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Bisection

A function was created based on equation 1.1 to solve roots using the bisection method

$$x_2 = (x_1 + x_3)/2 \quad \text{Equation 0-1}$$

$$f(x) = \sin(x) \sinh(x)$$

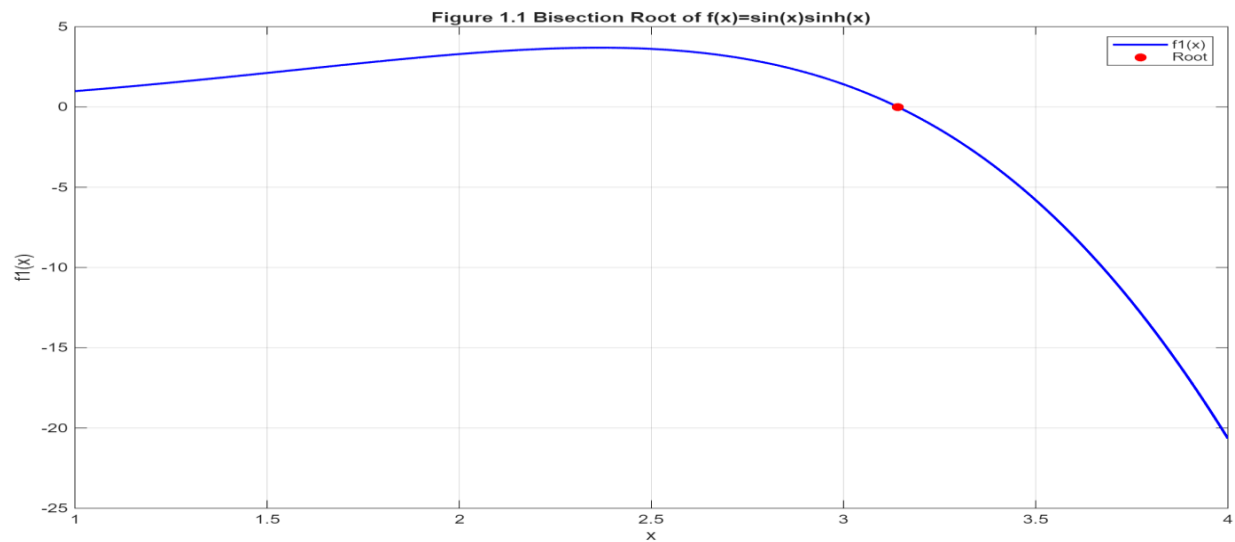


Figure 0-1 this shows the function labelled in the legend and the root

$$f(x) = (R^2 - x^2)^{1/2} - x \tan(x)$$

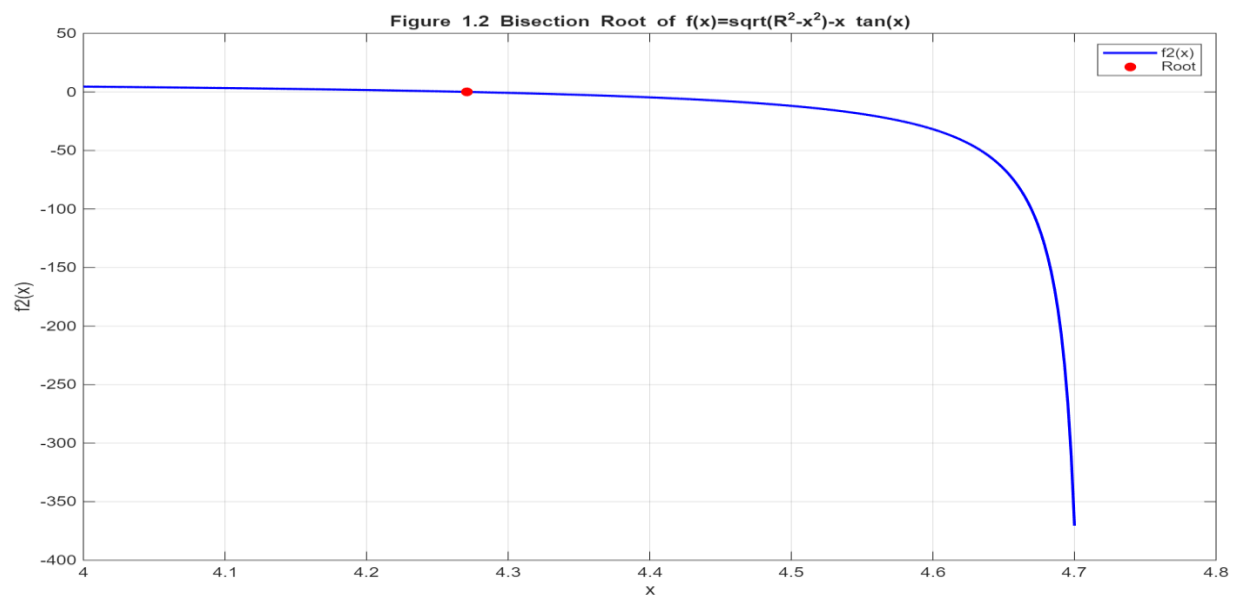


Figure 0-2 this shows the function labelled in the legend and the root

Newton's method

A function was made from equation 2.1 to evaluate roots with the newton method to use this method, functions and their derivative functions were created.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Equation 0-1

Newton b $f(x) = e^x - (1 + x + (x^2/2))$

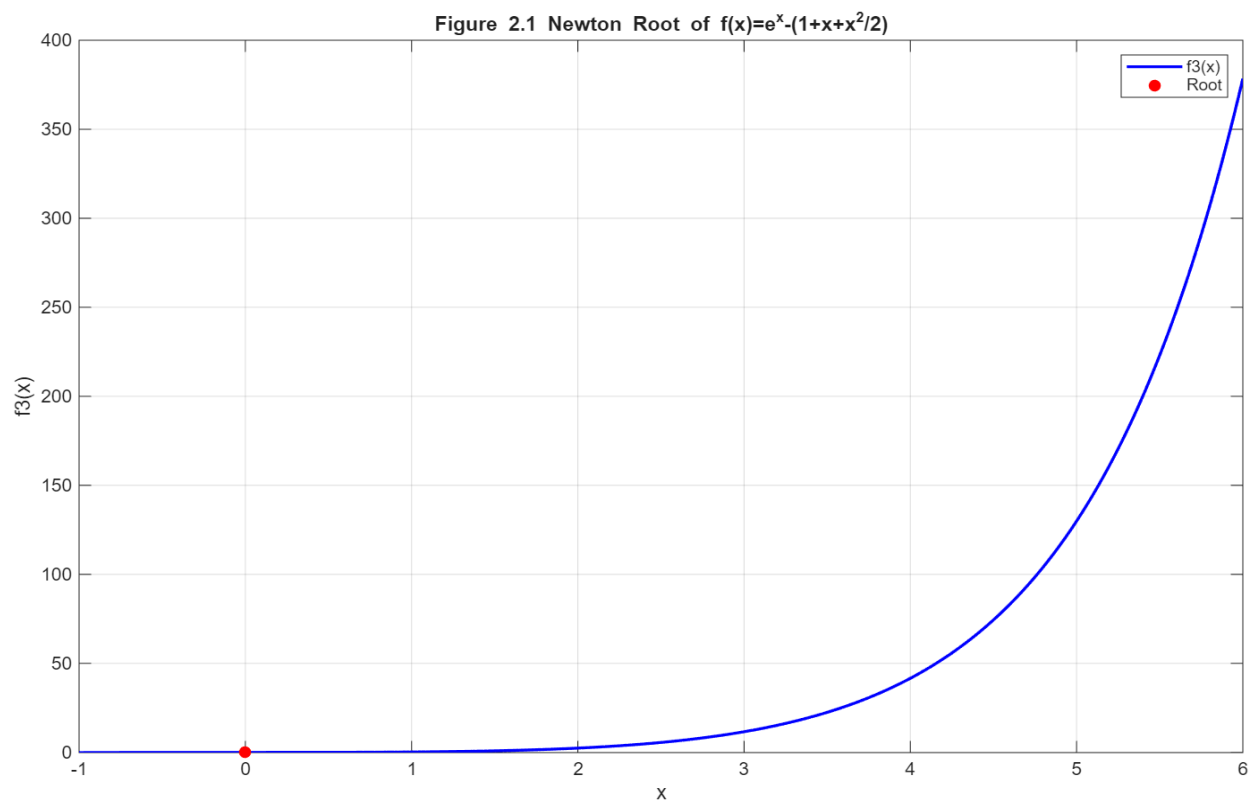


Figure 0-1 this shows the function labelled in the legend and the root

Newton c $f(x) = 10 \log(x) - x$

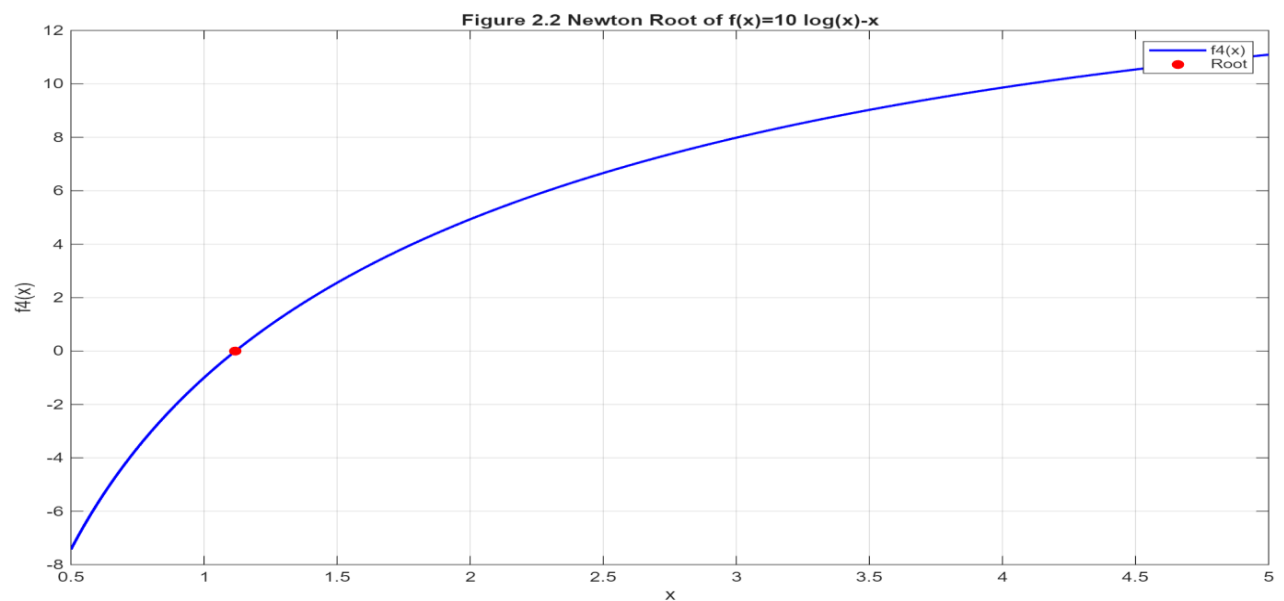


Figure 0-2 this shows the function labelled in the legend and the root

Intersect plot $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

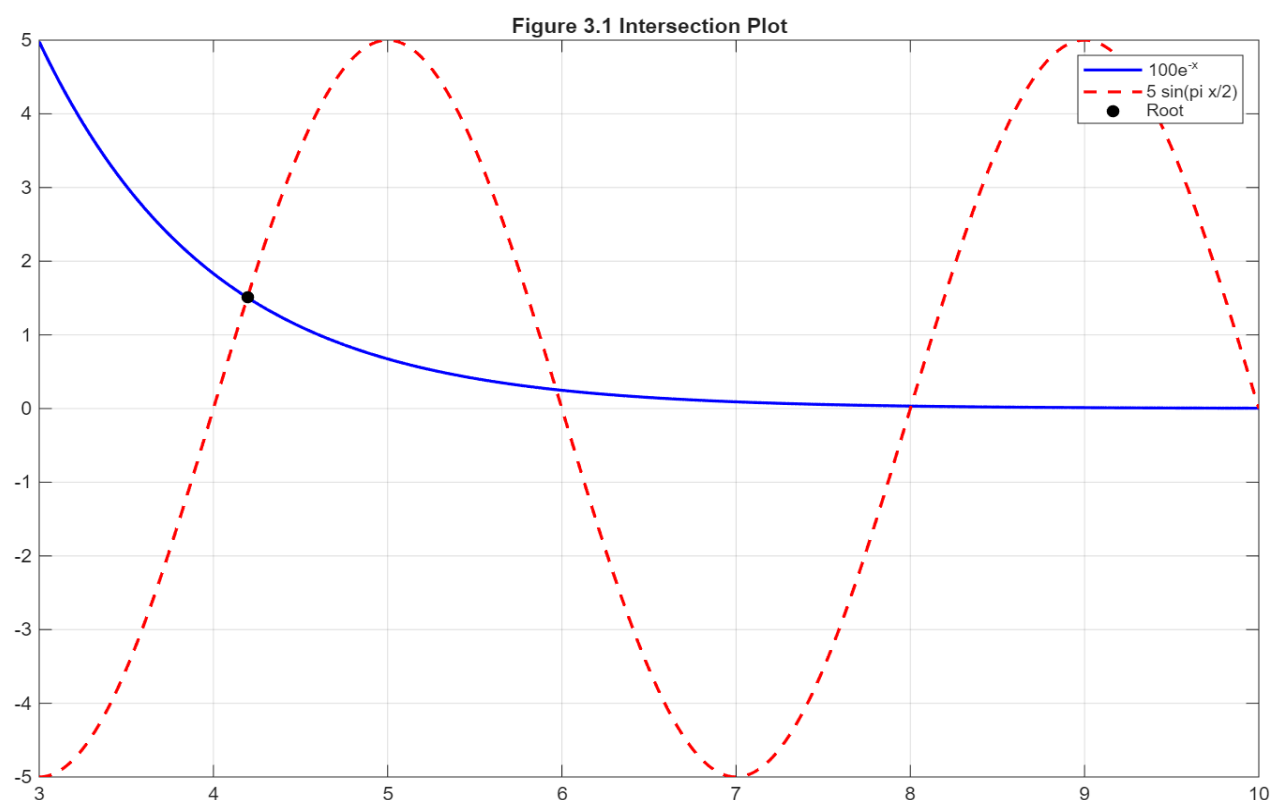


Figure 0-1 plot of function intersection

Bracketing

Bisect $f(x) = 100e^{-x} - 5\sin(\pi x/2)$



Figure 0-1 multiple roots Bisect for functions shown in legend

Newton $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

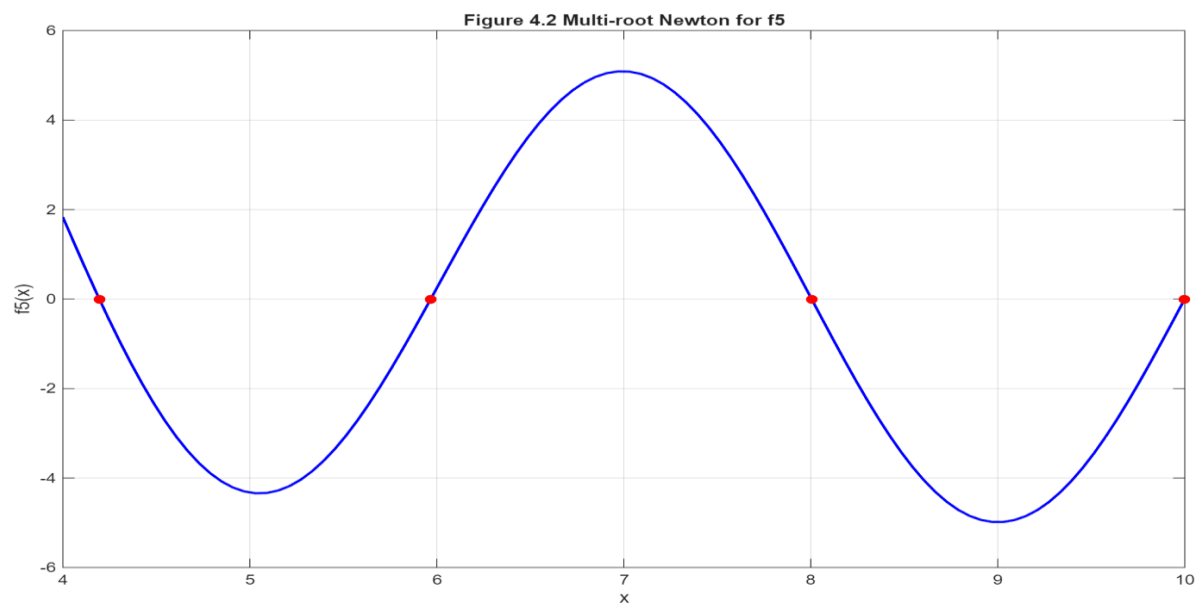


Figure 0-2 multiple roots Newton for functions shown in legend

Appendices

Convergence table Bisection $f(x) = \sin(x) \sinh(x)$

Iteration	a	b	c	f(c)	Error
1	1	1	4	2.5	1.5
2	2	2.5	4	-1.3931	0.75
3	3	2.5	3.25	2.3274	0.375
4	4	2.875	3.25	0.8428	0.1875
5	5	3.0625	3.25	-0.1718	0.0938
6	6	3.0625	3.1562	0.3602	0.0469
7	7	3.1094	3.1562	0.1005	0.0234
8	8	3.1328	3.1562	-0.034	0.0117
9	9	3.1328	3.1445	0.0336	0.0059
10	10	3.1387	3.1445	-0.0001	0.0029
11	11	3.1387	3.1416	0.0168	0.0015
12	12	3.1401	3.1416	0.0083	0.0007
13	13	3.1409	3.1416	0.0041	0.0004
14	14	3.1412	3.1416	0.002	0.0002
15	15	3.1414	3.1415	0.001	0.0001
16	16	3.1415	3.1416	0.0004	0
17	17	3.1416	3.1416	0.0002	0
18	18	3.1416	3.1416	0	0
19	19	3.1416	3.1416	0	0
20	20	3.1416	3.1416	0	0
21	21	3.1416	3.1416	0	0
22	22	3.1416	3.1416	0	0

Convergence table Bisection $f(x) = (R^2 - x^2)^{1/2} - x \tan(x)$

Iteration	a	b	c	f(c)	Error
1	4	4.7	4.35	-2.4692	0.35
2	4	4.35	4.175	2.0804	0.175
3	4.175	4.35	4.2625	0.2195	0.0875
4	4.2625	4.35	4.3062	-0.9881	0.0438
5	4.2625	4.3062	4.2844	-0.3553	0.0219
6	4.2625	4.2844	4.2734	-0.0612	0.0109
7	4.2625	4.2734	4.268	0.0808	0.0055

8	4.268	4.2734	4.2707	0.0102	0.0027
9	4.2707	4.2734	4.2721	-0.0254	0.0014
10	4.2707	4.2721	4.2714	-0.0076	0.0007
11	4.2707	4.2714	4.271	0.0013	0.0003
12	4.271	4.2714	4.2712	-0.0031	0.0002
13	4.271	4.2712	4.2711	-0.0009	0.0001
14	4.271	4.2711	4.2711	0.0002	0
15	4.2711	4.2711	4.2711	-0.0004	0
16	4.2711	4.2711	4.2711	-0.0001	0
17	4.2711	4.2711	4.2711	0.0001	0
18	4.2711	4.2711	4.2711	0	0
19	4.2711	4.2711	4.2711	0	0
20	4.2711	4.2711	4.2711	0	0

Convergence table Newton b $f(x) = e^x - (1 + x + (x^2/2))$

Iteration	x	f(x)	Error
0	0.1	0.0002	-0.0331
1	0.0669	0.0001	-0.0222
2	0.0448	0	-0.0149
3	0.0299	0	-0.0099
4	0.02	0	-0.0066
5	0.0133	0	-0.0044
6	0.0089	0	-0.003
7	0.0059	0	-0.002
8	0.0039	0	-0.0013
9	0.0026	0	-0.0009
10	0.0018	0	-0.0006
11	0.0012	0	-0.0004
12	0.0008	0	-0.0003
13	0.0005	0	-0.0002
14	0.0003	0	-0.0001
15	0.0002	0	-0.0001
16	0.0002	0	-0.0001
17	0.0001	0	0
18	0.0001	0	0
19	0	0	0
20	0	0	0

21	0	0	0
22	0	0	0
23	0	0	0

Convergence table Newton c $f(x) = 10 \log(x) - x$

Iteration	x	f(x)	Error
0	2.7	7.2325	-2.675
1	0.025	-36.9301	0.0924
2	0.1174	-21.5421	0.2558
3	0.3732	-10.2298	0.3966
4	0.7698	-3.3865	0.2824
5	1.0522	-0.5435	0.0639
6	1.1161	-0.0177	0.0022
7	1.1183	0	0
8	1.1183	0	0

Convergence table Bisect $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

Root #	Bisection Root	Newton Root
1	4.1949	4.1949
2	5.9674	5.9674
3	8.0043	8.0043
4	9.9994	9.9994

Convergence table Newton $f(x) = 100e^{-x} - 5\sin(\pi x/2)$

Root #	Newton Root (x)
1	4.1949
2	5.9674
3	8.0043
4	9.9994

Your code and functions

Bisect

```
function [root, table] = bisect(f,a,b,imax,tol)

xL = a;

xR = b;

fL = feval(f,xL);

fR = feval(f,xR);

if fL*fR > 0

    error('No root in interval')

end

% Preallocate table: iter, xL, xR, xmid, fmid, err

table = zeros(imax,6);
```

```
for iter = 1:imax

    xmid = (xL + xR)/2;

    fmid = feval(f,xmid);

    err = abs(xR - xL)/2;

    table(iter,:) = [iter, xL, xR, xmid, fmid, err];

    if fL*fmid < 0

        xR = xmid;

        fR = fmid; % now used properly

    else

        xL = xmid;

        fL = fmid;

    end

    if err < tol

        root = xmid;

        table = table(1:iter,:); % trim unused rows

        return;

    end

end

error('Exceeded max iterations in bisection');

end
```

Newton

```
function [root, table] = newton(x0,f,df,imax,tol)
```

```
x = x0;

% Preallocate table: iter, x_n, f(x_n), dx

table = zeros(imax,4);

for iter = 1:imax

    fx = feval(f,x);

    dfx = feval(df,x);

    if abs(dfx) < 1e-14

        error('Derivative zero');

    end

    dx = -fx/dfx;

    table(iter,:) = [iter-1, x, fx, dx];

    x = x + dx;

    if abs(dx) < tol

        root = x;

        table = table(1:iter,:); % trim unused rows

        return;

    end

end

error('Maximum Newton iterations exceeded');

end
```

f1

```
function y = f1(x)

y = sin(x).*sinh(x);

end
```

f2

```
function y = f2(x)

R = 10;

y = sqrt(R^2 - x.^2) - x.*tan(x);

end
```

f3

```
function y = f3(x)

y = exp(x) - (1 + x + x.^2/2);

end
```

f4

```
function y = f4(x)

y = 10*log(x) - x;

end
```

f5

```
function y = f5(x)

y = 100*exp(-x) - 5*sin(pi*x/2);

end
```

df3

```
function y = df3(x)

y = exp(x) - (1 + x);

end
```

df4

```
function y = df4(x)

y = 10./x - 1;

end
```

df5

```
function y = df5(x)

y = -100*exp(-x) - 5*(pi/2)*cos(pi*x/2);

end
```

Task2_Main

```
clc; clear; close all;
```

```
%% 1. BISECTION METHOD
```

```
disp('--- Bisection Method ---')
```

```
tol = 1e-6;
```

```
imax = 50;
```

```
%% (1.1) f1(x) = sin(x)sinh(x) over [1,4]
```

```
[root1, table1] = bisect(@f1,1,4,imax,tol);  
  
disp('Convergence table f1 (sin(x)sinh(x)):')  
  
disp(table1)  
  
  
x = linspace(1,4,300);  
  
figure(1)  
  
plot(x,f1(x),'b','LineWidth',1.6); hold on;  
  
plot(root1,f1(root1),'ro','MarkerFaceColor','r')  
  
title('Figure 1.1 Bisection Root of  $f(x)=\sin(x)\sinh(x)$ ')  
  
xlabel('x'); ylabel('f1(x)')  
  
legend('f1(x)','Root'); grid on  
  
  
%% (1.2) f2(x) with R=10 over [4,4.7]  
  
[root2, table2] = bisect(@f2,4,4.7,imax,tol);  
  
disp('Convergence table f2  $\sqrt{R^2-x^2}-x \tan(x)$ :')  
  
disp(table2)  
  
  
x = linspace(4,4.7,300);  
  
figure(2)  
  
plot(x,f2(x),'b','LineWidth',1.6); hold on;  
  
plot(root2,f2(root2),'ro','MarkerFaceColor','r')  
  
title('Figure 1.2 Bisection Root of  $f(x)=\sqrt{R^2-x^2}-x \tan(x)$ ')  
  
xlabel('x'); ylabel('f2(x)')  
  
legend('f2(x)','Root'); grid on
```

```
%% 2. NEWTON–RAPHSON METHOD
```

```
disp('--- Newton Method ---')
```

```
%% Newton (b) —  $f_3(x) = e^x - (1 + x + x^2/2)$ 
```

```
% Root is around  $x = 0$ , so start close to 0
```

```
[root3, table3] = newton(0.1, @f3, @df3, 50, 1e-6);
```

```
disp('Convergence table Newton f3:')
```

```
disp(table3)
```

```
x = linspace(-1,6,300);
```

```
figure(3)
```

```
plot(x,f3(x),'b','LineWidth',1.6); hold on;
```

```
plot(root3,f3(root3),'ro','MarkerFaceColor','r')
```

```
title('Figure 2.1 Newton Root of  $f(x)=e^x-(1+x+x^2/2)$ ')
```

```
xlabel('x'); ylabel('f3(x)')
```

```
legend('f3(x)','Root');
```

```
grid on
```

```
%% Newton (c) —  $f_4(x) = 10*\log(x) - x$ 
```

```
% Function has root near  $x = 2.7$  (rough estimate)
```

```
[root4, table4] = newton(2.7,@f4,@df4,50,1e-6);
```

```
disp('Convergence table Newton f4:')
```

```
disp(table4)
```

```
x = linspace(0.5,5,300);

figure(4)

plot(x,f4(x),'b','LineWidth',1.6); hold on;

plot(root4,f4(root4),'ro','MarkerFaceColor','r')

title('Figure 2.2 Newton Root of f(x)=10 log(x)-x')

xlabel('x'); ylabel('f4(x)')

legend('f4(x)','Root'); grid on

%% 3. INTERSECTION PLOT USING NEWTON

disp('--- Intersection Problem  $100e^{-x} = 5 \sin(\pi x/2)$  ---')

% f5 has multiple roots; start near 4 for the first intersection

[root5, table5] = newton(4,@f5,@df5,50,1e-10);

disp('Convergence table Intersection f5:')

disp(table5)

x = linspace(3,10,500);

figure(5)

plot(x,100*exp(-x),'b','LineWidth',1.6); hold on;

plot(x,5*sin(pi*x/2),'r--','LineWidth',1.6)

plot(root5,100*exp(-root5),'ko','MarkerFaceColor','k')

legend('100e^{-x}','5 sin(pi x/2)','Root')

title('Figure 3.1 Intersection Plot')

grid on
```

```
%% 4. BRACKETING MULTIPLE ROOTS (f5)
```

```
disp('--- Bracketing Multiple Roots ---')
```

```
x = linspace(4,10,100);
```

```
y = f5(x);
```

```
% Preallocate maximum possible size
```

```
brackets = zeros(length(x)-1,2);
```

```
k = 0;
```

```
% Find sign changes
```

```
for i = 1:length(x)-1
```

```
    if y(i)*y(i+1) < 0
```

```
        k = k + 1;
```

```
        brackets(k,:) = [x(i) x(i+1)];
```

```
    end
```

```
end
```

```
brackets = brackets(1:k,:); % trim unused rows
```

```
roots_bis = zeros(k,1);
```

```
roots_new = zeros(k,1);
```

```
for i = 1:k
```

```
roots_bis(i) = bisection(@f5, brackets(i,1), brackets(i,2), 50, 1e-6);

% For Newton, use midpoint of bracket as safer initial guess
x0 = (brackets(i,1)+brackets(i,2))/2;

roots_new(i) = newton(x0, @f5, @df5, 50, 1e-6);

end

disp('Bisection roots in [4,10]:')
disp(roots_bis)

disp('Newton roots in [4,10]:')
disp(roots_new)

figure(6)
plot(x,y,'b','LineWidth',1.6); hold on;
plot(roots_bis,f5(roots_bis),'ro','MarkerFaceColor','r')
title('Figure 4.1 Multi-root Bisection for f5')
xlabel('x'); ylabel('f5(x)'); grid on

figure(7)
plot(x,y,'b','LineWidth',1.6); hold on;
plot(roots_new,f5(roots_new),'ro','MarkerFaceColor','r')
title('Figure 4.2 Multi-root Newton for f5')
xlabel('x'); ylabel('f5(x)'); grid on
```