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Analytical methods
test 2

question 2

a) $Z_1 = 3 + 7j$, $Z_2 = 2 - 10j$, $Z_3 = -4 - j$

1 $Z_1 + Z_3$

$$\begin{array}{r} (3 + 7j) \\ 3 + 7j \end{array} + \begin{array}{r} (-4 - j) \\ -4 - j \end{array}$$

$$-1 + 6j$$

2 $Z_1 - Z_3$

$$\begin{array}{r} (3 + 7j) \\ 3 + 7j \end{array} - \begin{array}{r} (-4 - j) \\ -4 - j \end{array}$$

$$7 + 8j$$

3 $\frac{Z_1}{Z_2}$

$$Z_1 = (3 + 7j) \quad Z_2 = (2 - 10j)$$

Z_1	$\frac{3 + 7j}{2 - 10j}$	$\frac{2 + 10j}{2 + 10j}$	F
Z_2	$2 - 10j$	$2 + 10j$	O

i

L

$$\frac{6 + 30j + 14j + 70j^2}{4 + 20j - 20j + 100j^2}$$

$$\frac{6 + 44j + 70j^2}{4 + 100}$$

$$\frac{-64 + 44j}{104}$$

$\frac{-64}{104}$	$+$	$\frac{44j}{104}$
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4 $z_1 - z_2$

$$(3 + 7j) - (2 - 10j)$$

$$3 + 7j - 2 + 10j$$

$$1 + 17j$$

5 $(z_3)^3 - z_1$

$$(-4 - j)^3 (3 + 7j)$$

$$(-4 - j)(-4 - j)(-4 - j) - (3 + 7j)$$

$$b) \quad 8x^2 = -3x - 18$$

$$8x^2 + 3x + 18 = 0$$

a b c = 0

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 8 \times 18}}{2 \times 8}$$

$$x = \frac{-3 \pm \sqrt{9 - 576}}{2 \times 8}$$

~~2x8 16~~

$$x = \frac{-3 \pm \sqrt{-}}$$

$$x = \frac{-3 \pm \sqrt{1} \times \sqrt{16}}{16}$$

$$\frac{6+4j}{16}$$

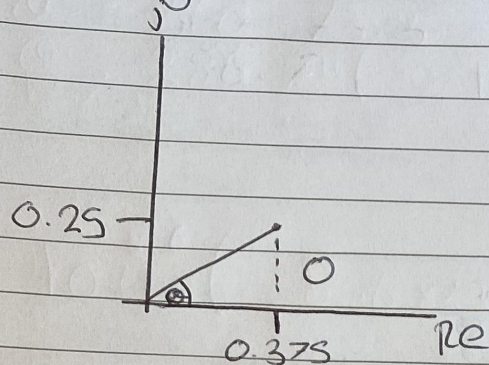
$$\frac{6-4j}{16}$$

$$0.375 + 0.25j$$

$$0.375 - 0.25j$$

cartesian form

Putting in to Polar form



$$0.375 + 0.25j$$

$$r^2 = 0.25^2 + 0.375^2$$

$$r = \sqrt{0.25^2 + 0.375^2}$$

$$r = \sqrt{0.141 + 0.0625}$$

$$r = \sqrt{0.2035}$$

$$r = 0.451$$

$$\tan \theta = \frac{0.25}{0.375}$$

$$\theta = \tan^{-1} \left(\frac{0.25}{0.375} \right)$$

$$\theta = 33.69^\circ$$

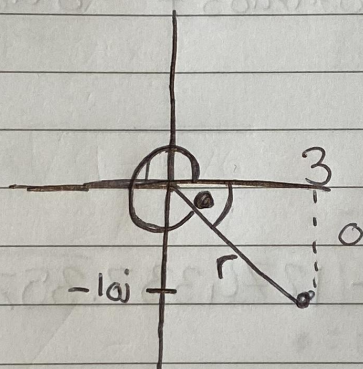
$$r \cos \theta + j \sin \theta$$

$$0.451 (\cos 33.69 + j \sin 33.69)$$

C de Moivre's theorem

$$1 (3-10j)^6$$

$$r^n (\cos n\theta + j \sin n\theta)$$



$$r^2 = 3^2 + 10^2$$

$$r = \sqrt{9 + 100}$$

$$r = \sqrt{109}$$

$$r = \underline{\underline{10.44}}$$

$$\tan \theta = \frac{10}{3}$$

$$\theta = \tan^{-1} \left(\frac{10}{3} \right)$$

$$\theta = 73.3^\circ$$

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$$\theta = 360 - 73.3$$

$$\theta = \underline{\underline{286.7}}$$

$$10.44^6 (\cos 6 \times 286.7 + j \sin 6 \times 286.7)$$

Polar form

$$\underline{\underline{1294800.898 (\cos 1720.2 + j \sin 1720.2)}}$$

*

$$1294800.898 (0.1770847403 + -0.984195608j)$$

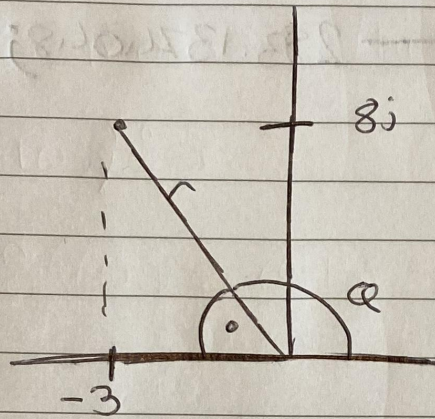
multiply out

$$= 229289.4808 - 1274337.357j$$

cartesian form

$$b) (-3 + 8j)^5$$

$$r^n (\cos n\theta + j \sin n\theta)$$



$$r^2 = 3^2 + 8^2$$

$$\tan \theta = \frac{8}{3}$$

$$r = \sqrt{9 + 64}$$

$$\theta = \tan^{-1} \left(\frac{8}{3} \right)$$

$$r = \sqrt{73}$$

$$\theta = 69.443$$

$$r = 4.179$$

$$\theta = 180 - 69.44 \text{ (3dp)}$$

$$\theta = 110.557^\circ$$

$$4.179^5 (\cos 5 \times 110.557 + j \sin 5 \times 110.557)$$

$$1274.564611 (\cos 552.835 + j \sin 552.835)$$

Polar form

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multiply out

$$1274.564611(-0.9750138361 + -0.222144411j)$$

$$-1242.718131$$

$$-283.1376048j$$

Cartesian Form

$$D \quad j^5 \times j^3$$

$$j^{5+3}$$

$$\underline{\underline{j^8}}$$

Question 1

A)

i)

$$A = \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix}$$

$r \times c$

3×3

$r \times c$

3×3

i) AB can be multiplied

$$-16 + -24 + 0$$

$$-10 + 4 + 0$$

$$-6 + 0 + 1$$

$$64 + -5 + -63$$

$$-40 + 10 + -45$$

$$12 + 0 + 0$$

$$24 + 0 + -18$$

$$4 + -2 + 7$$

$$-2.5 + 4 + 5$$

$$1.5 + 0 + 2$$

$$-18$$

$$-6$$

$$-5$$

$$-4$$

$$-75$$

$$6$$

$$9$$

$$6.5$$

$$3.5$$

AB

ii)

$$B = \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -4 & 1 \end{pmatrix} \quad A = \begin{pmatrix} 8 & -1 & 7 \\ -3 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$$

$3 \times 3 \qquad \qquad 3 \times 3$

can multiply

$$BA = \begin{pmatrix} -16 + 40 + 1.5 & 2 + 16 + 0 & -14 + 40 + 1 \\ 16 + -25 + 6 & -2 + 10 + 0 & 14 + 25 + 4 \\ 0 + 45 + 3 & 0 + -18 + 0 & 0 + -45 + 2 \end{pmatrix}$$

$$BA = \begin{pmatrix} 25.5 & 18 & 26 \\ -3 & 8 & 43 \\ 48 & -18 & -43 \end{pmatrix}$$

$AB \neq BA$ = non commutative (matrices)

iii) $A = \begin{pmatrix} 8 & -1 & 2 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix}$

3×3 3×2

~~AC cannot be multiply due to different rows and columns~~

iv) $C = \begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix}$ $A = \begin{pmatrix} 8 & -1 & 2 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$

2×3 3×3

~~Can multiply~~

AC

iii)

$$\begin{pmatrix} 16+6+7 & 40+0+84 & -10+(-12)+8 \\ -10+(-25) & -25+0+60 & \\ 6+0+2 & 15+0+24 & \end{pmatrix}$$

$$AC = \begin{pmatrix} 29 & 124 \\ -17 & 45 \\ 8 & 39 \end{pmatrix}$$

$$iv) C \begin{pmatrix} 2 & 3 \\ -6 & 0 \\ 1 & 12 \end{pmatrix}$$

$$3 \times 2$$

$$A \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$$

$$3 \times 3$$

cannot multiply

$$v) |A| \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$$

$$8 \begin{pmatrix} 4 & -0 \\ 2 \times 2 & + 0 \times 5 \end{pmatrix} - (-1) (15 - -10) + 7 (6 - 0)$$

$$= 8(4) + (5) + 7(6)$$

$$24 + 5 + 42$$

$$= \underline{\underline{71}}$$