

ACDB4007 Physics exercises 1

1 Questions

1.1 Question 1

Two hard spheres collide in a perfectly elastic collision shown in Figure 1. Determine the velocity of each ball after the collision. All calculated values to be expressed to 2 decimal places. (20 marks)

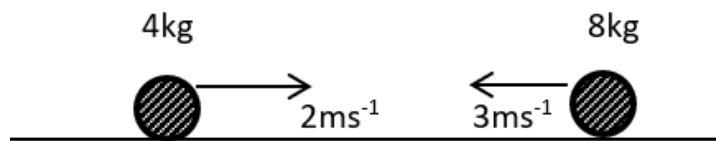


Figure 1

1.2 Question 2

A body is rotating at a speed of 5 revolutions per minute. Determine to 2 D.P. the angular velocity in radians per sec. Give your answer to 2 D.P. (4 marks)

1.3 Question 3

Find the resultant force for the system of forces shown in Figure 2. Give your answers to 2 D.P. (16 marks)

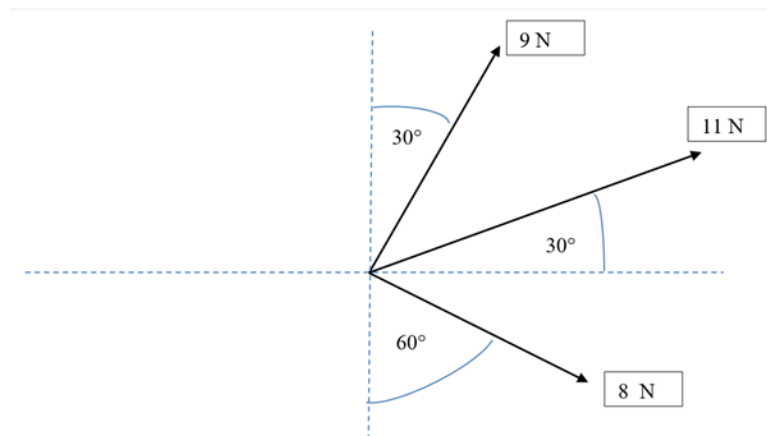


Figure 2

1.4 Question 4

A body of mass 300 kg is dropped from a vertical height of 1.3 m. Calculate the speed at impact on ground. Assume acceleration due to gravity $g = 9.8 \text{ ms}^{-2}$. Express your answer to two decimal places. (5 marks)

1.5 Question 5

Calculate, to 2 D.P. the Moment of Inertia of a 4 kg bar of uniform rectangular cross-sectional area, that rotates horizontally about the pivot point shown in Figure 3. (8 marks)

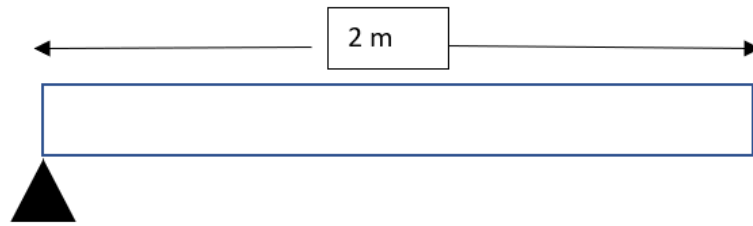


Figure 3

1.6 Question 6

A projectile of mass 30 kg penetrates a target to a depth of 0.6 m. If the speed at initial impact is 700 ms^{-1} determine the resistive force that the body exerts on the projectile, assuming uniform resistance. Express your answer to two decimal places. (5 marks)

1.7 Question 7

If a body of mass 10kg is travelling at 20 ms^{-1} on a rough horizontal surface with coefficient of friction $\mu = 0.2$, how long will it take for the body to come to rest? Assume $g = 9.8 \text{ ms}^{-2}$, express your answer in seconds to 2 D.P. (4 marks)

1.8 Question 8

Determine the reaction force on the 5kg box in the lift shown in Figure 4, if the lift accelerates upward at 2 ms^{-2} . (4marks)

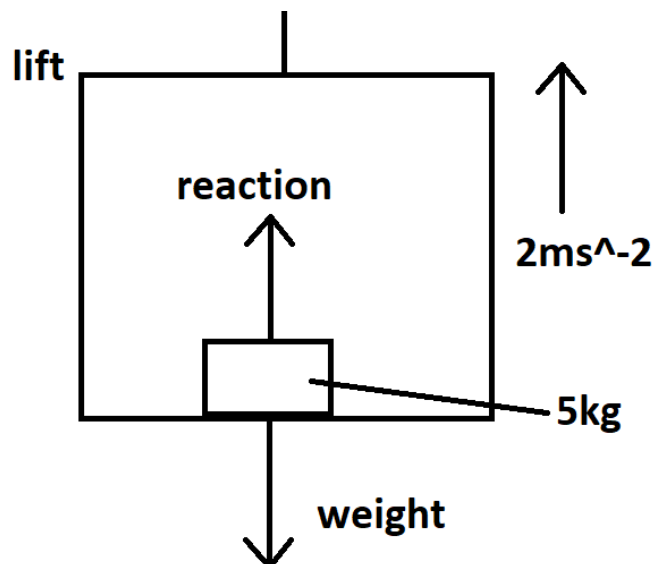


Figure 4

2 Solutions/ mark scheme

2.1 Explanation of mark schemes

The total marks that can be awarded for each question are shown at the end of each question (E.g [5 marks]) is the maximum number of marks that can be given for correctly answering that question.

The total marks on the exam (100 marks) is the sum of maximum marks for all questions.

The marks for each question are divided among the steps taken to reach the solution. Marks for each question are awarded for certain written lines of working.

Example:

Q: What is the acceleration produced when 5kg is pushed with 5N? (3 marks).

Solution:

$F = ma$ can be rearranged into the equation $a = F/m$ (1 mark)

$a = 5/5 = 1ms^{-2}$ (2 marks)

So, if you only write 1 line of the above correctly, you only receive the marks from that line. If you write down both steps correctly, you receive full marks. You won't get penalised for writing significantly more steps however if you need to.

If the answer is a value and you don't include units you will not get full marks.

E.g: If you write:

$F=ma$ can be rearranged into the equation $a = F/m$

$a = 5/5 = 1$

You will get 2 marks total rather than 3.

If you instead write:

$F=ma$ can be rearranged into the equation $a = F/m$

$a = 5/5 = 5ms^{-2}$

You will also get 2 marks, because while the number for the answer is wrong, the logic was correct, just a calculation error at the end.

2.2 Solutions to exercises

2.2.1 Question 1

This is a conservation of momentum question because we are asked to find the velocities after a collision. So we will need to use:

conservation of momentum, conservation of energy(elastic collision), therefore we also need the equations for finding momentum and KE. We will also need the quadratic formula for reasons we will see during the solution.

Let v_1 and v_2 be velocities of 4kg and 8kg masses after collision (1) This just identifies which balls the unknown velocities v_1 and v_2 are associated with.

Momentum $p = mv$

Conservation of Momentum Law: (1 mark)

Momentum before collision = momentum after collision (1mark)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

using u for initial velocities.

$$4 \times 2 - 8 \times 3 = 4v_1 + 8v_2 \text{ (1mark)}$$

Simplify terms:

$$-16 = 4v_1 + 8v_2 \text{ equation [1] (1mark)}$$

Rearrange for v_1 :

$$v_1 = -2v_2 - 4 \text{ equation [2] (1mark)}$$

Conservation of Energy Law: (1mark)

K.E. before collision = K.E. after collision (1mark)

$$KE = \frac{1}{2}mv^2$$

$$0.5m_1u_1^2 + 0.5m_2u_2^2 = 0.5m_1v_1^2 + 0.5m_2v_2^2$$

$$0.5 \times 4 \times 2^2 + 0.5 \times 8 \times (-3)^2 = 0.5 \times 4v_1^2 + 0.5 \times 8v_2^2 \text{ (2marks)}$$

Simplify:

$$8 + 36 = 0.5 \times 4v_1^2 + 0.5 \times 8v_2^2 \text{ (1mark)}$$

$$22 = v_1^2 + 2v_2^2 \text{ equation[3] (1mark)}$$

Substitute for v_1 in equation [3] from equation [2]:
 Substitute $v_1 = -2v_2 - 4$ into $22 = v_1^2 + 2v_2^2$

$$22 = (-2v_2 - 4)^2 + 2v_2^2$$

Expand brackets:

$$22 = 4v_2^2 + 16v_2 + 16 + 2v_2^2$$

Simplify:

$$0 = 6v_2^2 + 16v_2 - 6$$

$$0 = 3v_2^2 + 8v_2 - 3 \text{ (2marks)}$$

The following step was not explained in the lectures since we didn't come across an example so I will explain why we need to do it now.

The above is a quadratic equation: has one unknown v_2 but it includes a v_2^2 and a v_2 . Therefore, to find the solution of the equation we need to re-arrange the equation into the above form:

$0 = ax^2 + bx + c$ where x is v_2 here. The letters a and b are numbers that are multiplying the x^2 and x and c is a number on its own.

Once in that form we can use the quadratic formula to find the values of x .

The quadratic formula is :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We then substitute the number a, b, c to get the values for x . We get two different values for the two case where we either use $+$ in place of $\pm$ or $-$ in its place. Since $\pm$ just means $+$ or $-$.

So we use $a = 3, b = 8, c = -3$

$$v_2 = \frac{-8 + \sqrt{8^2 - 4 \times 3 \times -3}}{2 \times 3} = \frac{-8 + 10}{6}$$

$$v_2 = \frac{-8 - \sqrt{8^2 - 4 \times 3 \times -3}}{2 \times 3} = \frac{-8 - 10}{6}$$

From quadratic formula (1mark)

$$v_2 = (-8 \pm 10)/6 \text{ (2marks)}$$

$$v_2 = (-8 + 10)/6 = 2/6 = 1/3$$

$$v_2 = (-8 - 10)/6 = -18/6 = -3$$

$$v_2 = -3ms^{-1} \text{ or } v_2 = 1/3ms^{-1} \text{ (2marks)}$$

Substitute v_2 into $v_1 = -2v_2 - 4$ to get v_1 .

Therefore from equation [2]: $v_1 = -2v_2 - 4 = 2ms^{-1}$ or $v_1 = -14/3ms^{-1}$ (1mark)

Take second solution since first solution just replicates initial conditions. (1 mark)

2.2.2 Question 2

We will need to use the following equation: $\omega = 2\pi f$ since this is the definition of angular velocity. Note: this formula is not given to you in the formula booklet. Convert rotations per minute into rotations per second by dividing by 60:

$$frequency = rev/s = rpm/60 = 5/60 = 1/12 = 0.083s^{-1}$$

(2marks)

Now using the definition of angular velocity:

$$\omega = 2\pi f = 2\pi \times 0.083 = 0.52rad s^{-1}$$

(2marks)

2.2.3 Question 3

In this question we want to find the resultant force from the 3 forces in the diagram. We will therefore need the equations for the components of a vector force: I will give the components different names to distinguish them. The component of force that touches the angle made with the force: F_{adj} for adjacent force and F_{opp} for opposite force.

$$F_{adj} = F \cos(\theta)$$

$$F_{opp} = F \sin(\theta)$$

The equations for the opposite and adjacent sides of a triangle are given in the formula booklet(trigonometry section) in terms of sin and cos, but you will have to rearrange these for the opposite and adjacent respectively.

We need to find the vertical and horizontal components of each force individually, then find the resultant force vertically and horizontally. Finally we re combine the vertical and horizontal forces into one resultant force. We then need to state the magnitude and angle of the resultant force.

Resolving 9N force into vertical and horizontal: Since the angle touches the vertical axis, the adjacent component is the vertical component, and the opposite is the horizontal:

$$F_v = 9\cos(30) = 7.79N, \quad F_h = 9\sin(30) = 4.5N$$

(3marks)

Where F_v , F_h are the vertical and horizontal components respectively.

For the 11N force the angle touches the horizontal, so the adjacent is the horizontal and the opposite is the vertical.

$$F_v = 11\sin(30) = 5.5N, \quad F_h = 11\cos(30) = 9.53N$$

(3marks)

For the 8N force, again the angle touches the vertical, so we use the same sin and cos for each component as we used in the 9N force.

$$F_v = 8\cos(30) = 6.93N, \quad F_h = 8\sin(30) = 4N$$

(2marks)

However, we need to remember the vertical in this case points down not up.

We now find the resultant force in the vertical and horizontal directions. Adding up the components in the horizontal:

$$RF_h = 4.5 + 9.53 + 4 = 18.03N$$

(2marks)

In the vertical direction:

$$RF_v = 7.79 + 5.5 - 6.93 = 6.36N$$

(2marks)

We now need to find the resultant force of the two combined components. We find the magnitude with Pythagoras theorem:

$$a^2 = b^2 + c^2$$

Since we just want a not a^2 , we take the square root of a^2 .

$$F = \sqrt{RF_h^2 + RF_v^2} = \sqrt{18.03^2 + 6.36^2} = 19.12N$$

(2marks)

We now need to find the angle. We do that using the equation $\tan(\theta) = o/a$ which can be rearranged into: $\theta = \tan^{-1}(o/a)$. Therefore:

$$\theta = \tan^{-1}(6.36/18.03) = 19.43^\circ$$

(2marks)

2.2.4 Question 4

In this question we have to find the final velocity, and we are given the distance (1.3m), initial velocity (rest $0ms^{-1}$) and acceleration (g). We can either use the suvat equation:

$$v^2 = u^2 + 2as$$

Or we can simply use conservation of energy, since we can see that before dropped, all the energy is gravitational potential energy(GPE) and when impacting the ground all the energy is in kinetic energy.

I will follow the solution using the conservation of energy.

We can calculate the GPE at the start using the equation $GPE = mgh$. This is given in the formula booklet.

$$GPE = 300 \times 9.8 \times 1.3 = 3822J$$

(2marks)

The kinetic energy at the ground must be equal to this due to conservation of energy. The equation for kinetic energy is also given in the formula booklet.

$$KE = 3822 = \frac{1}{2}mv^2 = 150v^2$$

(1mark)

We then have: $3822J = 150v^2$.

Rearranging for v^2 :

$$v^2 = \frac{3822}{150}$$

Then square root to get v:

$$v = \sqrt{\frac{3822}{150}}$$

(1mark) This can be put into a calculator and should give:

$$v = 5.05ms^{-1}$$

(1mark)

2.2.5 Question 5

In this question we need to calculate the moment of inertia, which means we will need to use the equation for the moment of inertia given in the formula booklet:

$$I = \int_0^L r^2 dm$$

(1mark)

dm is a small unit of mass. Since the equation has r in between the integral sign and the dm, we need to change dm for dr multiplied by the mass dm per small length dr. We know that the mass per unit length is the total mass divided by the total length. So we can say that:

$$dm = \frac{\text{mass}}{\text{length}} dr = \frac{m}{L} dr$$

(2marks)

We can then substitute this into the equation:

$$I = \int_0^L r^2 \frac{m}{L} dr$$

(1mark)

Since the m and l are just constants we can bring them to the front of the equation and just multiply by them after we have done the integral.

$$I = \frac{m}{L} \int_0^L r^2 dr$$

The integral of r^2 with respect to r is given by:

$$\int_0^L r^2 dr = \frac{1}{3}L^3$$

(2marks) Since we raise the power by 1 and divide by the new power. Then replace the r with the L. Putting this into the equation for the moment of inertia we get:

$$I = \frac{m}{L} \int_0^L r^2 dr = \frac{m}{L} \frac{1}{3}L^3 = \frac{mL^2}{3}$$

(1mark)

So

$$I = \frac{mL^2}{3}$$

Substituting the values for m and L we get:

$$I = \frac{4 \times 2^2}{3} = 16/3 = 5.33kgm^2$$

(1mark)

2.2.6 Question 6

In this question we need to find the force pushing backwards against the motion of the projectile to slow it down from $700ms^{-1}$ to $0ms^{-1}$ in a distance of $0.6m$. It may be helpful to draw a diagram of the projectile, labelling the resistive force on it. We are given the initial velocity, the final velocity and the distance. But we need to find the force. Since we know that force equals mass times acceleration $F = ma$, we just need to find the acceleration to find the force.

We can use the equation for work or the suvat equation involving u, v, a and a . I will continue using the solution using work. Using the equation for Work given in the formula booklet:

$$W = \int F dx$$

This just means the work is equal to the force times the distance if the force is constant. So we can rewrite it as:

$$W = F \times d$$

(1mark)

The work is then the energy needed to counter all the kinetic energy. The KE is given in the formula booklet and is:

$$KE = \frac{1}{2}mv^2 = \frac{10 \times 700^2}{2} = 4,900,000J = 4.9MJ$$

(2marks)

Since the work must counter the KE they must be equal. So $W = 4.9MJ$ Rearranging the equation for work in terms of force gives:

$$F = \frac{W}{d} = \frac{4900000}{0.6} = 8,166,666N = 8.17MN$$

(2marks)

2.2.7 Question 7

In this question we need to find time and are given the initial velocity mass and coefficient of friction μ . Since we are given the coefficient of friction we will need to use the equation for friction:

$$F_f = \mu R$$

Where F_f is the friction force and R is the reaction force from the surface. Since the body is on a horizontal surface the reaction force must be equal and opposite to the weight. The weight is given by:

$$W = mg = 20 \times 9.8 = 196N$$

(1mark)

The friction force is then:

$$F_f = 0.2 \times 196 = 39.2N$$

(1mark)

Now we have the force, we can find the acceleration of the object using $F = ma$. Rearranging for a :

$$a = \frac{F}{m}$$

Therefore, substituting F and m :

$$a = \frac{39.2}{20} = 1.96ms^{-2}$$

(1mark)

Now we have the initial velocity, the final velocity and the acceleration, we can use the suvat equation:

$$v = u + at$$

To find the time. We rearrange for time: Minus u from both sides:

$$v - u = at$$

Divide both sides by a ;

$$t = \frac{v - u}{a}$$

Now substituting the values for v , u and a :

$$t = \frac{20 - 0}{1.96} = 10.20s$$

(1mark)

2.2.8 Question 8

In this question we have to find the reaction force. Since the surface of the lift is flat and horizontal, the reaction force just points in the opposite direction to the weight.

In order for the body to remain on the floor of the lift it must be accelerating upwards at $2ms^{-2}$.

If it is accelerating upward then the resultant force must be upwards and equal to mass times the upwards acceleration thanks to $F = ma$:

$$F = ma = 5 \times 2 = 10N$$

(1mark)

Both forces acting here are vertical so we just have to consider the vertical forces.

Since the weight and reaction oppose each other, the resultant force is the difference between them. Since the resultant force is upward, the reaction force which is upward must be the larger of the two forces. Therefore we can say:

$$F = R - W$$

(1mark)

Where F is the resultant, R reaction and W weight. We want to find the reaction so we rearrange the above for R by adding W to both sides:

$$W + F = R$$

We can find A since the weight using $W = mg = 5 \times 9.8 = 49N$ (1mark)

Substituting W and F calculated earlier:

$$R = 49 + 10 = 59N$$

(1mark)