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A practical evaluation on forces in equilibrium:
Spring elasticity and string angle

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1. Introduction

This experiment was performed to find out the relation between the forces in equilibrium, spring length and string angle attained by suspending mass from the string, which is connected to the springs.

2. Apparatus

The apparatus needed to perform this experiment includes:

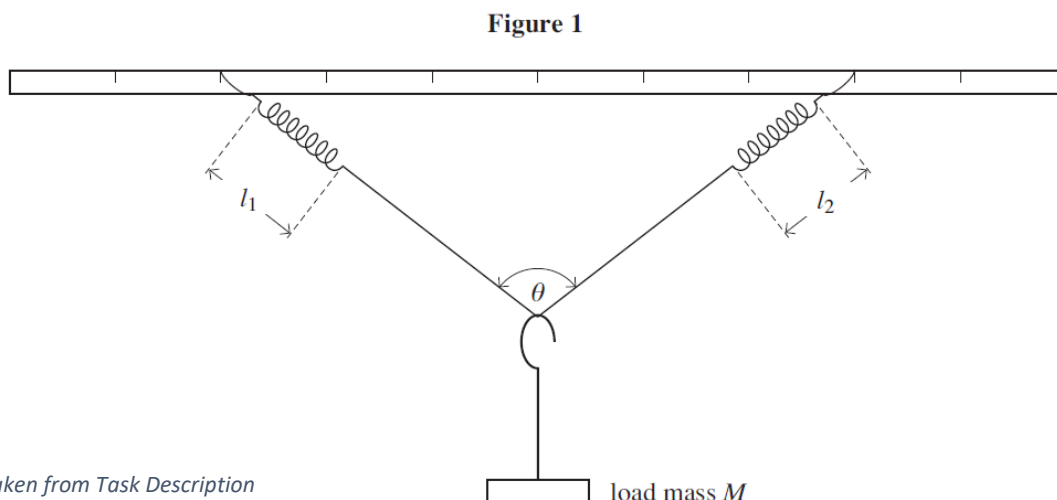
- 1 Retort stand
- 1 Boss head clamp
- 1 spirit level
- 1 1 metre ruler
- 1 piece of string
- Tape
- 2 springs
- 1 50g weight hanger
- 11 50g masses
- 1 Ruler which measures in millimetres
- 1 Protractor

2.1 Apparatus setup

These parts should be setup in the following way:

On a level surface clamp the ruler into the boss head clamp and unto the retort stand at eye level. Make sure the ruler is parallel to the table and use the spirit level to adjust the ruler, so it is perfectly level. Attach the pieces of string at the 20 centimetre and 80 centimetre mark on the ruler with tape. Connect these two springs with the piece of string. To this string attach the weight hanger.

Once this setup is finished it should look like this:



3. Instructions

- 1) Add a mass of 50g, so the total mass (M) is 100g, in the middle of the string and measure the lengths of the springs (l1 and l2) with the ruler and the angle of the string (θ) and the angle with the protractor and record the results in a table.
- 2) Repeat step 1 adding increments of 100g every time until a maximum mass of 600g is reached. Wait until the springs stop oscillating to get more accurate results.
- 3) Repeat the whole experiment to have a second set of data, to increase the reliability of the results.
- 4) Calculate the mean of the lengths of springs ($L = \frac{l1+l2+l3+l4}{4}$) (Equation 1) and the mean of the angle ($\theta_m = \frac{\theta1+\theta2}{2}$) (Equation 1-1) for every value of mass.
- 5) With θ_m calculate $\frac{M}{\cos(\frac{\theta_m}{2})}$ (Equation 2) for every value of mass.
- 6) Now create a scatter graph with the Formula on the x-axis and L on the y-axis. Draw the line of best fit.
- 7) Calculate the uncertainty and the uncertainty percentage of the spring length and string angle and portray them in tables.

4. Results

Table 1: Collected Data

	Reading 1			Reading 2			Derived Data		
Mass (g)	l1 (mm)	l2 (mm)	$\theta1$ (°)	l3 (mm)	l4 (mm)	$\theta2$ (°)	L (mm)	θ_m (°)	M/cos($\theta_m/2$)
0	50	50	180	50	50	180	50	180	0
100	80	70	110	85	70	115	76.25	112.5	179.995
200	110	100	100	115	95	101	105	100.5	312.774
300	140	125	92	145	124	95	133.5	93.5	437.839
400	170	145	89	165	146	89	156.5	89	560.813
500	190	170	81	190	165	80	178.75	80.5	655.108
600	210	195	79	210	190	79	201.25	79	777.58

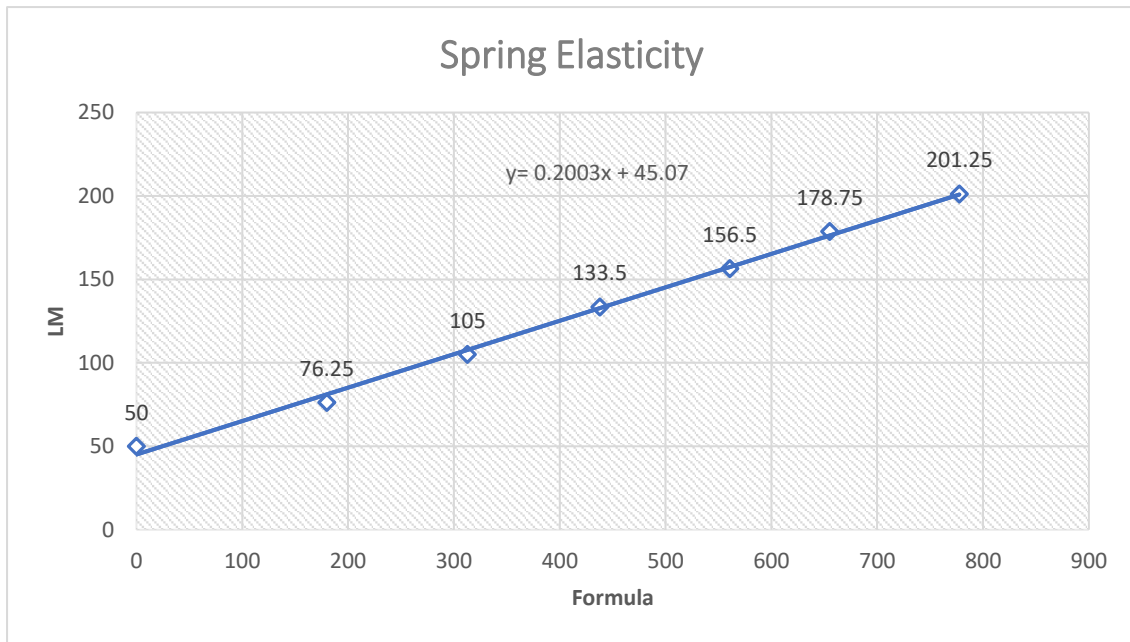


Fig.2: Scatter Graph with line of best fit from table 1

Table 2: Uncertainty of spring length

M (g)	Range (mm)	Uncertainty (mm)	Percentage Uncertainty
0	0	± 0	0
100	15	± 7.5	9.84%
200	20	± 10	9.52%
300	21	± 10.5	7.87%
400	25	± 12.5	7.99%
500	25	± 12.5	6.99%
600	20	± 10	4.97%

Table 3: Uncertainty of string angle

M (mm)	Range (mm)	Uncertainty (mm)	Percentage Uncertainty
0	0	± 0	0%
100	5	± 2.5	2.22%
200	1	± 0.5	0.50%
300	3	± 1.5	1.60%
400	0	± 0	0.00%
500	1	± 0.5	0.62%
600	0	± 0	0.00%

5. Analysis

Looking at the equation 2: $\frac{M}{\cos(\frac{\theta_m}{2})}$ we can conclude that we are looking at the relation of the angle of the string towards the length of the springs.

Table 1 contains all the collected results from the experiment, where l1 and l3 are the measured lengths of spring one and l2 and l4 are the measured lengths of spring two. The measured angle of the string is Θ_1 and Θ_2 . Both a mean of the spring lengths (L) and of the angle (Θ_m) have been calculated as well.

A relation that can be seen looking at L and Θ_m in Table 1 is, that when L increases Θ_m decreases.

Furthermore, looking at Table 1, the length of the second spring (l2, l4) is always shorter. This could mean that either sloppy work was conducted while collecting data and that the weight was not hung perfectly in the middle of the string or that the second spring is stiffer than the first.

Figure 2 is a scatter graph with the mean Length on the y-axis and the resulting data from $\frac{M}{\cos(\frac{\theta_m}{2})}$ on the x-axis. Its maximum is at 201.25mm and its minimum is at 50mm.

Looking at it one can see that the line of best fit is linear with a gradient of $y=0.2003x+45.07$ (Equation 3). The gradient can be calculated by choosing two points on the line of best fit and putting them in $m = \frac{y_2-y_1}{x_2-x_1}$ (Equation 4). The value from m can then be put into $y=mx+c$ (Equation 4.1).

Using one of the previously used points the equation 4.1 can then be transposed into $c=y-mx$ to find out the y-intercept resulting in the gradient of $y=0.2003x+45.07$ (Equation 3). A lot of points are very close to the line of best fit, leading to the conclusion that there is a linear relation between spring length and string angle.

For the data to be more reliable the uncertainty and the percentage uncertainty of the length of the springs and the angle of the string must be calculated.

Looking at the uncertainty of the spring length (Table 2) one can see that the percentage uncertainty is quite big ranging from ~10% to ~5%. This could be due to the second spring being stiffer than the first leading to a bigger range or that the measuring equipment was not precise enough, mis calibrated or misused.

Table 3 is more consistent with the percentage uncertainty only going up to ~2%. This means that precise equipment was used effectively.

5.1 Improvements

In hindsight improvements could have been made collecting the data by being more patient and using more precise measuring equipment, for example a Vernier calliper, so statistical inaccuracies like table 2 could be prevented. Another way to have been more precise is to have marked the middle of the string as to know where to hang the masses. An interesting exercise could have been to measure the string so as to know the full length of string +spring and to calculate the surface of the created triangle between the 1m ruler, the springs and the string.

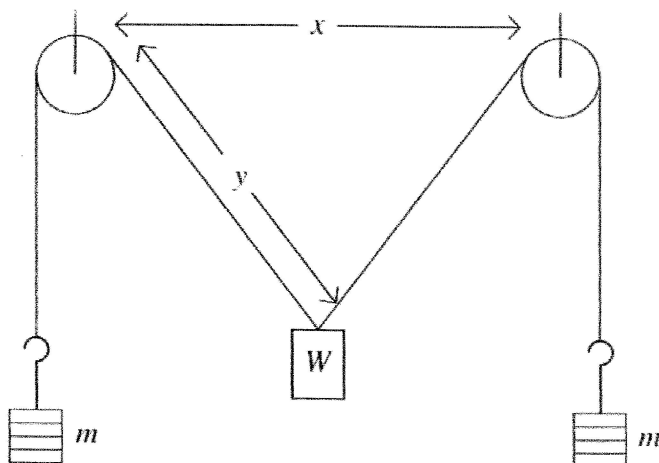
6. Conclusion

Results from this experiment show that there is a linear relation between spring length and string angle.

2

In an experiment similar to the one you performed in Stage 1 an unknown load, of weight, W , was supported by two strings kept in tension by equal masses, m , hung from their free ends, with each string passing over a frictionless pulley. The arrangement was symmetrical and is shown in **Figure 1**.

Figure 1



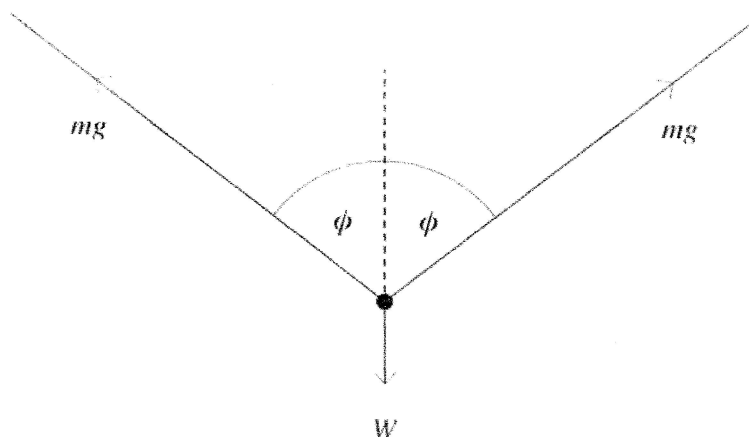
The distance x was kept constant throughout the experiment. The length y was measured for different values of m . The results are shown in the table.

The distance between the strings at the pulleys, $x = 0.500$ m

m/kg	y/m	$(\frac{x}{y})^2$	$\frac{1}{\sqrt{4 - (\frac{x}{y})^2}}$
0.250	0.736	0.462	0.532
0.275	0.484	1.067	0.584
0.300	0.400	1.563	0.641
0.325	0.360	1.929	0.695
0.350	0.339	2.175	0.740
0.375	0.322	2.411	0.793
0.400	0.308	2.635	0.856

- 2 (a) **Figure 2** shows the three forces acting through the point at which the strings are attached to the load. The weight of the load is W and the tension in each string is mg , where g is gravitational field strength.

Figure 2



- 2 (a) (i) By resolving the forces vertically show that $m = \frac{W}{2g\cos\phi}$ where ϕ is the angle between each string and the vertical.

$$\cos \phi = \frac{W}{2mg}$$

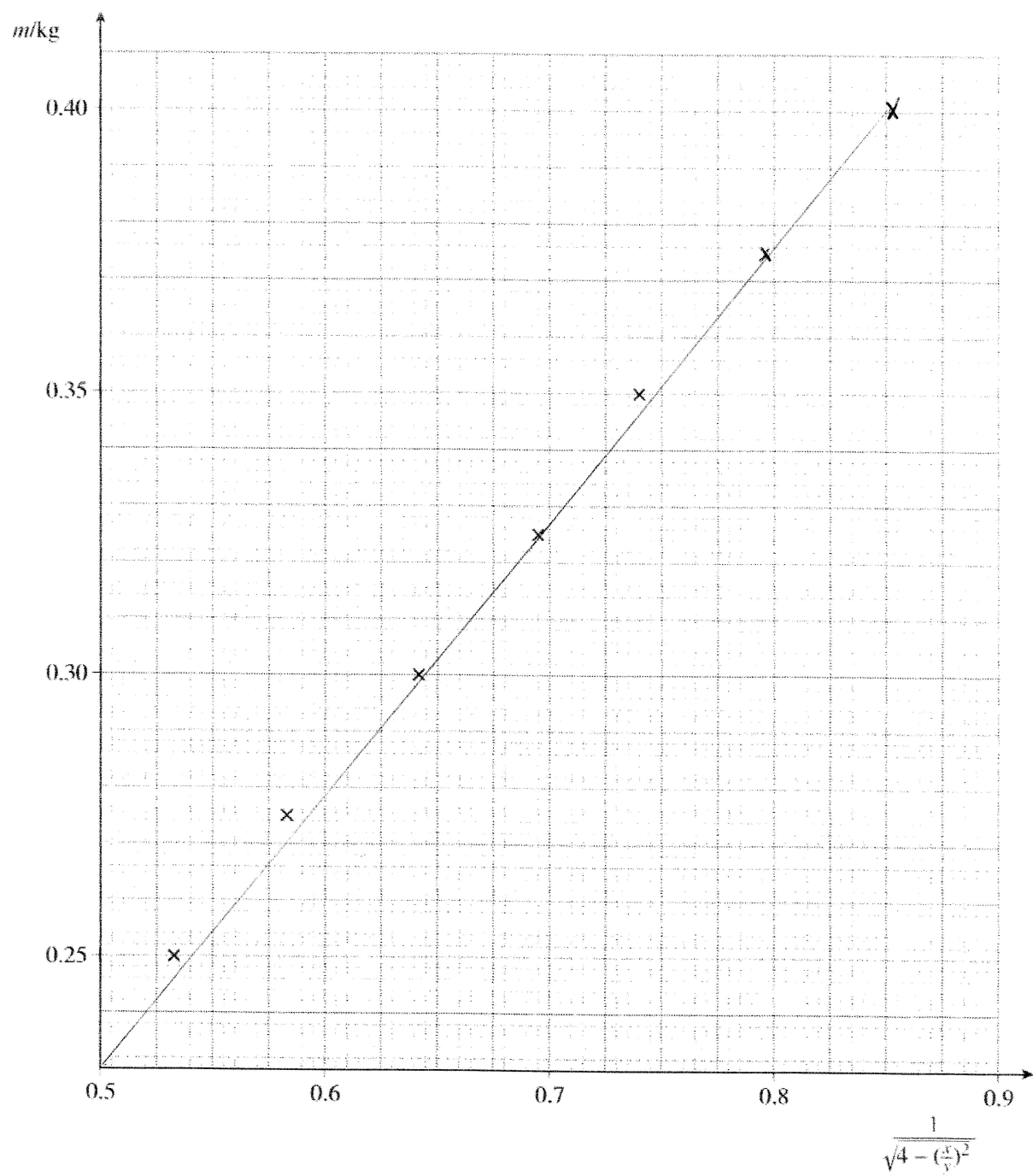
(1 mark)

- 2 (a) (ii) Complete the table on page 4.

(2 marks)

- 2 (a) (iii) Complete the graph on the following page by plotting the missing two points and drawing a straight line of best fit.

(2 marks)



- 2 (b) (i) Determine the gradient of your graph.

$$y = mx + c$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0.29 - 0.23}{0.625 - 0.5} = 0.48$$

$$~~y = mx~~ \quad y = 0.48x + c$$

$$0.23 = (0.48)(0.5) + c$$

$$c = -0.01$$

$$y = 0.48x - 0.01$$

(3 marks)

- 2 (b) (ii) The equation for the straight line is $m = \frac{W}{g} \times \frac{1}{\sqrt{4 - (\frac{x}{y})^2}}$ $\left| \frac{1}{\sqrt{4 - (\frac{x}{y})^2}} \right| \cdot g$

Given that $g = 9.81 \text{ N kg}^{-1}$, determine a value for W .

$$W = m \cdot \sqrt{4 - (\frac{x}{y})^2} \cdot g$$

$$W = (0.48)(0.532)(9.81) = 2.51 \text{ for } m(\text{kg}) = 0.250$$

(2 marks)

- 2 (c) The uncertainty in the measurement of x was $\pm 1 \text{ mm}$ and the uncertainty in the measurement of y was $\pm 2 \text{ mm}$. Calculate the percentage uncertainty in $(\frac{x}{y})^2$ for $m = 0.300 \text{ kg}$.

$$\text{Range} = 2.173$$

$$u = \pm 1.087$$

$$\frac{2.173}{2} = 1.087$$

$$\text{Mean} = 1.749$$

$$\%u = 0.61\%$$

$$\frac{1.087}{1.749} = 0.61$$

(3 marks)

2 (d) (i) Explain the term *systematic error*.

Systematic errors are errors that are not determined by chance but are introduced by an inaccuracy inherent to the system.

(1 mark)

2 (d) (ii) There may be a systematic error in this experiment because of friction in the pulleys. When the measurements were taken, increasing values of m were used. State and explain how friction in the pulleys would have affected the measured values of y .

Friction is a force that affects motion. It works in the opposite direction of the motion. This would produce skewed results in a real life scenario.

(2 marks)