

Matthew Williams

Q1.

	ver	⊥	Hor
S			
u	$70 \sin 60$	$70 \sin 30$	$70 \cos 30$
V			
A	-9.81	$-9.81 \cos 30$	0
T			

$$90 - 60 = 30^\circ$$

a) $\perp$

$$S = ut + \frac{1}{2} at^2$$

$$0 = t(u + \frac{1}{2} at) \quad t = 0 \quad \text{or} \quad u + \frac{1}{2} at = 0$$

$$= \frac{70 \sin 30 + (-9.81 \cos 30^\circ)t}{2}$$

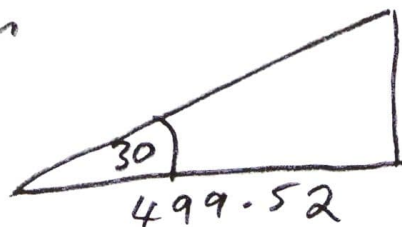
$$2(70 \sin(30)) = (9.81 \cos 30) t$$

$$t = \frac{-2(70 \sin(30))}{(9.81 \cos 30)} = 8.24 \text{ seconds}$$

b) $S = ut$

$$S = 70 \cos(30) \times 8.24$$

$$= 499.52 \text{ m}$$



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Q1 cont

$$\cos \theta = \frac{ad}{Hyp} \quad \therefore \quad Hyp = \frac{Ad}{\cos \theta}$$

$$Hyp \quad \frac{499.52}{\cos(30)} = 576.796m$$

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Q2. ① $S = 3t^2 + 10t - 8$

vel $S = 6t + 10$

acc $S = 6$

② $S = 7t^3 + 4t^2 + 4$

vel $S = 21t^2 + 8t$

acc $S = 42t + 8$

③ $S = 10t^3 - 30t + 2$

vel $S = 30t^2 - 30$

acc $S = 60t$

a) $B = 3 \times A$

$$21t^2 + 8t = 3 \times 6t + 10$$

$$21t^2 + 8t = 18t + 30$$

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Q2 cont

b) Acc B = C

$$42t + 8 = 60t$$

$$8 = 60t - 42t = 18t$$

$$t = \frac{8}{18} \quad 0.444 \text{ m/s}^2$$

c) Distance $t=0$ to $t=4$

$$A = S = 3t^2 + 10t - 8$$

$$S = 3(4)^2 + 10(4) - 8$$

$$S = 80 \text{ meters}$$

$$B = S = 7t^3 + 4t^2 + 4$$

$$S = 7(4)^3 + 4(4)^2 + 4$$

$$S = 516 \text{ meters}$$

$$C = S = 10t^3 - 30t + 2$$

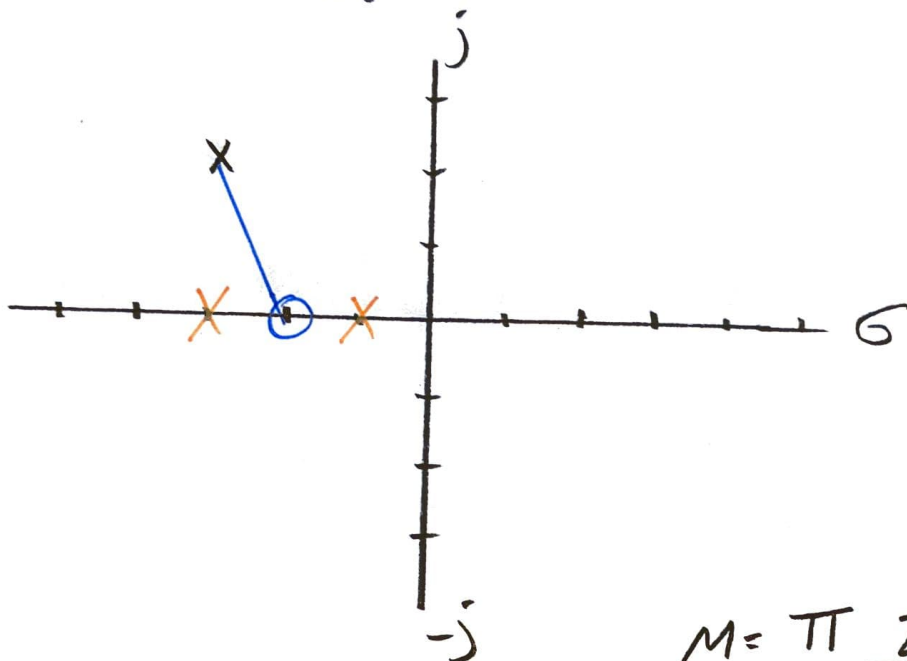
$$S = 10(4)^3 - 30(4) + 2$$

$$S = 522 \text{ meters}$$

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Q3

$$G(s) = \frac{(s+2)}{s(s+1)(s+3)}$$

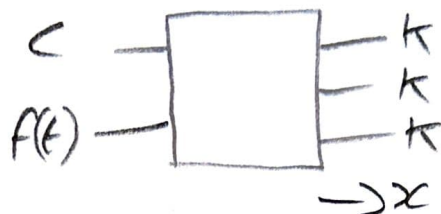


$$M = \frac{\pi \text{ Zero Lengths}}{\pi \text{ Pole Lengths}}$$

$$= \sqrt{2^2 + 1^2} = 2.236$$

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Q4.



$$a) \quad f(t) - k_1 x(t) - k_2 x(t) - k_3 x(t) - c_1 \dot{x} - c_2 \dot{x} = M \ddot{x}$$

$$b) \quad f(t) - 6x(t) - 4\dot{x} = 6\ddot{x}$$

$$f(t) = 6\ddot{x} + 4\dot{x} + 6x(t)$$

$$\downarrow \quad F(s) = 6s^2 X(s) + 4s X(s) + 6X(s)$$

$$F(s) = X(s) (6s^2 + 4s + 6)$$

$$TF = \frac{X(s)}{F(s)} = \frac{1}{6s^2 + 4s + 6}$$

Second Order
Standard Form:

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{\frac{1}{6}}{s^2 + \frac{4}{6}s + 1}$$

$$\omega_n^2 = 1 \quad \sqrt{1} = 1$$

$$\omega_n = 1 \text{ rad/s}$$

$$\therefore \frac{1}{2\pi} = \underline{0.159 \text{ Hz}}$$

$$\cancel{\frac{4}{6}s} = \cancel{2\zeta\omega_n s}$$

$$\text{left with } \frac{2}{6} = \zeta \times \omega_n$$

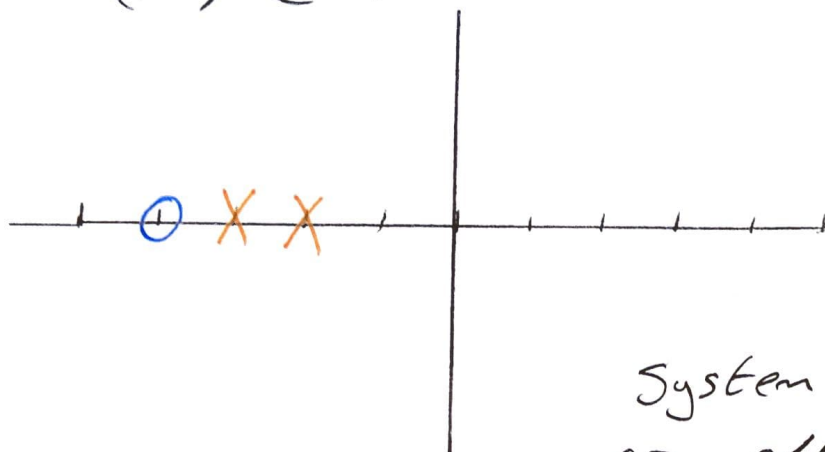
$$\therefore \frac{2}{6} = \zeta \times 1 \quad \therefore \zeta = \frac{1}{3} \text{ or } 0.333$$

$\therefore$ Under 1 $\rightarrow$ UNDER DAMPED

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(Q5) $G(s) = \frac{4(s+1)}{(s+2)(s+3)}$ $\leftarrow$ zeros -4
 $\leftarrow$ Poles -2, -3

a)



System is **STABLE**
as all poles in LHP
plane

b) $\frac{1}{s} \left(\frac{4(s+1)}{(s+2)(s+3)} \right) = \frac{4(s+1)}{s(s+2)(s+3)} = \frac{4}{s}$

$$= \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+3)}$$

$$\frac{4(s+1)}{s(s+2)(s+3)} = \frac{A(s+2)(s+3) + B(s)(s+3) + C(s)(s+2)}{s(s+2)(s+3)}$$

So $4(s+1) = A(s+2)(s+3) + B(s)(s+3) + C(s)(s+2)$

$s=0$

$$4 = A(2)(3)$$

$$A = \frac{4}{6} \leftarrow \text{DC Response}$$

$s=-2$

$$4(-1) = B(-2)(1)$$

$$-4 = -2 \therefore B = \frac{-4}{-2} = 2$$

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Q5 cont

$$s = -3$$

$$4(-2) = C(-3)(-1)$$

$$-8 = 3 \quad \therefore C = \frac{-8}{3}$$

$$\text{Response} = \frac{4}{6}s + \frac{4}{2}(s+2) - \frac{8}{3}(s+3)$$

$$\text{Laplace} = \frac{4}{6} + \frac{4}{2}e^{-2(t)} - \frac{8}{3}e^{-3(t)}$$

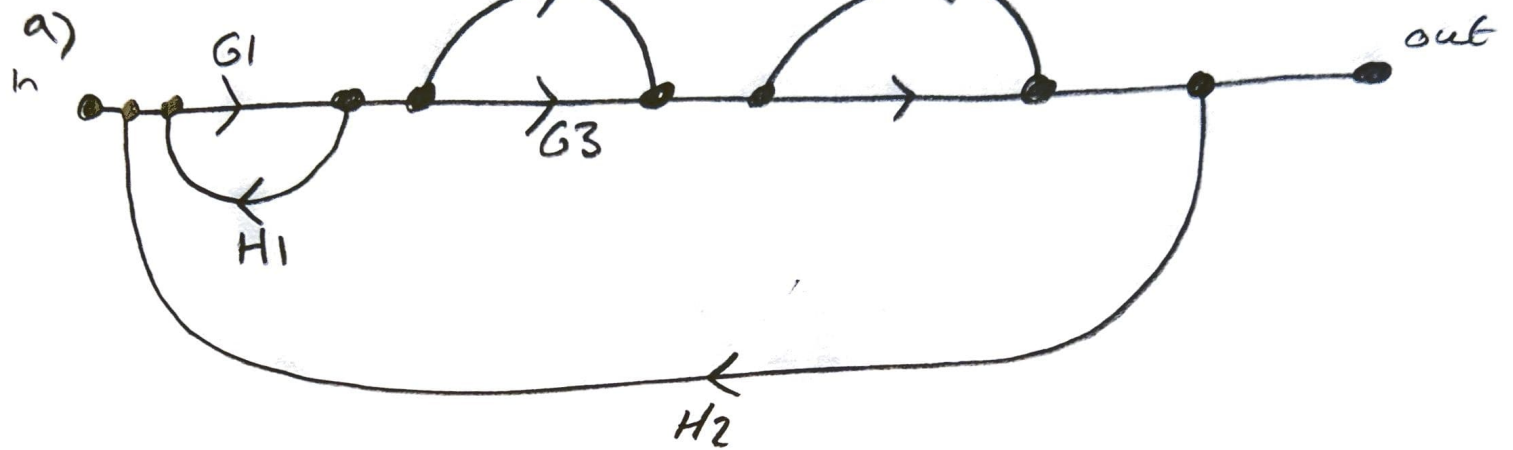
@ 0.8 seconds

$$\frac{4}{6} + \frac{4}{2}e^{-2(0.8)} - \frac{8}{3}e^{-3(0.8)}$$

$$= \underline{\underline{0.829}}$$

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(Q6)



b)

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^n P_i \Delta_i}{\Delta}$$