d^2 y}{dt^2} = \frac{V^2 R^2}{\{R - (h - y)\}^3}$$

$$\frac{d^2 y}{dt^2} = V^2 \frac{R^2}{\{R - (h - y)\}^3}$$

So we can now see that the advancement of the wheel corresponds to the advancement of the motorcycle, where the velocity and vertical acceleration of the wheel are:

$$\dot{y} = V \sqrt{\left(\frac{R}{R - (h - y)} \right)^2 - 1} \quad \ddot{y} = V^2 \frac{R^2}{[R - (h - y)]^3} \quad 0 < y < h$$

When suspension is included the time taken to climb the step depends on the suspension set up and the pressure in the tyre so this example will only work for non-suspension motorcycles (eh what?).

Example: moving up a step

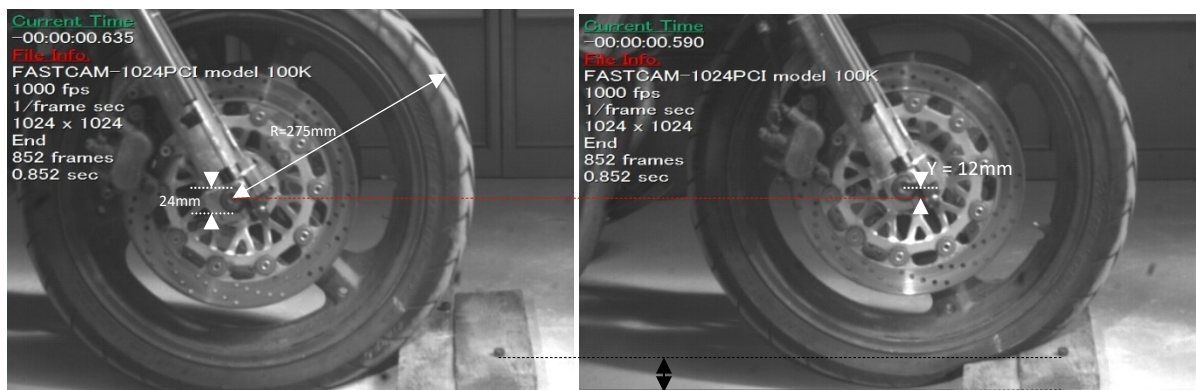
Consider Tim's Honda Super Four. Wheel radius is 275mm and the step is 40mm high. In test 1, run 1 the bike was moving at a velocity of 3.18ms.

$$s = \sqrt{0.04 (2 \times 0.275 - 0.04)}$$

$$s = \sqrt{0.04 (0.55 - 0.04)}$$

$$s = 0.0204$$

Y is the distance the wheel has moved, measured by the movement vertically of the spindle on the wheel. Using the high-speed camera, the spindle movement can be seen; knowing that the radius of the wheel is 275mm the spindle displacement can be measured.



The spindle bolt is 24mm so this measurement can be used to gauge the distance the spindle has travelled vertically. This gives the value of y. By lining up the photos from the camera it can be seen that the spindle moved a vertical height of 12mm.

Using this information, the 1st and 2nd differentials of y can be calculated;

$$\dot{y} = 3.18 \sqrt{\left(\frac{0.275}{0.275 - (0.04 - 0.012)}\right)^2 - 1}$$

$$\dot{y} = 3.18 \sqrt{\left(\frac{0.275}{0.247}\right)^2 - 1} = 1.556$$

$$\dot{y} = 1.556 \text{ ms}^{-1}$$

And to move on to the acceleration;

$$\ddot{y} = 3.18^2 \frac{0.275^2}{[0.275 - (0.04 - 0.012)]^3}$$

$$\ddot{y} = 10.11 \times \frac{0.076}{0.015} = 51.23$$

$$\ddot{y} = 51.23 \text{ ms}^{-2}$$