

① a) Force on half cylinder = $l \times$ projected area.
 $= p \times dL$

Total resistive force = $2\sigma_H \times Lt$

$$2\sigma_H Lt = p dL$$

$$\therefore \sigma_H = \frac{pd}{2t}$$

b) Longitudinal stress is ~~half~~ half of hoop stress.

c) $1.0265 \times 10^3 \text{ kg/cm}^3 = 1026.5 \text{ kg/m}^3$

d) $p = \rho gh$ $d = 4.6 \text{ m}$ $t = 4.6 \text{ mm} = 0.0046 \text{ m}$
 $\sigma_H = 200 \times 10^6 \text{ Pa}$

$$\sigma_H = \frac{pd}{2t} \quad \sigma_H \quad p = \frac{2\sigma_H t}{d}$$

$$p = \frac{2 \times 200 \times 10^6 \times 0.0046}{4.6} = 400 \text{ kPa}$$

$$400 \times 10^3 \times 2.1 = 840 \times 10^3 \text{ Pa}$$

$$400 \times 10^3 = \rho gh = 1.0265 \times 9.81 \times h$$

$$840 \times 10^3 = \rho gh = 1026.5 \times 9.81 \times h$$

$$h = \frac{840 \times 10^3}{1026.5 \times 9.81} = 83.416 \text{ m}$$

e) $400 \times 10^3 + 0.1 \times 10^6 = \frac{500 \times 10^3}{1.05 \times 10^6} \times 2.1 = 840 \times 10^3 \text{ Pa}$

$$h = \frac{840 \times 10^3}{1026.5 \times 9.81} = 83.416 \text{ m}$$

$$104.27 \text{ m}$$

① f) $400 \times 10^3 + 0.2 \times 10^6 = 600 \times 10^3 \text{ Pa}$

$$600 \times 10^3 \times 2.1 = 1.26 \times 10^6 \text{ Pa}$$

$$h = \frac{1.26 \times 10^6}{1026.5 \times 9.81} = \underline{\underline{125.12 \text{ m}}}$$

g) Ideally you would want to seal the submarine at as high internal pressure as possible to allow for much greater depths to be achieved while remaining below the yield stress of the skin.

h) Both the temperature of the water and the salinity would change at different depths, so the density of the water would also change. This would mean a calculation with 1 density would give a different maximum depth than with another water density. In reality the submarine may also be pressurised at a much higher pressure allowing it to go to much greater depths.

② a)

A: Define the problem;

- Establish customer needs
- Develop a problem statement
- Create a criteria for success

B: Gather information;

- Is the problem real and is statement accurate?
- What are the existing solutions to the problem?
- What is wrong with the way the problem is currently being solved?
- How much will customers pay for a solution to the problem?

C: Generate multiple solutions;

- Creative phase involving brainstorming
- Design teams should be free to take risks.
- People from different disciplines should be involved.

D: Analyse and select a solution;

- Carry out functional analysis
- Do mechanical / strength analysis
- Analyse product safety and reliability
- Analyse regulations and standards.

E: Test and implement a solution;

- Prototyping and testing under real conditions should be carried out.
- Create documentation, for an audit for example.
- Apply for patents to protect the design.

② b) Step C, Generating multiple solutions could benefit from using FEA as ~~leading~~ ^{leading} with CAE is much quicker in the design process rather than starting with CAD and then developing CAE afterwards.

Step E, testing and implementing a solution could also benefit from the use of FEA as Major Loadpaths would have been developed early in the development process allowing for a much quicker testing process.

$$\textcircled{3} \text{ a) } M_{xx} = \frac{w}{2} \left(\frac{L}{2} - x \right) = \frac{wL}{4} - \frac{wx}{2} = EI \frac{d^2 y}{dx^2}$$

$$\textcircled{2} \quad \therefore EI \frac{dy}{dx} = \frac{wL}{4} x - \frac{wx^2}{4} + A \quad \textcircled{1}$$

$$\textcircled{3} \quad EI y = \frac{wLx^2}{8} - \frac{wx^3}{12} + Ax + B \quad \textcircled{2}$$

$$\textcircled{4} \quad \text{when } x=0 \quad \frac{dy}{dx} = 0 \quad \therefore A = 0 \quad \text{(from ①)}$$

$$\textcircled{5} \quad \text{when } x = \frac{L}{2} \quad y=0 \quad \therefore 0 = \frac{wL}{8} \left(\frac{L}{2} \right)^2 - \frac{w}{12} \left(\frac{L}{2} \right)^3 + B \quad \text{(from ②)}$$

$$\therefore B = \frac{wL^3}{96} - \frac{wL^3}{32}$$

$$B = \frac{-wL^3}{48}$$

$$\textcircled{6} \quad \therefore y = \frac{1}{EI} \left[\frac{wLx^2}{8} - \frac{wx^3}{12} - \frac{wL^3}{48} \right]$$

$$\text{b) } y_{\max} \text{ is when } x=0$$

$$y_{\max} = \frac{-wL^3}{48EI}$$

③ c) $\gamma_{\max} = 10.5 \text{ mm} = 0.0105 \text{ m}$
 $W = 47 \times 10^3 \text{ N}$
 $E = 225 \times 10^9 \text{ Pa}$
 $I = 70 \times 10^{-6} \text{ m}^4$
 $L = ?$

~~W~~ $\gamma_{\max} = \frac{-WL^3}{48EI}$

$$L = \frac{2 \sqrt[3]{6EI} \sqrt[3]{\gamma_{\max}}}{\sqrt[3]{W}}$$

$$L = \frac{2 \sqrt[3]{6 \times (225 \times 10^9 \times 70 \times 10^{-6})} \times \sqrt[3]{0.0105}}{\sqrt[3]{47 \times 10^3}}$$

$$L = 5.5276$$

Max safe span = 5.52 m

d) There could be multiple reasons why this above result would differ in real life, first of all the steel would have ~~corrosion~~ corroded over time so the value of 225 GPa for E ~~might~~ would not be entirely accurate. The value for W may change as the train would likely be loaded with freight or passengers.

e) The speed of the train and whether the train will need to brake at this point as braking will introduce more forces onto the railway tracks.